Sources and Studies in the History of Mathematics and Physical Sciences

Clemency Montelle
Kim Plofker

Sanskrit Astronomical Tables

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Clemency Montelle
Department of Mathematics & Statistics
University of Canterbury
Christchurch, Canterbury, New Zealand

Kim Plofker
Department of Mathematics
Union College
Schenectady, NY, USA

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To our graduate students, past and present,
and to Lily, who was born into this book!

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Chapter 1
Introduction

The history of computational algorithms, numerical methods, and data analysis has long been understudied in the history of mathematics, particularly outside the area of early modern and modern Western mathematics. Many founders of the modern discipline of history of mathematics in the nineteenth and twentieth centuries shared a professional bias in favor of identifying the essence of mathematics with elegant and rigorous demonstration of abstract propositions, which was viewed by most of their contemporaries as the highest form of mathematical endeavor. They largely disregarded the evolution of more plebeian or mechanical mathematical activities, such as approximating numerical parameters, constructing tables of function values, and optimizing computational performance. It has only recently started to become apparent how much interesting material the pioneers of history of mathematics overlooked when they neglected the arts of "number-crunching." The distorting impact of this historiographic bias on our understanding of the development of mathematics has been particularly severe in the case of non-Western mathematical traditions that emphasized computational methods over proof structures.

The historical role of numeric-array tables in texts treating quantitative sciences and the computational methods used to generate such tables have begun to receive increased scholarly attention in the last few decades, particularly from historians of mathematics and astronomy. Edited collections such as Campbell-Kelly et al. (2007), Husson and Montelle (2014), and the forthcoming Husson et al. (Forthcoming) and Tournès (Forthcoming) bring together specialized studies on scientific tables in various periods and cultures of inquiry ranging from pre-classical antiquity to the twentieth century and from East Asia to North America.

Our knowledge of the role and nature of numerical tables within scientific traditions is farther advanced in some areas of specialization than in others. For instance, research on construction and use of tables in the Islamic world has benefited from both broad surveys and investigations into specific aspects, but there remains an enormous amount of unstudied and unpublished material: of the approximately 200 extant Islamic astronomical handbook/table texts (*zījes*),

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only a handful have been scrutinized in detail. The study of numerical tables in European manuscripts and early printed works is another "growth field" just beginning to tackle its immense store of resources. Likewise, the use of tables in ancient Mesopotamia, Greece, Egypt, and China has recently been addressed, but on the whole the genre remains extremely understudied.[1]

Like other types of scientific texts, numerical tables in Sanskrit and other Indic languages tend to be more numerous, and even more neglected, than their counterparts in non-Indian textual traditions. Some early Indologists lumped together all Sanskrit astronomical texts as "tables," while others distinguished more carefully between tabular and theoretical works but deplored contemporary Indian astronomers' preference for the former over the latter. Well into the twentieth century, this disapproving attitude kept the study of Indian exact sciences somewhat on the defensive. Research tended to focus on the many aspects of Sanskrit texts attesting to theoretical rigor, brilliant discoveries, geometric models, and other qualities conforming to the idealized depictions of Hellenistic and early modern European intellectual traditions that shaped modern conceptions of "science." Sanskrit treatises with a strongly computational bent, and the creative numerical techniques used to produce them, received far less attention from historians.[2]

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[1] For Islamic tables, besides the publications cited above, overviews of the corpus or parts of it can be found in Kennedy (1956), King and Samsó (2001), and Samsó (2011): a revised and greatly expanded version of the former is currently in preparation by Benno van Dalen, building on van Dalen (1993). Studies of individual *zījes* or their features include, among many others, Nallino (1899–1907), Debarnot (1987), Van Brummelen (1991, 1998). Many more such studies are underway, particularly in the ongoing *Ptolemaeus Arabus et Latinus* project (ptolemaeus.badw.de), which also explores a number of Greek table texts. By far the most important of these Greek works, the *Handy Tables* of Ptolemy, has been analyzed most recently in Tihon (2011) and Mercier (2011); the somewhat fragmentary remainder of the Greek table corpus is discussed in, e.g., Jones and Perale (2013), Jones (1999), Sidoli (2014), and Van Brummelen (1993). Much of the very prolific genre of cuneiform astronomical tables has been examined in Neugebauer (1983) and Ossendrijver (2018), and their investigation continues in works such as Steele (2006, 2010). Egyptian tables were first systematically surveyed in Neugebauer and Parker (1960–1969); much of this information, along with a great deal of related research—e.g., Symons (2016)—is summarized and organized in the Ancient Egyptian Astronomy Database (aea.physics.mcmaster.ca). Some of the sources treating Chinese tables include Martzloff (2000, 2006, 2016), Yabuuti Kiyosi (1974/1994), and chen (1992). The vast genre of Latin and other medieval/early modern European astronomical tables is not discussed at all here due to lack of space; but see, for example, Chabás and Goldstein (2012) and the analyses of many of their trigonometric and logarithmic tables at the LOCOMAT project (locomat.loria.fr).

[2] The indiscriminate assignment of all Sanskrit astronomical works to the category of "tables" is partly due to earlier scholars' use of that term to refer to any astronomical system, and partly to the ignorance of many of them about this genre (as when H. Poleman in his catalogue of Indic manuscripts in North America describes (Poleman 1938, p. 246) the "Miscellaneous Bundle" at Columbia University, containing several complete handbooks and table texts as well as individual tables, as "[a] collection of several hundred miscellaneous folios, mostly tables not important enough and not bearing sufficient information to identify at all"). Even some Indologists who avoided such errors and were otherwise quite sympathetic to Sanskrit *jyotiṣa* maintained a rather dismissive attitude towards table texts; consider, for example, the remark of the early nineteenth-century student of *siddhāntas* Lancelot Wilkinson to the effect that "the foundation of such little

Not until the work of David Pingree (occasionally in collaboration with Otto Neugebauer) beginning in the mid-1960s did Indological scholarship undertake systematic and comprehensive analysis of Sanskrit astronomical tables. After the initial groundbreaking catalogues and studies, this subject was to some extent displaced by more general projects for the documentation and study of manuscripts in all fields and genres of the exact sciences in Sanskrit. In the meantime, recognition of the importance of the astronomical tables genre as a witness to the aims and activities of Sanskrit science in the second millennium has continued to grow, along with the propagation of ever more powerful and versatile digital tools for analyzing tables.[3] If the ongoing and emerging successors to Pingree's endeavors are to continue progressing in their mammoth task of mapping and rationalizing the vast corpus of Sanskrit scientific manuscripts, they will need an up-to-date, more accessible guide to Sanskrit astronomical tables as far as they are currently understood. Such a guide is what the present work seeks to provide.

We begin in Chapter 1 with an overview of Indian mathematical astronomy and its literature, including table texts, in the context of history of pre-modern astronomy in general. Chapter 2 discusses the primary mathematical astronomy content of table texts and describes the various attempts at a taxonomy of this genre. Chapter 3 sketches the broad outlines of their representation in the Sanskrit scientific manuscript corpus, based on estimates and extrapolation from sampled manuscript repositories. In this chapter we also address the characteristics of table manuscripts as physical documents within established scribal traditions, as well as their notation conventions and specialized technical vocabulary. In Chapter 4 the major categories of individual tables compiled in these texts are surveyed, with brief analyses of some of the methods for constructing and using them. The chronological evolution of the table-text genre over the course of the second millennium, and the impacts of its changing role on the discipline of Sanskrit *jyotiṣa*, are treated in Chapter 5, while Chapter 6 extends these investigations to related topics in current and future research. Finally, Appendix A inventories all the identified individual

knowledge as they display in predicting eclipses and the like, has, from [...] the almost universal practice amongst the jyotishís, of making all their calculations from tables and short formulae, couched in enigmatical verses, been allowed to fall into a state of utter oblivion" (Wilkinson 1834, p. 507). Emphasis on the achievements of Sanskrit mathematical astronomy according to modern criteria of precise geometric models, etc., is a constant theme in analyses of the corpus; see, for instance, Dikshit (1981, pp. 119–120) and Arkasomayaji (1980, p. xxi).

[3]The primary studies and surveys of Sanskrit table texts by Pingree are Pingree (1970), Neugebauer and Pingree (1967), and Pingree (1968, 1973, 1987b, 1989). Much additional information about them appeared together with data on other *jyotiṣa* works in his manuscript catalogues such as Pingree (1984, 2003, 2004), and above all the immense *Census of Exact Sciences in Sanskrit* (Pingree 1970–94). Manuscripts on *jyotiṣa* are also described in Sarma and Sastry (2002) as well as innumerable catalogues of Sanskrit manuscripts in general in collections all over India, especially the magisterial *New Catalogus Catalogorum* project (Raghavan et al. 1968–2007). Various digital tools for storing and analyzing tabular data have been developed by Benno van Dalen (www.bennovandalen.de/Programs/programs.html). More recent database projects include "Table Analysis Methods for the history of Astral Sciences" (TAMAS) (tamas.hypotheses.org) led by Matthieu Husson and CATE (Computer Assisted Table Editing) (uc.hamsi.org.nz/cate/), which automates parts of the critical editing process.

works in this genre currently known to us, while Appendix B provides reference information about the details of all the notational, calendric, astronomical, and other classification systems that we invoke in this study. Appendix C is a glossary of the relevant Sanskrit terms, and Appendix D supplies a list of abbreviations, reference works and image credits for the manuscript collections containing the source materials discussed herein.

1.1 Overview of pre-modern astronomy

None of the major literate traditions of mathematical astronomy within the pre-modern Afro-Eurasian "Ecumene" can be fully understood in isolation from its neighbors.[4] In fact, it would be over-optimistic to describe any of these highly complex and incompletely preserved scientific disciplines as *fully* understood by any modern scholar. But much of what we do know about them is derived from piecing together the evidence of their interactions in encounters between trading partners, political rivals and allies, etc. Although each of these traditions devised its own unique concepts and methods for interpreting and predicting the appearance of the heavens, they all communicated at least to a limited extent with their counterparts elsewhere, as well as with other knowledge systems in various languages in their local culture areas. They all likewise drew on a common stock of observed fundamental astronomical facts. Hence we begin this introduction with a very rough sketch of the interlocking structure of the most influential traditions of mathematical astronomy in the pre-modern "Old World." (For those traditions where neither their regional extent nor their textual corpus fits neatly into a single familiar geographic or linguistic category, we have somewhat awkwardly identified them using both their core cultural/regional identity and the dominant language of their scientific literature.)

1.1.1 Mesopotamian and Egyptian

Thanks to its wealth of surviving original documents in the form of clay tablets incised with cuneiform writing, the Akkadian-language astronomy of Mesopotamia in the last two millennia BCE is probably the best attested science of antiquity. By

[4] "Mathematical astronomy" here means computational systems that rely on the (observed and assumed) periodicity of various celestial phenomena and some form(s) of spatial reference system and standardized time-units to predict quantitatively the approximate future course of (some) astronomical events. Some other studies restrict the term to systems of astronomical computation meeting particular criteria for abstraction, accuracy, mathematical complexity, etc. In this narrower sense, mathematical astronomy is generally held to begin with Late-Babylonian arithmetic models around the middle of the first millennium BCE; see, e.g., Neugebauer (1975, vol. 1, p. 1) and Ossendrijver (2012, p. 1). According to these criteria, many simple algorithms from earlier centuries such as those relating seasonal length of daylight to the lengths of gnomon shadows would not count as "mathematical" astronomy.

the early second millennium, scribes and diviners relied on a luni-solar calendar cycle with a year of 12 synodic (lunar) months. The day began at sunset, the month at the first visibility of the new lunar crescent, and the year at the start of the month associated with the vernal equinox. A "leap month" would be sometimes intercalated by administrative fiat to keep the first month more or less aligned with the start of the seasonal year. Each calendar month contained either 29 or 30 days depending on when the following new crescent was sighted at the end of it. A simpler scheme for administrative calculations assumed a more regular "ideal year" of twelve 30-day months (Brack-Bernsen 2007).

Shorter time-units could be of seasonally varying length, such as the "watch" equal to one-third of the interval between sunset and sunrise on a given night, or of constant length, such as the *uš* equal to one-sixtieth of an average day-and-night period. Time was measured by the outflow of a standard volume of water from a perforated container, by the lengths of gnomon shadows, or by the motion of designated stars across the night sky. The changing length of day and night over the course of the year was modeled by increasing and decreasing arithmetic progressions ("linear zigzag functions") that determined the length of shadow or volume of clock-water appropriate to a particular time in a given month. The ratio in constant time-units of the year's longest and shortest days was conventionally taken to be 1½ (Hunger and Pingree 1999, pp. 81–83; Ossendrijver 2012, pp. 32–33). Timekeeping computations were apparently handled, like other complicated arithmetic, with the standard Babylonian base-sixty ("sexagesimal") place-value cuneiform number system including sexagesimal fractions (see, e.g., Hunger and Pingree 1999, pp. 44–45).

Beyond its immediate practical purposes of calendar regulation, mathematical astronomy was rooted in Mesopotamian society's view of the cosmos and its systems of divination. The traditional cosmology of Akkadian literature depicted a flat earth which the struggles of the gods had separated from the overarching heaven and the watery depths below. The sun, the moon, and the five visible star-planets (called the "wild sheep," from their shifting positions among the fixed stars), along with some stars and constellations, were associated with particular deities in the Akkadian pantheon. For instance, Venus was assigned to the fertility goddess Ishtar and Jupiter to Marduk, the patron deity of Babylon.

Foretelling future events from observed ominous phenomena was a central concern (see, e.g., Reiner 1995, pp. 3–14 and Rochberg 2004, pp. 44–97). The appearance and movements of human beings, animals, household objects, weather events, and celestial bodies were all considered fateful. The configurations of the sun, moon, and planets, being observable on the grand scale of the heavens and directly linked with the personae of the gods, were held to be especially significant, bearing divine messages about the fate of kings and realms. Although increasing theoretical power over time made their complicated periodic recurrences more precisely predictable, their importance as omens does not seem to have diminished. Observing, recording, interpreting, predicting, and neutralizing astral portents continued for many centuries to be a specialized profession underwritten by the governing apparatus of palace and temple. Of primary interest were the following events: planetary "synodic phenomena" including the moments when they "retrograde" or change direction in the sky, their first appearance after sunset and last appearance before dawn; conjunctions of planets with one another or with the sun or moon or certain stars; and most important of all, the dreaded lunar and solar eclipses.

A basic celestial reference system from the second millennium BCE combined the four cardinal directions with three parallel "paths" or bands running east to west across the sky: the central path associated with the god Anu, the northern "Path of Enlil," and the southern "Path of Ea." It was recognized that the sun and other planets periodically swung north and south in a roughly regular track through these three paths as they traveled eastward relative to the fixed stars. The sequence of asterisms lying along the planets' track was known as the "Path of the Moon" (which later became the standard western zodiac, see, e.g., Rochberg 2010, pp. 41–42) including such familiar constellations as "Twins," "Crab," "Lion," "Scorpion," and "Fishes." Several dozen other stars and constellations were also assigned names and approximate locations within the three heavenly "paths." Observed distances between celestial objects in this "sidereal" or star-based reference system were specified in linear units such as cubits (forearm lengths) and digits (finger-breadths). (See, for example, Hunger and Pingree 1999, pp. 57–63 and 148–151.)

In the eighth century, the Babylonian bureaucracy instituted a program of daily observations that recorded lunar phases, planetary positions and synodic phenomena, meteorological and economic data, and other information relevant to calendrics and astral omens. This practice, continuously maintained until near the beginning of the Common Era, was accompanied by a number of technical developments that made Akkadian astronomy of the last few centuries BCE much more comprehensive and predictively powerful (Steele 2015a). Leap months were determined in accordance with a regular schedule of seven intercalations in 19 years (nowadays widely known as the "Metonic cycle"), rather than as an *ad hoc* fix for calendar slippage (Steele 2015b). For computational convenience, all months were uniformly divided into 30 notional "days" each equal to one-thirtieth of an average synodic month (Ossendrijver 2012, p. 33). Cycles of recurring phenomena were combed out of observational records to construct period parameters for planetary motion, and scribes recorded predicted as well as observed positions and times of astronomical events. By the mid-fifth century, the "Path of the Moon" had been schematized to 12 equal fixed intervals named after 12 of its original constellations and subdivided into 30 equal parts apiece, which we now call the signs and degrees of the zodiac. Planetary positions were specified by a combination of these zodiacal degrees and a perpendicular measurement roughly corresponding to celestial latitude: this combination was the forerunner of the well-known orthogonal coordinate systems of spherical astronomy (Steele 2007).

The mathematical models that enabled the Late-Babylonian quantitative prediction of celestial events included some very sophisticated algorithms for determining, e.g., the occurrence and appearance of eclipses, or whether and when the new-month crescent would be visible when the moon was in a given part of the zodiac at a given time of year. Such algorithms were built up from various piecewise linear functions (zigzag or step functions) derived from the period parameters for the planetary phenomena, particularly their synodic phenomena. All these techniques relied on cuneiform sexagesimal place-value arithmetic procedures, often carried out to many fractional places. Surviving cuneiform texts do not address any physical or geometric theory for their computational models, such as the rotating spheres familiar from classical and medieval astronomy.[5]

The Late-Babylonian astronomical corpus consists of so-called procedure texts (Akkadian *epūšu*), specifying algorithms for astronomical computation, and table texts (*tērsītu*), listing in numerical row-and-column format various computed astronomical quantities, such as the dates and positions of occurrences of a given type of celestial event.[6] Synodic phenomena of the moon and planets, including first and last visibilities, syzygies, eclipses, etc., remained the most important types of event, though tables of planetary positions for consecutive days, or ephemerides, were also computed based on interpolation between successive synodic phenomena. The various numeric-array tables constructed by Babylonian astronomers appear to have been the ancestors from which many later genres of astronomical tables are directly descended.

This ability to specify and calculate positions of the celestial bodies at any arbitrary moment, combined with the ancient heritage of celestial divination literature, gave rise to the Late-Babylonian notion of the individual horoscope or birth omens. This discipline expanded the practitioners' objectives from the traditional task of determining the time of an expected ominous event to include analysis of the ominous significance of the planets' configuration at an arbitrary time. Such analyses later became the core of genethlialogy or nativity astrology in post-Babylonian astral sciences (Rochberg 2004, Chapters 3–5).

Other ancient West Asian calendars, including Hebrew and (pre-Islamic) Persian ones, shared and subsequently perpetuated some characteristics familiar in Akkadian calendrics, e.g., sunset day-reckoning and schemes for leap-month intercalation.[7] The Babylonian deities and celestial omens compiled by their priests survived for centuries as part of the astrology and ritual of the polytheistic Sābians. But the most enduring and prominent legacy of Mesopotamian astronomers, besides the now-universal sexagesimal units they devised for measuring time and celestial position, lay in their transformation of astronomy and astrology in Hellenistic science.

The neighboring culture of Egypt from the Middle Kingdom to the Late Period also adopted and transmitted some aspects of Mesopotamian astronomy, as well as devising concepts and techniques of its own. The ancient Egyptian lunar calendar

[5]An analysis of a geometric interpretation used to motivate arithmetic computations of orbital velocity, albeit not involving a geometric model of the orbits themselves, is discussed in Ossendrijver (2016).

[6]For discussion of the term *tērsītu*, see, e.g., Neugebauer (1983, vol. 1, pp. 12–13), Evans (1998, pp. 318–320) and Ossendrijver (2012, p. 599).

[7]For the origins of Hebrew calendars, see, e.g., Kelley and Milone (2011, pp. 219–220). For the origins of Iranian calendars, see, e.g., Panaino et al. (1990) and Sołtysiak (2015).

reckoned synodic months from the last sighting of the pre-dawn crescent rather than from the first appearance of the post-sunset one, and its intercalation schedule was regulated according to the dawn rising of the star Sirius. In the third millennium BCE this seasonal calendar was supplemented, and for administrative purposes replaced, by a civil calendar with a uniform 365-day year containing twelve notional “months” of 30 days or three ten-day “decades” apiece, plus five extra “epagomenal” days at the new year. By the late second millennium the daylight and nighttime periods were conventionally divided into 12 parts each, the origin of the 24 “seasonal” or unequal hours in a nychthemeron that gave rise to our present universal system (Rochberg 1989, p. 147). Time was measured by means of water-clocks and gnomon shadows (Symons and Khurana 2016).

A system of stellar calendric timekeeping was devised by the early second millennium which employed the risings of 36 stars or small asterisms (called by modern scholars “decans”) lying more or less equally spaced near the sun’s path through the sky over the course of the year. Because the sun appears to move eastward through the stars from one day to the next, these decans gradually shift their position from night to night. That is, if one decan is located, say, at the eastern horizon just before sunrise at the start of decade 1, the following night it will be slightly above the horizon just before sunrise, and so on until the next decan to the east in its turn sits on the eastern horizon just before the first sunrise of decade 2. Thus the changing positions of the chosen decan stars through the night and through the year roughly quantify the passage of time.

1.1.2 Hellenistic: Greek-primary discourse

Hellenic cosmology as late as the early first millennium BCE appears to have resembled worldviews found elsewhere in the ancient Mediterranean region. The known world was separated by the great “river” Ocean from legendary or supernatural realms beyond it, and above it the gods dwelling in Olympus ruled the planets and arranged the stars. More prosaically, the seasonal and monthly cycles were described with a variety of agrarian and administrative calendars. Ancient Greek calendars considered the equinoxes and solstices (“tropics”) key moments in the year. But there was no standard administrative calendar in place throughout the Greek world and no centralized bureaucratic apparatus in support of astral sciences to impose one; its diverse collection of eras, intercalation schemes, and calendar formats was ultimately subordinated to (but not eradicated by) the official Julian calendar under Roman rule (see, e.g., Hannah 2005; Jones 2007).

The intersection of calendrics problems with philosophical and cosmological speculations in ancient Greece starting around the middle of the first millennium BCE produced the first known systems of spherical astronomy. It is not clear how these heavily geometrized models first emerged, but certainly by the time of Plato in the fifth century philosophers were familiar with the fundamental concepts of the spherical form of the geocentric cosmos and the circularity of the motion of heavenly bodies, as well as the sphericity of the earth itself.[8]

In the work of Aristotle about a hundred years later, the assumptions of geocentrism, geostability, and the uniformity of circular celestial movements appear as part of a physical theory that derives them from essential characteristics of the nature of matter and motion. The philosophical concept of the “perfection” of the superlunary universe, as an innately and supremely regular system approximated by the imperfect (but improvable) models of astronomy, became a core principle of scientific thought. The development in the last few centuries BCE of a mathematical system of rigorous deductive demonstration, crowned by the geometrical treatises of Euclid, Archimedes, and others, vindicated Hellenistic scholars’ praise of the power of geometric models.

This does not imply that scholars were able to agree on the exact form of the geometric models that best represented the motion of the heavens. In fact, all such models remained quite unsatisfactory in their descriptive accuracy or their physical consistency or both. Eudoxus in the fourth century, followed by some others including Aristotle himself, attempted to construct “homocentric” systems of multiple nested spheres exactly centered on the earth and revolving in different directions to qualitatively reproduce planetary patterns of pausing and reversing. Such mechanisms, however, were not quantitatively accurate enough to be usefully predictive. Physically unorthodox and counterintuitive hypotheses proposed by Aristarchus in the third century reassigned the daily revolution of the stars to the earth itself, and even suggested the possibility of doing the same with the yearly cycle of the sun, but these far-fetched notions were taken seriously by only a very few ancient astronomers (Heath 1981).

In the meantime, the absorption of Babylonia and Egypt into the empires of Alexander and his successors brought into Greek science some of the astral knowledge from Akkadian and Egyptian sources. This included the 12-sign and 360-degree division of the ecliptic circle of longitude, with its perpendicular coordinate latitude; numerical parameters and arithmetic methods for computing planetary positions and various ominous events; the 365-day Egyptian administrative calendar and day of 24 seasonal hours (later modified to “equinoctial” or equal hours); and nativity divination from planetary positions, which evolved into Hellenistic horoscopic astrology. Place-value sexagesimal fraction notation was used for astronomical calculations, although the sexagesimal digits themselves were represented by “Ionian” alphabetic numerals ($\alpha = 1, \beta = 2, \ldots \iota = 10, \kappa = 20, \ldots$ etc.), as were integers. These calculations included determination of key parameters such as the length of the “tropical” year measuring the time from one vernal equinox to the next (which is slightly shorter than the sun’s “sidereal” year or annual return to the same fixed star because of the precession of the equinoxes, i.e., the slow shifting of the ecliptic relative to the equator). The 7-day week and the 24 Egyptian

8For a survey of the emergence of this cosmological conception, see, for instance, Heath (1991), Dicks (1960) and Evans (1998, esp. 289–392).

hours were assigned for astrological purposes to the sequence of the seven planets in order of their orbital distances, i.e., Saturn, Jupiter, Mars, Sun, Venus, Mercury, and Moon. That is, the first of the 168 hours of the week was allotted to the sun, the second to Venus, and so on cycling through the sequence so that the moon ruled the 25th hour, Mars the 49th, etc.

The complex process of synthesizing various Babylonian observational records and quantitative techniques with Greek spherical geometry and physics is not very thoroughly documented in surviving texts, but certain developments mark its general path (Jones 2015, 1991). Spherical reference systems with respect to the horizon and the celestial and terrestrial equators measured their coordinates in the same sexagesimally divided degrees known from ecliptic latitude and longitude (Evans 1998, pp. 99–105). Angles between great circles on the sphere (e.g., the ecliptic and the celestial equator, the ecliptic or equator and the observer's horizon, etc.) created various spherical and plane triangles to be solved by a combination of geometric and numeric methods known as trigonometry. (The trigonometric relation of triangle angles or arcs to their sides for Hellenistic astronomers was based on the concept of the chord of the arc, rather than its sine (Van Brummelen 2009).) In particular, trigonometric methods refined practical arithmetic schemes for calculating "rising times" or "ascensions," the intervals required for certain arcs of the ecliptic to rise above the local horizon.

Modified orbital models attempted to account for the wealth of observational data about celestial motions while remaining physically comprehensible and consistent, at least insofar as possible. Babylonian astronomy's period relations specifying so many returns of a celestial body to the same star in so many years could be very naturally represented by Aristotelian uniform circular motion, producing good estimates for the planets' average or "mean" motions. But since constant mean motion on a simple geocentric circular orbit is a rather crude approximation of a planet's actual progress on an elliptical orbit with the sun at one focus, Hellenistic astronomy added more circles to account for observed fluctuations in velocity, retrogradation, variation in latitude, and other apparently non-uniform "true" motions.

The main types of circular mechanism used for this purpose were the eccentric (a circular orbit whose center is displaced from the center of the earth by a distance called the eccentricity) and the epicycle (a small circle whose center revolves on a concentric circular orbit, and upon which the planet itself revolves). They can be used in equivalent ways for modeling a planet's apparently non-uniform speed in its revolution. This non-uniformity is known as the ecliptic anomaly and arises from the fact that the revolution actually occurs on an ellipse with its orbital focus displaced from its center, rather than on a circle. The orbit's heliocentricity produces the more noticeable synodic phenomena such as retrogradations, explained by the so-called synodic anomaly represented by an epicycle device synchronized with the sun's motion.

Using these concepts, the fundamental uniform circular motion demanded by Aristotelian principles could be displaced from an unsatisfactory geocentric circular orbit onto some non-homocentric circle. Then the mean position predicted by a planet's simple period relation could be corrected by some slight adjustment based on the trigonometry of the configuration formed by the planet, the earth, and one or more reference points on its combination of circles. These corrections from mean to true positions are generally called "equations," in the sense of a term that equalizes or makes accurate an approximate value.

The middle of the second century CE generated the most detailed and comprehensive sources we possess about Hellenistic astronomy, partly due to their having driven most of their predecessors and contemporaries to extinction: namely, the works of Ptolemy of Alexandria. Ptolemy's *Mathematical Syntaxis* or *Almagest* on theoretical mathematical astronomy, his *Handy Tables* for astronomical calculation, his *Tetrabiblos* "Four Books" on astrology, and his *Geography* all dominated their respective fields in Greek and subsequently Latin astral sciences for hundreds of years.

The *Almagest* is the first surviving treatise to explain in detail (though not without numerous quantitative inaccuracies and physical inconsistencies) the observed motions and appearances of the planets by means of a reasonably self-consistent though highly complicated combination of orbital circles placed between the spherical earth and the spherical cosmos. Its chief importance for our present purpose, however, is in its enabling the creation of Ptolemy's equally magisterial *Handy Tables* (*Procheiroi kanones*), a comprehensive set of astronomical tables for facilitating the computation of the astronomical quantities whose theory he explained in the *Almagest*. The *Handy Tables* (see, e.g., Tihon 2011 and Mercier 2011) were largely responsible for the propagation of Hellenistic spherical astronomy models and trigonometric methods throughout the Byzantine, Islamic, and Latin worlds.

1.1.3 East Asian: Chinese-primary discourse

The classical Chinese science of astronomy, like its Babylonian counterpart, is known today mostly through records of the governmental institutions that sponsored it and preserved its collections of observational data. Although originating independently of Babylonian astral sciences, Chinese astronomy was similarly supported by state interest in the maintenance of an official luni-solar calendar and prognostication from celestial phenomena, and it similarly based its predictive mathematical models primarily on arithmetic schemes rather than geometric constructions. Over the centuries of the Common Era many aspects of these models spread with other elements of Chinese culture to, e.g., Japan, Korea, and parts of Southeast Asia.[9]

[9] For the most ancient known Chinese astronomy, see Pankenier (2013), Raphals (2013, ch. 2), Smith (2011), and Cullen (2006). The imperial Chinese *lifa* models are discussed in, e.g., Sivin (2011) and Martzloff (2000).

Astral sciences in imperial China were descended from a long tradition of sky-watching and calculation about timekeeping. Celestial observation practices for determining solstices appear to have been codified as early as the late third millennium BCE. Even before a bureaucratic apparatus existed for calendar maintenance—at least since the late second millennium BCE—a standard cycle was employed to keep track of the passage of days. This "sexagenary" cycle was compounded from the simultaneous progress of one smaller cycle with ten elements and another cycle with twelve: thus the day-numbers proceeded through all sixty elements of the sequence $(1, 1), (2, 2), \ldots, (10, 10), (1, 11), (2, 12), (3, 1), \ldots, (9, 11), (10, 12),$ before recommencing with $(1, 1)$. Surviving records from the first millennium BCE attest to observations of more extraordinary phenomena such as eclipses and comets. The application of the sexagenary cycle to years provided a consistent system of chronological reckoning with a 60-year period that coexisted with later regnal and dynastic epochs.

Political unification under the Han dynasty resulted in imperial administrative support for a complex official system of astronomical and astrological practices. The ideology of the emperor as the "son of heaven" or direct link between earthly and celestial realms made it crucial for the state bureaucracy to correctly forecast and interpret celestial events. This requirement along with the principle that heavenly patterns of recurrence were constantly though slowly changing over time made it necessary to periodically revise the mathematical models used for astronomical prediction. About 50 such revisions were effected at irregular intervals from the Han through the Qing dynasties, sometimes based more on political imperatives than on empirical verification. The *lifa* or calendar-system texts describing the models were incorporated into imperial archives, and the yearly ephemerides or almanac produced according to them, with its predictions of various luni-solar and planetary phenomena and their ominous implications, was officially presented to the emperor in an annual ceremony.

These models mostly employed a common standard metrology and structure underlying their respective modifications of some computational procedures. The winter solstice marked the beginning of the cycle of seasons, and the (invisible) conjunction of sun and moon indicated the start of the month. The new year of the civil calendar began from the new moon of some month designated in relation to the winter solstice: initially this was the month in which the solstice actually fell, but by the start of the Common Era it was conventionally assigned to the second month following the solstice (although astronomers continued to calculate years using the earlier convention). Celestial positions were marked with reference to 28 canonical lunar constellations and later also with respect to 12 "zodiac" constellations. Reference circles were divided into 365.25 units corresponding to the days of the year, signifying (solar) time rather than angle measurement.

Various cycles used in the predictive models were combined into a nominal "greater epoch" or temporal zero-point of the system, which could extend as much as millions of years into the past. These relations were adjusted algebraically to produce fine-tuned algorithms equivalent to astronomical functions that could be represented either by computational procedures or by tabulated values of astronomical quantities such as solar position and day-night length.

Spherical geometric models and the trigonometric tools for using them appeared after the middle of the first millennium CE in some systems influenced by Indian spherical astronomy, and in the early second millennium in *lifa* works on Greco-Islamic astronomy. A separate astronomical bureau was even established for the use of Muslim astronomers and the Chinese officials who worked with them in the Ptolemaic tradition. But all these "foreign" innovations remained secondary to the traditional Chinese *lifa* systems until the introduction and gradual assimilation of heliocentric models starting with the arrival of Jesuit missionaries in the sixteenth century.[10]

1.1.4 South Asian: Sanskrit-primary discourse

The orally transmitted Old Indo-Aryan Vedas contain the first verbal record of Indian ritual practices requiring the maintenance of a luni-solar liturgical calendar: namely, half-yearly and monthly oblations at solstices and syzygies, as well as daily rituals at twilights.[11] This tropical-year cycle is schematized as containing 12 synodic months of 30 days each, with occasional intercalation of a 13th month to keep seasons and months aligned. Standard names for the 12 months are derived from 12 of the 27 lunar constellations or *nakṣatras* marking the moon's path through the stars; each month is named after the *nakṣatra* occupied by the full moon during that month.

Occasional appearances of terms such as "*nakṣatra*-observers" indicate that regulating the calendar to conform to observed celestial positions was a recognized part of liturgical practice. The five star-planets play no role in the luni-solar calendar and there are no unambiguous references to them in the Vedic *saṃhitās* "collections" of sacred texts.[12] Various natural phenomena including eclipses are perceived as having ominous significance but explicit systems of celestial divination or astrology are not attested. Occasional references to numbers indicate an exclusively decimal system of integers with terms for common fractions such as the half and the third (see, e.g., Plofker 2009, pp. 13–14).

The Vedic Sanskrit word *jyotis* "light," especially "celestial light, luminary," gave rise to the standard Sanskrit term *jyotiṣa* for astronomy and calendrics, notably

[10] See Ōhashi (2008) and van Dalen (2002b) for overviews of Chinese adaptations of Indian and Islamic astronomy, respectively, and Martzloff (2016) for the impact of Jesuit astronomy.

[11] A survey of the origins and aspects of calendar in this context is discussed in Yano (2003).

[12] Some Vedic narratives of deities and sages have been proposed by historians as mythologized descriptions of planetary phenomena (Kak 2005). But the persistent uncertainty about specifics of the chronological, historical, and geographic context of the Vedas makes it impossible to establish a definitive interpretive framework for such readings (Plofker 2009, pp. 33–35).

as one of the six *vedāṅgas* “supports of the Vedas” or sciences for maintaining Vedic rites. A late Vedic or post-Vedic verse composition called *Jyotiṣavedāṅga* or *Vedāṅgajyotiṣa* contains the earliest known quantitative description of this liturgical calendar. Its official year of 366 days is equated to twelve solar months or two *ayanas* (tropical courses of the sun), and commences at the new moon around the winter solstice; the abovementioned sequence of *nakṣatras* serves as the reference system for the positions of sun and moon. Five of these calendar years constitute the period of the *yuga* “yoking, conjunction” or intercalation cycle containing two extra synodic months. Each synodic month is divided into 30 “lunar days” or *tithis*, a time-unit of crucial significance throughout the history of classical *jyotiṣa*.

Thus 5 years or $366 \times 5 = 1830$ civil days correspond to $62 \times 30 = 1860$ *tithis*, a fairly rough approximation that was doubtless empirically adjusted as successive *yugas* gradually slipped out of moon/sun/season alignment. Simple arithmetic approximations quantify intercalation rules and the seasonal variation in length of a day, attesting to various smaller time-units including the *muhūrta*, one-thirtieth of a nychthemeron, and the *ghaṭikā*, one-half *muhūrta*. The name *ghaṭikā*, from *ghaṭī* “pot, water-jar,” indicates the use of some sort of water-clock for timekeeping purposes.[13]

Several Indian works on subjects ranging from civil administration to cosmology dating from the late first millennium BCE, in the “second urbanization” period when writing had become well established in both Indic vernaculars and Sanskrit, shed some light on contemporary astral sciences. The *jyotiṣa*-related topics they touch on include systems of time metrology, involving variable (seasonal) units such as the *bhāga*, one-eighth of a daylight or nighttime period, as well as constant units like the *ghaṭikā* and *muhūrta*. Synonyms for *ghaṭikā* such as *nāḍikā*, *nālikā* “tube, stalk” also suggest water-clock devices; indeed, mention is made of timekeeping schemes using water-clocks, as well as gnomon shadows, with linear approximation algorithms to relate their observed measures to time-units. Complex and diverse systems of sub-units are employed in the metrology of time (and other quantities), e.g., *kṣana* “instant” for $1/6$ *ghaṭikā*, *kalā* “fraction” for $1/40$ *ghaṭikā*, and so on (Hayashi 2017). The written numerals used to keep track of all these equivalences in practical computations are non-place-value glyphs retaining the traditional decimal base.

There are also allusions in this literature to the star-planets and to very long cycles of cosmic creation and destruction, sometimes called *yuga* like the aforementioned

[13]It has long been debated whether the similarities between first-millennium Babylonian mathematical astronomy and some concepts in the *Jyotiṣavedāṅga*—e.g., the 30-fold division of the synodic month, the 60-fold division of the day, arithmetic schemes to express length of daylight variation, etc.—may indicate some transmission of Babylonian astral sciences to South Asia, possibly via the Achaemenid Empire in the sixth or fifth century. But since no direct evidence of such transmissions has been found, and there is no way of conclusively determining when the existing *Jyotiṣavedāṅga* was composed or what parts of it may record astral knowledge acquired in earlier periods, these hypotheses remain uncertain.

shorter calendric periods and sometimes *kalpa* "creation." The *kalpa* was eventually codified as 4,320,000,000 years, composed of *mahāyugas* of 4,320,000 years. Each *mahāyuga* in turn comprises four successively degenerating lesser *yugas*, similar to (and perhaps a source for) the Golden, Silver, Bronze, and Iron Ages of the classical world. All identifiable human history is encompassed by the most recent iteration of the last and least of these ages, called the Kaliyuga (González-Reimann 2009). Quantitative cosmological models in sacred literature are spatially as well as temporally huge, with Mount Meru, the dwelling of the gods, in the midst of concentric rings of continents and oceans on the immense flat disk of the earth suspended in the "cosmic egg" between heavens and underworlds.

Other features of contemporary *jyotiṣa* were developed through interaction with the Indo-Greek and subsequently Indo-Scythian (Śaka) cultures following Alexander's incursions in the northwest of the subcontinent. Several surviving Sanskrit works produced in the centuries around the start of the Common Era reflect a synthesis of traditional Indian astronomy, especially in calendrics and timekeeping, with some features of Hellenistic astronomy, trigonometry, and astrology.

Their computational models assume a spherical earth at the center of a spherical universe with a unified system of sun, moon, planets, and stars revolving in circular orbits. The models' geometry and numerical parameters determine the predicted motions of these bodies (including eclipses and other synodic phenomena) by their trigonometric relations, expressed not in the chords of Greek trigonometry but in more computationally efficient sine-based quantities. Celestial positions are denoted using Babylonian-Hellenistic systems of spherical coordinates with 360 sexagesimally divided degrees in a circle, e.g., latitude and longitude with respect to the earth's equator and with respect to the celestial ecliptic marked out by twelve 30° zodiacal signs. The ecliptic obliquity or angle between ecliptic and celestial equator is assigned a standard value of $24° = 1440'$. The time-unit *ghaṭikā* or sixtieth of a day is now conventionally subdivided into sixtieths called *vighaṭīs*, *vināḍīs* or *palas*, which in turn contain sixty *vipalas*.

Predictive schemes of celestial divination, strongly influenced by the horoscopy of the Babylonian-Greek tradition, became a main field within *jyotiṣa* (or *jyotiḥ-śāstra*, a term reflecting the discipline's status as one of the Classical Sanskrit *śāstras* or didactic sciences as well as an ancient *vedāṅga*). The rise of astrology sparked increased attention to the five star-planets in astronomical calculation, as did the accompanying adoption of the seven planetary weekdays in their Hellenistic order. In fact, the Greek influence on Indian exact sciences, probably reflecting the knowledge and interests of the Indo-Greeks who transmitted it, seems to have been strongly weighted towards the practical mechanisms of astrological computation.

None of the elaborate geometry edifices of Euclid, Archimedes, Menelaus, etc., have left identifiable traces in early Common Era Sanskrit texts, and neither have the astronomical systematizations due to Ptolemy. Hellenistic formats of prose treatises and numeric-array tables remained either unknown or unappealing as a medium for Indian astral sciences, which continued to be documented, like other *śāstras*, in Sanskrit metrical verse.

The Greek alphanumeric number system likewise did not transfer to the Indian milieu: instead, Indian astronomers at some point before the mid-first millennium CE modified their existing decimal integers to a fully place-value system, including the use of a zero symbol. They employed these same numerals to represent the sexagesimal divisions of Babylonian-Greek metrology, much as we use the modern decimal place-value descendants of them for sexagesimal hours, minutes, and seconds (Plofker 2008). Composing scientific literature in metrical verse required more flexible ways of expressing numerals, the most common being the so-called object-numeral (*bhūtasaṃkhyā*) encoding in which a standard number-word can be replaced by a word for any object or concept conventionally associated with that number, such as “moon” or “earth” for “one,” “eye” or “hand” for “two,” etc. South Indian astronomers in particular often preferred the alphabetic or *kaṭapayādi* system in which the 33 consonants of the *nāgarī* alphabet are mapped onto the ten decimal digits, so that actual Sanskrit words and sentences can be read as numeral sequences (see, e.g., Plofker 2009, pp. 43–48, 72–77). The dependence of Hellenistic astronomy techniques on some identifiable epoch point of time may have influenced the adoption of historical eras, as opposed to the traditional endlessly cyclic *yuga* of the liturgical calendar, in Sanskrit astronomy texts. Two such epochs in particular became standard in later *jyotiṣa*: the Śaka or Śālivāhana era corresponding to 78 CE, named for its (unclear) associations with Śaka/Indo-Scythian rulers and/or the legendary king Śālivāhana; and the (Vikrama) Saṃvat (*saṃvatsara* “year”) era of 57 BCE, associated with another king of ancient legend, Vikramāditya.[14] At this point in its development, Indian mathematical astronomy begins to take shape as the full-fledged “classical *jyotiṣa*” or “*siddhānta* astronomy” to which most of the technical literature discussed in this book belongs, and which is described in more detail in Sections 1.2 and 2.1. Its influence on the Pahlavi-language astronomy and astrology of Sasanian Iran was profound, although most of the historical details are now unrecoverable. Features of this Sanskrit *śāstra* also subsequently appeared in the exact sciences as practiced in the Islamic world and many other cultures in the central, southeastern, and eastern regions of Asia.

1.1.5 Islamicate: Arabic-primary discourse

The rapid spread of Islam from the Arabian peninsula throughout much of western Asia and the Mediterranean region in the seventh and eighth centuries brought Arabic-speaking scholars into contact with several “foreign” scientific traditions. The branch of astral science that emerged from these contacts is generally designated “Islamicate” rather than “Arab” or “Islamic,” since its practitioners included many non-Muslims and non-Arabs. Muslims across the world from Spain to China along with, e.g., Christians, Ṣābians and Jews contributed to it books written not

[14] For the possible astronomical significance of these epoch choices, see, e.g., Falk (2004).

only in Arabic but also in Persian, Hebrew, Syriac, Turkish, and other languages. However, the political and cultural dominance of Islam was a common feature to most of the areas where this corpus was created, and Arabic persisted as its shared language of learning. Other specifically Islamic features of this science included the strictly lunar calendar of 12 synodic months, shorter than a solar year, whose cycles were counted from the Hijrī epoch at the start of the month Muḥarram in 622 CE and whose days began with sunset. The 7-day week was marked as a religious cycle for Muslims, as for Jews and Christians.

The first Arabic works on astronomy and astrology in the Islamic period reveal the influence of Hellenistic, Sasanian Persian, and Indian sources. The Pahlavi *zīk*, or set of tables for astronomical computations accompanied by explanatory instructions, became the model for the standard *zīj* genre of Islamic astronomy (King and Samsó 2001). Sanskrit sources were known both through the medium of Pahlavi texts and in some directly transmitted works. Forms of the decimal place-value "Indian numerals" and arithmetic techniques for using them spread throughout the Islamic world. But the non-place-value *abjad* numerals expressed in the letters of the Arabic alphabet, akin to the Greek alphabetic numerals although probably adopted from Aramaic ones, remained the standard representation of, e.g., tabulated numbers in *zījes* (Glick 2011, p. 49).

The so-called translation movement of the eighth and ninth centuries centered in the ʿAbbāsid capital of Baghdad made much of the Hellenistic corpus of exact sciences accessible to Arabic-literate astronomers (Gutas 1998). Hellenistic mathematics and the orbital models of Ptolemy became the foundation of the prolific mathematical astronomy tradition commonly known as "Greco-Islamic," mostly supplanting earlier works adapted from Sanskrit and Pahlavi originals (Takahashi 2010) (although the Indian trigonometry of sines was still generally preferred to the original Greek chords).

The pagan origins of these foreign treatises, along with the theologically suspect pretensions of the astrological doctrines that accompanied them, generated some controversy in Islamic intellectual circles. In consequence, between about the tenth and twelfth centuries Muslim astronomers tended to emphasize the rational, material nature of their discipline and its usefulness in orthodox religious applications, such as determining prayer times (*mīqāt*) and the local direction towards Mecca (*qibla*), or predicting the first visibility of the crescent moon that commenced each month. Such rebranding, coupled with strong critiques of astrology as blasphemous and fraudulent, established astronomy on a firmer doctrinal footing in Muslim contexts (see, e.g., Saliba 1982 and Ragep 2001).

Certain assumptions originating in Aristotelian physics, such as the natural uniformity and circularity of celestial motion, survived this "de-paganization" of Greco-Islamic astronomy. Attempts by astronomers to make their models more physically consistent with these assumptions required eliminating the geometric kluges employed by Ptolemy and replacing them with truly homocentric alternatives, in some cases very theoretically sophisticated ones.

During these developments the Islamic world overall kept expanding, geographically, economically, and culturally. Astronomical professions ranged from the design and production of observational and navigational instruments through instruction of students and religious timekeeping to theoretical research and text production at observatories or courts—and inevitably, despite religious disapproval, a wide variety of astrological forecasting. Exponents of Greco-Islamic astronomy worked at institutional observatories in, e.g., Beijing and Maragha in the thirteenth century and Samarqand in the fifteenth, as well as innumerable schools and courts. Consequently, the conversion of dates in the Islamic calendar to corresponding ones in the Jewish, Christian (Julian), Persian, and Indian calendars was one of the major topics in *zījes*. The cosmopolitan nature of this science meant that over a period extending from roughly the twelfth through the seventeenth century, various parts of it were encountered, translated, and adopted by a number of western European, Byzantine, Chinese, and Indian scholars. It is the interaction of this Islamicate learning with Sanskrit *jyotiṣa* in the Indo-Islamic empires of the second millennium that will be treated in Section 1.5.2.

1.2 Overview of classical Indian mathematical astronomy

This description is centered on what are sometimes called the "classical" and "early medieval" periods of Indian history, beginning around the middle of the first millennium CE with the composition of some of the earliest comprehensive treatises on *jyotiṣa* that still survive in full. The Gupta and Pallava dynasties controlled much of northern and southern India, respectively, side by side with numerous smaller kingdoms. Modified forms of the Vedic rites and their exegesis in various philosophical schools of thought had coalesced into the wide array of practices and beliefs now generally called "Hinduism," whose *brāhmaṇa* priests formed hereditary groups of scholars and ritual officiants. Learned and literate exponents of other belief systems rejecting the authority of the Vedas, most prominently Buddhism and Jainism, worked chiefly in the great monastic schools of their faiths.

Courts, communities, temples, schools, etc., all required calendars and other forms of administrative computation provided by their scholars. And almost everyone took for granted the importance of being able to identify and calculate the times of auspicious and inauspicious events in the heavens. These requirements, together with ancient traditions of divination from random observed events or characteristics, nourished the three intertwining *skandhas* or branches of *jyotiḥśāstra*: *saṃhitā* "collection" or interpretation of omens (not necessarily celestial in nature) according to some taxonomic authority on their interpretation; *gaṇita* "calculation" or the mathematical models and methods used in astronomical and astrological prediction; and *horā* "horoscopy" (from the Greek word for "hour"), the astrological principles and conventions for identifying and interpreting celestial phenomena as predictors of human fortunes (Pingree 1981, pp. 1–2).

1.2.1 Astronomy and mathematics

Mathematical astronomy and mathematics "proper" are closely linked in the Classical Sanskrit *śāstras*. As noted above, the computations required for astronomical prediction and calendar maintenance in *jyotiḥśāstra* are called *gaṇita* "calculation," a word also used for quantitative reasoning in almost any context.[15] Non-astronomical *gaṇita* topics are generally classified either as *pāṭī* "board[-computation], arithmetic," dealing with fundamental arithmetic operations, series, proportions, geometry formulas, and so on, or as *bīja* "seed, algebra," concerning positive and negative quantities and the solution of equations for unknown quantities (see, e.g., Hayashi 1997b, 2008a,b). Both *pāṭī* and *bīja* are treated in individual works in Sanskrit didactic literature, but some astronomy texts also contain chapters dedicated to one or both of them.

The mathematical topic of trigonometric relationships between angles and sides of triangles (usually designated by *jyā* "sine" or some other Sanskrit term for a particular trigonometric quantity) appears to occupy a sort of hybrid position between astronomy and mathematics in Sanskrit science. Formulas and tabulated values expressing such relations are presented, with or without some amount of demonstrative exegesis, in almost all *jyotiṣa* treatises, but not in *pāṭī* or *bīja* works (see Section 2.1.8 for more detail).

1.2.2 The topics of astronomical computation

The oft-quoted closing verse of the ancient *Jyotiṣavedāṅga* of Lagadha (see Section 1.1.4) explains the sacred ultimate aim of astronomical calculation (Sarma and Sastri 1985, p. 26):

> *vedā hi yajñārtham abhipravṛttāḥ*
> *kālānupūrvā vihitāś ca yajñāḥ* ||
> *tasmād idaṃ kālavidhānaśāstraṃ*
> *yo jyotiṣaṃ veda sa veda yajñān* ||

> The Vedas came forth for the sake of the ritual oblations, and the ritual oblations were ordained in accordance with the times [of their performance]. Therefore, one who has understood *jyotiṣa*, this science of regulating time, has understood the ritual oblations.

[15] Mahāvīra (mid-ninth century) discusses the scope and applicability of *gaṇita* in the *Gaṇitasārasaṅgraha* (Plofker 2007, p. 442, verses 1.9–17). Bhāskara I lists subdivisions of *gaṇita* in his commentary on the *Āryabhaṭīya* (Keller 2006, vol. 1, p. 8). In particular, some authors draw a distinction between computation with *avyakta* "unmanifest" quantities, i.e., algebra, and with *vyakta* "manifest" ones; see below (Dhammaloka Forthcoming). There seems to be no clear consensus as to whether *gaṇita* counts as a *śāstra* in its own right or even a supercategory of *jyotiṣa*.

As noted previously, this luni-solar calendric foundation eventually supported a vast superstructure of quantitative astronomical prediction and/or astrological interpretation of events ranging from eclipses to planetary conjunctions to nativities. But the underlying purpose of tracking and regulating a complex system of time-units remained fundamental to the practice of *jyotiṣa*.

Classical Sanskrit verse astronomy texts generally followed more or less the following sequence of topics in their first few chapters explaining their core calculations.[16]

Mean motions. Both the ancient liturgical *yuga* and Hellenistic mathematical astronomy utilized the fundamental concept of astronomical cycles, in which a given configuration of celestial bodies recurs after a given amount of time. This was combined in classical *jyotiṣa* with the immensely long eons from sacred legend, such as the *kalpa* of 4,320,000,000 years or the *mahāyuga* of 4,320,000 years. That is, the mean angular velocities of the planets and stars were formulated as ratios of an integer number of revolutions about the earth to the number of years in a *mahāyuga* or *kalpa*. These revolutions were geometrically conceived as uniform motion upon a geocentric circular orbit, starting from the sidereal zero-point of longitude on the ecliptic among the stars. For computational convenience, these ratios could be reduced to approximate values of mean velocity in units of arcminutes per day.

The astronomer needed to know the mean positions of the planets at some specified initial moment—either the very beginning of the eon or some more recent epoch—as well as the interval between that epoch and some desired time. Usually that interval was expressed in days, the so-called *ahargaṇa* "day-accumulation," which could be multiplied by a planet's mean daily velocity and then added (modulo complete revolutions) to the epoch longitude to obtain the mean longitude at the desired time. That mean longitude also had to be adjusted for the *deśāntara* "place-difference," or time interval corresponding to the terrestrial longitude of the user's locality east or west of the prime meridian.

True motions. The slightly irregular cycles of sun and moon can be successfully represented with one eccentric or epicyclic motion (similar to the Hellenistic concept of ecliptic anomaly and called *manda* "slow" in Sanskrit), but the apparent vacillations of the star-planets require a second one as well (the *śīghra* "fast" inequality corresponding to the synodic anomaly).

Each of these orbital inequalities is represented in *jyotiṣa* as a celestial entity in its own right (the apogee or *ucca* of its circle) with its own revolution-number and velocity. The relative positions of the mean planet, the apogee, and the earth at a given time determine a right triangle whose solution gives the appropriate *phala*

[16] Some of the technical details of these fundamental calculations as well as some of the subsequent ones in the following list are discussed in Chapter 4. Comprehensive explanations of all or most of these topics are provided in the commentaries accompanying many editions and translations of Sanskrit astronomical texts, such as Dvivedī (1901–1902), Shukla (1976), Arkasomayaji (1980), Shukla (1986), Chatterjee (1981), and Ramasubramanian and Sriram (2011).

"result" or equation, i.e., correction to the planet's mean longitude. The sizes of the planet's orbit and eccentricity or epicycle radius fix the maximum value of the equation. For the star-planets, their two equations are numerically combined to produce the total adjustment resulting in the true longitude. Corresponding adjustments to the planet's mean velocity are determined in order to obtain the true velocity.

These procedures require this second chapter of an astronomical work to provide the numerical parameters of the orbital models as well as one or more versified tables of trigonometric function values to use in solving triangles.

'Three questions.' When the geocentric true positions of celestial bodies are known for a specific time at a specific locality, determining their appearance in the local sky requires trigonometric conversions involving terrestrial latitude and the horizontal coordinate system of altitude and amplitude. So does the computation of true local time from the shadow of a gnomon. The related problems of finding direction, place, and time for a given event at a given locality constitute the *tripraśna* "three questions" topics typically assigned to the third chapter of a *jyotiṣa* work.

Eclipses. Predicting the occurrence and appearance of an eclipse, one of the most ominous events in Indian astrology, requires knowledge of the sun and moon's true celestial positions and their conversion to horizontal coordinates, as well as the lunar latitude and the luminaries' true velocities. From these data the apparent diameters of their disks and of the earth's shadow are found. And from those in turn the astronomer calculates the amount of obscuration, the duration of the eclipse and of its totality (if any), and the *valana* "deflection" or inclination of the eclipse path to the cardinal directions.

For a lunar eclipse (usually treated in the fourth chapter), the color of the eclipsed orb is determined by the amount of obscuration. For a solar eclipse (fifth chapter), the user of the text also needs to find the components of lunar parallax due to the size of the earth and the closeness of the moon. The *parilekha* "drawing" or diagram of an eclipse's appearance is also explained in eclipse chapters.

The subsequent several chapters of a comprehensive work on *jyotiṣa* generally discuss some or all of the following subjects, though their sequence is less clearly defined.

Synodic phenomena. This topic treats the risings, retrogradations, stations, etc., of the star-planets that depend on their positions with respect to the sun. Canonical values for the planets' approximate synodic elongations associated with the various phenomena are refined by more precise calculations of their true longitudes with the requisite constraints. The importance of this inquiry, as of the ones in the remaining chapters, is primarily owing to its astrological significance.

Orientation and projection of the moon's crescent. This relates to the orientation and dimensions of the illuminated portion of the moon during its changing phases, and techniques for setting up a diagram or a sighting-tube based on its predicted position and appearance, in order to impress a viewer.

Non-solar shadows. The calculation of the shadow cast by the moon, and in theory even those of other planets, may be described. It is essentially equivalent to the corresponding procedures for the shadow of the sun in the third chapter, and is suggested for nocturnal timekeeping.

Planetary conjunctions. The definition of "conjunction" in the case of planets is a matter of importance in the ninth chapter; simply computing the time of equality of the planets' longitudes is apparently not enough to identify a conjunction, which requires the planets to be on the same secondary to the ecliptic. As in Western astrology, conjunction is considered to have a profound effect on the influences of the celestial bodies, so it is crucial to identify the event correctly.

Conjunctions of planets and stars. This chapter identifies the positions of some fixed stars (generally the junction-stars of the *nakṣatra* constellations) in polar coordinates and gives a rule for the computation of their declinations. The conjunction of a planet with a star is computed as for that of two planets.

Computation of *mahāpātas*. The ominous events known as *pātas*, which depend on the relationship between the declinations of the two luminaries, are predicted in a fashion similar to that for the conjunction of planets.

The arrangement of the topics described above, while fairly representative, is by no means uniform or complete. Often the given topics are presented in a different arrangement (although the order of the first five chapters is more or less standard); often other subjects are included among them, and alternative formulas provided; and often an additional section, with additional chapters on a variety of subjects, is appended. Some of the most frequently encountered supplementary topics are the following.

Instruments. The text may describe the construction and use of various instruments for time-keeping, such as a gnomon or different forms of water-clocks or sundials; for observation, such as a perforated sighting-ring; or for theoretical study, such as an armillary sphere.

The sphere. A summary of the technical computations for the celestial and/or terrestrial sphere, often including questions of geography and cosmology, may be included as a separate chapter.

Objections to opposing theories. An author may devote a chapter to contradicting or refuting assertions that differ from his own, whether they originate with other astronomers or in traditional Purāṇic cosmology. Physical considerations, irrelevant to the computational procedures that constitute most of the treatise, are sometimes discussed here, e.g., in demonstrations that the earth does not rotate or that the moon is below the sun.

Problems. A selection of sample problems may be provided to test the reader's knowledge of the procedures explained in the text.

Arithmetic and algebra. The non-astronomical mathematical categories of *pāṭī* and *bīja*, dealing respectively with procedures for operating with known and unknown quantities, are sometimes allotted an individual chapter or chapters in a *jyotiṣa* work.

1.2.3 The *pakṣas* or parameter schools

The general similarities of model and method described here did not prevent classical *jyotiṣa* works from significantly diverging in some aspects of their content. In particular, different choices about their fundamental parameters, mostly in the values of the celestial bodies' revolution-numbers and consequent mean velocities, streamed most such works into one of the following few categories, generally called *pakṣa* "wing, side" or more loosely "school."[17]

Brāhmapakṣa. The "astronomy of *Brāhma*" is mentioned as a source at the close of the *Āryabhaṭīya* of Āryabhaṭa, composed about 500 CE. We have no definitively identified earlier text specifying the Brāhmapakṣa's parameters; the somewhat fragmentary *Paitāmahasiddhānta* has been dated to the fifth century in Pingree (1967–8), but a later date is argued for it in van der Waerden (1980). The seminal surviving exponent of this system, in which time begins at sunrise at the start of the *kalpa*, is the *Brāhmasphuṭasiddhānta* of Brahmagupta composed in 628; the canonical *Siddhāntaśiromaṇi* of Bhāskara II in the twelfth century also follows it. Its main region of influence appears to have been northern and western areas of India, although the *Siddhāntaśiromaṇi* was famed throughout the subcontinent.

Āryapakṣa. The abovementioned author of the *Āryabhaṭīya* developed in it his own set of parameters, like the Brāhmapakṣa using a sunrise epoch, but with a different division of the *mahāyuga*. This became the dominant *pakṣa* in southern India.

Ārdharātrikapakṣa. The same Āryabhaṭa in the sixth century also developed in a different *siddhānta* another parameter suite known as the "midnight" (*ardharātra*) *pakṣa*, from its choice of epoch. No canonical treatise for this system survives, but it is partially documented in the *Khaṇḍakhādyaka* of Brahmagupta, notwithstanding his emphatic critiques of many aspects of Āryabhaṭa's astronomy. Especially in

[17] These canonical "school" divisions in *jyotiṣa* are discussed in Plofker (2014, pp. 1–2) and Pingree (1978a, pp. 534, 629–630). To the best of our knowledge, the earliest references to them in Sanskrit sources include Dinakara's *Candrārkī* (verse 2) as well as Mallāri in his commentary to Gaṇeśa's *Grahalāghava* (Jośī 1994, pp. 44–45, verse 1.16). The terms *pakṣa* and *pratipakṣa* are well-known terms in philosophy, meaning opposing positions in an argument or a debate, but the etymological path for the evolution of the general concept of a *pakṣa* as a school of opinion and/or intellectual lineage in Sanskrit remains to be explored. The *pakṣa* identities of individual table texts, where known, are specified in Appendix A, and the characteristic parameters of the various schools in Appendix B.

northern India some astronomy manuals were composed within the Ārdharātrika-pakṣa.[18]

Saurapakṣa. Another midnight-epoch system, the Saurapakṣa or "sun (*sūrya*) school" is embodied in a recension of the *Sūryasiddhānta* dating to around the eighth century. It gave rise to many works especially in central and eastern India, and like the Brāhmapakṣa seems to have been the basis of many table texts.

Gaṇeśapakṣa. This parameter system was set out in the handbook *Grahalāghava*, which was composed by Gaṇeśa in 1520 and was widely adopted in northern and central India. It is a modified combination of parameters from other *pakṣas* based on Gaṇeśa's own observational tests of their accuracy.

1.3 Astrological aspects of *jyotiṣa*

Astral omens about eclipses, the appearance and position of the moon, and other celestial phenomena appear in Sanskrit texts as early as the first millennium BCE. As discussed in Section 1.1.4, the spread of Hellenistic genethlialogy near the turn of the millennium added to these omen corpora highly mathematized predictions about planetary positions and their impacts on terrestrial events, which became inextricably linked with ritual calendrics as part of the fundamental purpose of *jyotiṣa*.[19]

1.3.1 Divination from omens or *saṃhitā*

Terrestrial omens, such as the movements of animals and birds, bodily traits and markings, and everyday incidents and accidents (including dreams) dominate the tradition of divination from omens. But many meteorological and celestial events too, most significantly eclipses but also more routine phenomena like planetary risings and settings, the moon's occupation of a particular *nakṣatra*, and the orientation of the lunar crescent, are identified as auspicious or inauspicious in omen texts.[20]

[18] This school is represented in southeast Asian calendric traditions as well; Ōhashi (2008, p. 323) suggests that its diffusion may reflect a particular affinity for the Ārdharātrikapakṣa among Buddhists.

[19] For an account of the development of celestial omens in the ancient near east and their spread to other early cultures of inquiry, see Pingree (1997). For general surveys of astral omens and other branches of astrology, see Gansten (2010, 2011).

[20] The classical authority for astral omens in Sanskrit is Varāhamihira's *Bṛhatsaṃhitā* (Kern 1865). A detailed list of editions and translations is given in Pingree (1981, pp. 72–73, note 36).

1.3.2 Celestial forecasting and horoscopes

The basic notion of a horoscope or prediction of life outcomes from the configuration of stars and planets at the native's birth, and its relation to the terrestrial location of the birth via the ascendant or point where the ecliptic crosses the local eastern horizon, was fundamental to Hellenistic astral sciences. It merged with Indian divination traditions in the early first millennium CE to launch the complicated genethlialogical systems known as *jātaka* "nativity."

The predictive capacities of horoscopes were speedily sought in many situations other than nativity forecasting. The practice of casting a horoscope for the moment when a questioner inquires about a particular matter of uncertainty (such as a lost object or an imminent danger), and predicting its resolution based on that horoscope, was called *praśna* "question," i.e., the astrology of interrogations. The reverse practice of identifying auspicious moments to perform particular actions, inspired by Hellenistic catarchic astrology, was known as *muhūrta* "moment," a blend of horoscopy with omen interpretation. Two very common subjects of inquiry in *muhūrta* were marriage (*vivāha*) and military expedition and travel (*yātrā*), which evolved into astrological subfields in their own right.

1.3.3 Fundamental astrological concepts

The single most important entity in Indian astrology is the horoscope or forecast derived from a complex of celestial data, particularly the nativity birth-chart or *janmapattra*.[21] The horoscope's fundamental basis is the positions of the planets in the zodiac at the moment of the native's birth or some other crucial time. A planet (*graha*) is one of the nine celestial bodies including the luminaries, the five star-planets, and the moon's nodes. The luminaries and star-planets are assigned pairwise relationships as friends (*mitra*), neutral (*sama*), or enemies (*śatru*), e.g., the sun is in friendship with the moon, Mars, and Jupiter, in enmity with Venus and Saturn, neutral with respect to Mercury, and so forth.

These seven planets are allotted special "lordship" over one or two individual zodiacal signs (*rāśis* or 30° arcs of the ecliptic) considered to belong to them. In addition, each of these seven has an "exaltation" (*ucca*) or sign in which its power is intensified, especially in a particular degree of that sign, as well as a sign and degree of "dejection" (*nīca*) diametrically opposite the *ucca*. The 12 signs with their planetary lords and their degrees of exaltation for a designated planet are shown in Table 1.1.

[21] Various astrological concepts are covered in *Bṛhajjātaka* (Jhā 1934) including divisions of zodiacal signs (1.6–12), exaltation/dejection (1.13), friendships/enmities of planets (2.15–18), *daśās*/*antardaśās* (chapter 8), *aṣṭakavarga* (chapter 9), moon in *nakṣatras* and zodiacal signs (chapters 14–15), *dṛṣṭis* (chapter 19), *bhāvas* (chapter 20), and *vargas* (chapter 21).

Table 1.1 Zodiacal signs with their planetary lords and degrees of exaltation (*ucca*) (Jhā 1934, pp. 15–16).

Sign	1	Aries	2	Taurus	3	Gemini	4	Cancer
Lord		Mars		Venus		Mercury		Moon
ucca	10°	Sun	3°	Moon			5°	Jupiter
Sign	5	Leo	6	Virgo	7	Libra	8	Scorpio
Lord		Sun		Mercury		Venus		Mars
ucca			15°	Mercury	20°	Saturn		
Sign	9	Sagittarius	10	Capricorn	11	Aquarius	12	Pisces
Lord		Jupiter		Saturn		Saturn		Jupiter
ucca			28°	Mars			27°	Venus

Planetary influences were held to depend not just on their positions with respect to the zodiac and one another, but also on the terrestrial locality from which they were perceived. The core concept in this regard is the ascendant or horoscopic point (*lagna*), where the ecliptic intersects the eastern horizon of the locality. This point determines the division of the ecliptic into 12 houses (*bhāva* or *gṛha*). The first *bhāva* is usually equated to the entire sign containing the ascendant. Alternatively, it may be considered as a 30° arc of the ecliptic measured east from the ascendant, or the first one-third of the ecliptic arc between the ascendant and the prime meridian, etc. Consequently, the tenth *bhāva*, around 270° from the house of the ascendant, will be the house containing midheaven.

The nature and power of a planet vary greatly depending on which house, sign, and/or part thereof they occupy. These variations include "aspect" or *dṛṣṭi*, in which a planet affects a house standing at a certain angle from its own. (A planet may also aspect another planet depending on their relative positions.) A planet's transit (*gocara*) from one house or sign to another is also considered significant. Moreover, various combinations of planet, house, and aspect configurations are designated *yogas* (literally "conjunction" or "combination"), which have special impacts of their own.

Along with its division into 12 signs, the ecliptic is apportioned among the 27 *nakṣatras* or lunar constellations (see Table B.3). Each of these can affect the influence of the moon or some other body that occupies it, as well as interacting with the configuration of the houses to modify their effects. The zodiac is further complicated by the schemes known as *vargas* or *rāśi-bheda* "dividing of signs," which partition each sign into a certain number of equal parts or *aṃśas*. There are traditionally 16 such *vargas* based on the divisors 1, 2, 3, 4, 7, 9, 10, 12, 16, 20, 24, 27, 30, 40, 45, and 60, but other divisors may also be used.[22] These "micro-zodiacs" impart ominous characteristics of the zodiac as a whole to subdivisions of a single sign. A few of the most important *vargas*, which together with the base category of *rāśi* or sign make up the "six *vargas*" (*ṣaḍvarga*), are shown in the table below.

[22] See *Bṛhajjātaka* 1.6–7, 1.9 (Jhā 1934) and *Horāśāstra* (chapter 6) (Kapoor 1991).

horā	*dreṣkāṇa*	*navāṃśa*	*dvādaśāṃśa*	*triṃśāṃśa*
2 × 15°	3 × 10°	9 × 3°; 20	12 × 2°; 30	30 × 1°

The so-called *aṣṭakavarga* or "*varga* of eight" is somewhat different from these *rāśi*-division *vargas*. It is a sort of point-scoring system for assessing the cumulative effects of the planets' positions on one another and on that of the ascendant. Each planet is held to have special benefic or malefic influences in certain houses separated by given intervals from the house occupied by some other planet, by the ascendant, or by itself. The assignment of these auspicious or inauspicious house-intervals for a given planet changes depending on which object it is influencing. So each planet has its own *aṣṭakavarga* chart listing its house-interval effects for each of the seven planets and for the ascendant (hence the "eight" in its title; an example of such a chart is shown in Figure 4.72). Malefic and benefic impacts are designated "points" (*bindu*) and "lines" (*rekhā*), though sources do not always agree on which is which.[23]

The diverse features of a planet's circumstances at a given moment determine its *avasthā* or "situation/status," one of a group of states such as "power" or "happiness" or "misery" that have corresponding impacts upon the horoscope. Further complexities are added to the system by the concept of planetary "conditions" or "life stages" that correspond to, and exert influence on, the lives of humans. All the planets' effects are further modified by being filtered through the system of *daśās* or life stages. Various terms of years in a native's life are assigned to different planets depending on the *nakṣatra* occupied by the moon at the moment of birth. Then the predictions of the horoscope are allocated to the appropriate life stage(s) based on the *daśā* result. Since there are also numerous subdivision schemes for the *daśās* (*antardaśās* or "inner *daśās*"), each with its own modifications of the predicted influences, the system is sufficiently intricate to accommodate virtually any apparent discrepancy that may arise in predictions.

1.4 Text genres in classical *jyotiṣa*

Any type of Sanskrit *jyotiṣa* work may be generally designated *grantha* or *pustaka*, "book, composition." Additionally, in the classical and late medieval periods there developed several distinct textual genres for various practical and pedagogical purposes which have lent their names to the titles of many works representing them. The following sections briefly survey the main characteristics of these genres.

[23] While Bhaṭṭotpala explains in his commentary on the *Bṛhajjātaka* (verse 2.9) that points are benefic and lines are malefic (Jhā 1934, p. 143), in his *Horāśāstra* (verse 66.14–15), Parāśara associates points with malefic effects and lines with benefic effects (Kapoor 1991, p. 844).

1.4.1 The comprehensive treatise or *siddhānta*

The Sanskrit word *siddhānta* (literally "doctrine," "treatise") appears to be the earliest standard term for detailed verse expositions of mathematical astronomy. It is characterized especially by the convention of stating its mean-motion parameters in terms of immense numbers of integer revolutions during a very long period, usually a *kalpa* or lifetime of the universe. Some sources distinguish the *tantra* ("model" or "system") as a similar but separate genre or subgenre typified by the use of the shorter eon or *mahāyuga* instead of the *kalpa*.[24] Using one of these eons instead of a historical epoch date as its zero-point of time, a *siddhānta* or *tantra* is theoretically equally applicable to any time from the remote past to the far future. In fact, this characteristic of timelessness can make it difficult to identify the date of composition of such treatises to within a few decades or even centuries, especially ones ascribed to a divine author.

The *siddhānta* is usually divided into about one or two dozen chapters (*adhyāya* or *adhikāra*) on various topics, beginning with the fairly canonical sequence described in Section 1.2.2 for the fundamental astronomical computations. These initial chapters are frequently lumped together as a main section or "volume" (also conventionally called an *adhyāya*) under the name *gaṇita* "calculation." Their approach generally prizes comprehensiveness over concision, for example, in supplying multiple techniques for computing the same quantity in mathematically exact or conveniently approximate form.

Later chapters may be devoted to any of a wide variety of astronomical and/or mathematical topics, ranging from non-astronomical arithmetic and algebra to the construction and use of observing instruments to the combinatorics of Sanskrit prosody. In particular, a *siddhānta* often contains an expository chapter or multi-chapter *adhyāya* called *gola* ("sphere" or "spherics") discussing the mathematical rationales for spherical cosmological models and the algorithms based on them.

The pedagogical nature of the *siddhānta* is highlighted by its emphasis on understanding such mathematical rationales and theoretical aspects of astronomy in general. Exhortations to students about the importance of studying *gola* explanations as well as just memorizing *gaṇita* algorithms, in order not to be found wanting in "assemblies of the learned," will be recognized by mathematics teachers across the centuries.[25]

Prominent works in this genre include what are probably the three most famous Sanskrit mathematical works within modern history of mathematics: the *Āryabhaṭīya* of Āryabhaṭa composed around 500 (Shukla 1976), the 628 *Brāhma-*

[24] For the distinction of the *tantra* based on the period, see Chatterjee (1981, vol.II, p. 3) or Pingree (1981, pp. 48–51).

[25] For passages by Sanskrit authors which underscore the importance of studying the *gola*, see, for instance, Brahmagupta in his *Brāhmasphuṭasiddhānta* (verse 21.1) (Dvivedī 1901–1902, p. 359), Vaṭeśvara in his *Vaṭeśvarasiddhānta* (*Gola* verses 1.1–6) (Shukla 1986, vol.II, pp. 613–4), Nityānanda in his *Sarvasiddhāntarāja* (*Gola* verse 2) (Misra 2016, pp. 105–106).

sphuṭasiddhānta of Brahmagupta (Dvivedī 1901–1902), and the 1150 *Siddhānta-śiromaṇi* of Bhāskara II (Śāstrī 1989) (see Section 1.2.3). The *Śiṣyadhīvṛddhida-tantra* of Lalla around the eighth century (Chatterjee 1981), the *Siddhāntaśekhara* of Śrīpati in the mid-eleventh century (Misra 1932, 1947), and the 1500 *Tantra-saṅgraha* of Nīlakaṇṭha in the south Indian school of Mādhava (Ramasubramanian and Sriram 2011) are also important exponents of the *siddhānta/tantra* tradition. A group of texts composed probably in the late first up to the early second millennium is known as the "divine (*apauruṣeya*) *siddhāntas*" from the ascription of their authorship to various deities (Dikshit 1981, pp. 26–27; Pingree 1981, pp. 26, 30).

1.4.2 The astronomical handbook or *karaṇa*

An equally important type of Sanskrit verse astronomy text, apparently developed contemporaneously with the *siddhānta* and in many cases by the same authors, is the more practically convenient genre called *karaṇa* "doing," "making," "calculation." Almost always attributed to a human rather than divine author, a *karaṇa* employs a contemporary epoch date and approximate parameter values in place of the "universal" parameters of the *siddhānta* genre, and its trigonometry uses cruder sine values and/or algebraic approximation formulas to simplify calculations. Descriptions of *karaṇa* texts frequently emphasize their ease of use and compact size.[26]

Significant examples of this genre include the *Khaṇḍakhādyaka* by the above-mentioned Brahmagupta (Chatterjee 1970) and the *Karaṇakutūhala* of Bhāskara II (Mishra 1991), as well as the *Laghumānasa* of Muñjala composed in the tenth century (Shukla 1990) and the 1092 *Karaṇaprakāśa* of Brahmadeva (Dvivedī 1899). Probably the most renowned work in this category besides the *Karaṇakutūhala* is the 1520 *Grahalāghava* of Gaṇeśa (Jośī 1994).

1.4.3 The annual calendar or *pañcāṅga*

The annual calendar or *pañcāṅga* "fivefold" synthesizes essential aspects of the ancient ritual timekeeping, historical eras, astronomical models, and astrological purposes described above into a unified though complicated system. A complete calendar for a given year would list the sequence of all the days in the year, along with the specific times during each day marking the transitions of the other time-

[26] This is exemplified by the criteria laid down in Vaṭeśvara's early tenth-century *siddhānta* for composing a *karaṇa* work: "A *karaṇa* is to be made quite concise, not apparent to others, easily used by the stupid." (Shukla 1986, vol. I, p. 56).

units, and the occurrence of prescribed religious rites and festivals along with particularly auspicious or inauspicious celestial events such as eclipses.

The name of this ephemeral genre refers to the five chief time-units whose passage the calendar tracks. All of these can be approximated by mean or average versions based on assumptions of uniform angular motion for the sun, moon, and lunar node. But the construction of a calendar generally requires taking into account their deviation from uniform motion, so that, e.g., sunrise or full moon will correspond (more or less) to the observed moment of the astronomical phenomenon in question, and not just to the passage of an average time-unit. The calendar typically references all of the following time-units in addition to the familiar *māsa*, (synodic) "month," and *saṃvatsara*, (solar) "year"[27]:

tithi This is the fundamental unit of the Hindu ritual calendar, equal on average to one-thirtieth of a synodic month. Natal anniversaries, *pūjā* (worship) ceremonies, and so forth are typically celebrated with reference to a given *tithi* of a given month rather than to a civil day. The astronomical definition of the *tithi* is the time period during which the elongation or angular separation between the longitudes of the sun and moon increases by exactly 12°. Since the irregularities of the sun's and moon's motion cause variations in their apparent speed, the length of a true *tithi* is constantly changing, generally falling a little short of but sometimes exceeding the length of a mean civil day. A professional *pañcāṅga*-maker is required to determine at exactly which times in which successive days a given *tithi* starts and ends, and which *tithis* fall entirely within a single civil day ("omitted" *tithis*) and which extend beyond a given civil day on both sides ("additional" *tithis*).

vāra The *vāra* or weekday indicates a given date's position in the familiar regular 7-day cycle. It is never (deliberately) interrupted or intercalated.

nakṣatra This is a purely lunar time-unit, indicating the time required for the moon to traverse an arc of 13° 20′ in longitude or 1/27 of the ecliptic. Again, its true value depends on the varying speed of the moon and requires more complicated calculation than the uniform interval of the mean *nakṣatra*.

yoga The *yoga* (distinct from the astrological *yoga* or conjunction mentioned in Section 1.3.3) is the interval during which the sum of the sun's and moon's longitudes increases by 13° 20′. There are 27 of these units in a month.

karaṇa A true *karaṇa* (here distinguished from the handbook/manual genre of the same name) is exactly one-half of the true *tithi* within which it falls. There are four fixed *karaṇas* in a lunar month, which occur in the last *tithi* of one month and the first of the next; after the first *tithi* comes a sequence of eight repetitions of the seven "moveable *karaṇas*."

pakṣa The *pakṣa* "side, half" (not in the previously mentioned sense of "astronomical school," sometimes translated as "fortnight") is half a synodic month or

[27]See more detailed descriptions of these quantities and the calendars that unite them in Plofker and Knudsen (2011), Sewell and Dikshit (1896), and Chatterjee and Chakravarty (1985).

15 *tithis*. The waxing or "bright" *pakṣa* begins at new moon and the waning or "dark" *pakṣa* at full moon.

***saṃkrānti*, solar transit** This refers to the moment of entry of the sun into a zodiacal sign, whose successive recurrences constitute the 12 solar months. As with days and *tithis*, solar months and lunar months proceed roughly in one-to-one correspondence most of the time, but sometimes an interval of one kind will fall completely within an interval of the other kind.

śuklapratipad This refers to the instant of new moon which begins the first *tithi* of the synodic month (in the *amānta* convention; the *pūrṇimānta* alternative starts the lunar months at full moon at the beginning of the waning *pakṣa*).

1.4.4 Astrological and miscellaneous works

Although the immensity of Sanskrit astrological literature—mostly written in metrical verse, like other didactic texts—precludes describing it in detail here, we enumerate a few characteristic elements helping to identify it within *jyotiṣa* in the larger sense. Section 1.3.3 mentions many astrological technical terms which often appear in the titles of works treating them. In particular, because the nativity horoscope is the chief desideratum of Sanskrit astrological practice, any text whose title contains *janma* or *jātaka* probably relates to it. The foremost early classic of this genre is the *Bṛhajjātaka* of Varāhamihira composed in the sixth century. A form of horoscopy inspired Persian/Arabic sources is called *tājika*, which includes the popular topic of annual horoscopes or *hāyana*.

Interpretation of omens (*saṃhitā*), whether celestial or mundane, always remained a key component of *jyotiṣa* divination practices, as illustrated by the seminal work *Bṛhatsaṃhitā* by the aforementioned Varāhamihira. Some prominent subdivisions of this topic relate to animal and bird signs (*śākuna*), bodily marks (*sāmudrika*, especially *hasta-sāmudrika* or palmistry), jewels (*ratna*), breathing (*svarodaya*), physical sites and buildings (*vāstu*), and dreams (*svapna*). A later form of divination derived from Islamic geomancy is called *ramala*.

Propitiatory and magical practices often involve concepts and methods from astrological and omen-related forecasting. Apotropaic rituals called *śānti* are often addressed to the planets (*graha*), sometimes specified as nine in number including the lunar nodes (*nava-graha*). Diagrams for magical effect or divinatory interpretation may be called *cakra* or *yantra*.[28]

Several of the topics typically treated in the later chapters of a *siddhānta* work may also have independent texts devoted to them. This is especially true of non-astronomical *gaṇita* applications. A standalone work may also be concerned with the exposition of a particular observational instrument or *yantra*, which

[28] For the history of these topics, see the general surveys in Pingree (1978c, 1981, 1997), as well as the canonical *Bṛhajjātaka* (Aiyar 1905).

can sometimes be difficult to distinguish in a list of titles from works on the abovementioned magical or astrological *yantra*.

1.4.5 The astronomical table text

At some point probably in the early second millennium, the canon of Sanskrit mathematical astronomy expanded to include many texts consisting solely or primarily of two-dimensional arrays of numerical values. They are generally accompanied by instructions on how to use the values to perform simplified versions of astronomical calculations, in place of the more laborious parameter-and-algorithm procedures in *siddhāntas* and *karaṇas* (Rupa K. 2014, p. 37)

Such two-dimensional arrays acquired the technical names *koṣṭhaka*, literally "granary, treasury, compartments" and *sāraṇī*, literally "stream, path, line," echoing the etymology of its classical synonym "canon"; see Appendix C.2.1 for a discussion of the use of such terms. The rest of this book is devoted to describing the emergence, development, and legacy of this genre of astronomical tables in Sanskrit *jyotiṣa*.

1.5 Tables and Sanskrit astral sciences in the second millennium

Indian astronomy in the first few centuries of the second millennium, like its counterparts in the Islamic and European worlds, was firmly situated in a geocentric spherical universe whose fundamental models and parameters had been worked out centuries ago, but whose complexity still inspired ceaseless variation in astronomical techniques. However, this variation was not primarily displayed in the genre of comprehensive treatises. After the composition of the hugely influential *Siddhāntaśiromaṇi* and the approximately contemporaneous "divine *siddhāntas*" with no attributed historical human author, only a few major *siddhāntas* were composed in north India before the twentieth century. During the same time, the school of Mādhava in south India continued the *siddhānta* tradition with some remarkable works such as Nīlakaṇṭha's *Tantrasaṅgraha*. This section explores some of the other developments, primarily in the literature from the northern half of the subcontinent, that help fill in the blanks of the story told by the surviving traditional *siddhānta* works.

1.5.1 Origins and development of Sanskrit tables

Table texts in ancient astronomy are familiar as a continuous descent with modification extending through the Babylonian, Greek, and Islamic traditions successively, and eventually issuing in the manifold tables of modern astronomy in its precomputer period. The genesis of the Indian *koṣṭhaka*/*sāraṇī* or astronomical tables appears to be collaterally related to this lineage, but with a substantial amount of independent development.

Sequences of astronomical data in Sanskrit texts go back as far as the lists of constellations, months, etc., in some Vedic texts, whose date is very uncertain but which almost certainly cannot be younger than the late second millennium BCE. In the earliest surviving quantitative astronomy texts, computations relied on versified algorithms rather than extended sequences of tabulated data. Lists of key function values for astronomical/calendric calculations, such as daylight lengths corresponding to given times of the year or gnomon-shadow lengths corresponding to given times of a day, were also worked into the verses of metrological texts in the late first millennium BCE (Kangle 1960–1965, vol. 2, pp. 138ff). A few hundred years later, treatises on spherical astronomy routinely included brief sequences of stellar celestial coordinates and values of trigonometric functions, planetary orbital parameters, etc. (Plofker 2012). Such "versified tables" persist throughout the later history of *jyotiṣa* literature as the primary means of incorporating numerical data into verse texts. Scribes often repeated such data in displayed numeric-array tables accompanying the verses. Since most surviving Sanskrit manuscripts date from the last few centuries of the second millennium, we cannot trace the history of this scribal convention very far back, although there are examples of other displayed numeric data formats in a mathematical manuscript from several centuries earlier (Hayashi 1995).

It seems highly likely that most astronomers constructed much more detailed lists or tables of function values for their own use, without reproducing them in full in compositions meant for dissemination, see, e.g., Shukla (1986, vol.II, pp. 227–228, verse 2.5.2). The compilation of such tables for distribution to others was perhaps in part a response to the spread of paper technology in early second-millennium northern India (Ghori and Rahman 1966; Rezavi 2014). In this way the chief locus of astronomical computation began to shift from the astronomer's *pāṭī* or other erasable working surface to the cells of the row-and-column grids permanently inked on a page (or as permanently as the typical life expectancy of a manuscript allowed, at least).

South Indian *jyotiṣa* and the *vākya* system

A different type of verbally encoded table data is found in the South Indian *vākya* or "sentence" texts, whose first explicitly dated surviving examples go back to the early second millennium but which are traditionally ascribed to an inventor several centuries earlier. They record celestial positions of the planets not by means of number words as in the *Brāhmasphuṭasiddhānta* verse table cited above, but rather by assigning each of the consonants of the Sanskrit *nāgarī* alphabet to one of the ten decimal digits, and then making up a short Sanskrit sentence for each given data value that contains consonants chosen and ordered to correctly represent its numerals. (See Pai et al. 2017 for a detailed discussion of one such *vākya* text.)

1.5.2 The influence of Islamic tables

A detailed portrait of what Indian astral sciences looked like to a non-Indian observer at the very beginning of the second millennium is provided in the *India* of the Muslim scholar-scientist al-Bīrūnī. Not being an accomplished Sanskritist (perhaps not at all literate in the language) (Pingree 1975), al-Bīrūnī was dependent on commentaries and local pandits to explain the contents of the texts he discussed. Though the impressions he received of this material were almost certainly somewhat garbled and his comprehension of them was never complete, his descriptions of them constitute a unique and highly valuable survey of a textual corpus still very imperfectly known.

In al-Bīrūnī's depiction (which of course is further constrained by the characteristics of the particular regional variant of Sanskrit astronomy that he encountered in the northwest of the subcontinent), the Indian exact sciences are still profoundly engaged with "classic" treatises from several centuries earlier. The most notable of these are the works of Āryabhaṭa and Brahmagupta, from the early sixth and early seventh centuries, respectively.

No identifiable Sanskrit numeric-array tables are described by Bīrūnī, who was of course thoroughly familiar with such tables in the *zījes* of his own Greco-Islamic astronomy, and even constructed several in the *India* to aid his systematic explanations of features of Indian astrology.[29] He does refer in the *India* to "a leaf of a canon (*zīj*) composed by Durlabha of Multân [*warqa... min zīj ʻamalahu Durlaba al-Mūltānī*] which I have found by chance." But it seems highly unlikely that this work of Durlabha actually included any *sāraṇī*-style numerical tables of astronomical function values computed for, say, each degree of argument, after the fashion of a typical *zīj* text. The only indication to that effect is al-Bīrūnī's application of the term *zīj* to Durlabha's composition, and he uses the same word in exactly similar ways to refer to, e.g., the sixth-century *Pañcasiddhāntikā*, the seventh-century *Khaṇḍakhādyaka*, and other *karaṇa* works that apparently contain no such tables (Pingree 1981, pp. 32–34). That is, these *karaṇas* resemble the (mostly later) Islamic *zīj* texts only in being handbooks for astronomical computation, not in providing extensive tables of pre-calculated values as a substitute for astronomical computation. Consequently, the so-called canon of Durlabha should be presumed to have been a typical *karaṇa* text rather than an early form of *koṣṭhaka/sāraṇī* work. By the same token, these same works described on al-Bīrūnī's authority as "Sanscrit" *zīj* texts (Kennedy 1956, p. 138) should not be mistaken for table texts.

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[29] Kennedy's classic introductory survey of such texts defines them as follows (Kennedy 1956, p. 123): "A *zīj* consists essentially of the numerical tables and accompanying explanation sufficient to enable the practising astronomer, or astrologer, to solve all the standard problems of his profession, i.e., to measure time and to compute planetary and stellar positions, appearance, and eclipses." Originally derived from the "Handy Tables" of Ptolemy, they were called *zīj* from Persian *zīg* or "cord," a name which apparently associates the rows and columns of numerical tables with the warp and weft cords of a loom (and which may or may not have inspired the somewhat similar Sanskrit name *sāraṇī*).

In fact, the first Indian works fully corresponding to our usual notion of an astronomical table text are thought to have been initially inspired by Islamic *zījes* like those known by al-Bīrūnī.[30] The inference is supported by fundamental similarities between some Indian and Islamic table types, along with the presence of many Islamic *zījes* in second-millennium India. The chronology of this presumed cross-cultural adaptation of the *zīj* into the *koṣṭhaka*, however, remains very uncertain.

As with the origins of the *siddhānta* and *karaṇa* genres nearly a millennium earlier, there is no textual evidence documenting the initial steps of this development: the earliest surviving *koṣṭhaka*/*sāraṇī* works already exhibit mature specimens of the Sanskrit table-text format, although some of them combine it with characteristics of the *karaṇa* (see Chapter 5). Most of the surviving works in this genre date from the sixteenth and seventeenth centuries.

1.5.3 Concurrent developments in *jyotiṣa*

In the couple of centuries following Bīrūnī's account, Indian *jyotiṣīs* began employing the so-called *bījas* (not exactly the same as, though not unrelated to, *bīja* in the sense of "algebra") or sets of parameter modification constants. The construction and application of these adjustments is still not fully understood, but they seem to have been designed both to update the values prescribed in old texts for better agreement with current results and to convert results obtained using one set of *pakṣa* parameters to values consistent with a different *pakṣa*. Although they are first explicitly attested in an eleventh-century table text (see Section 5.1), they are firmly entrenched in north Indian *jyotiṣa* works of all kinds by the twelfth century.[31]

The previously mentioned group of "divine *siddhāntas*" also emerged approximately during the early second millennium. This timing suggests the tentative speculation that their re-centering of the view of *jyotiṣa* as a sacred tradition in the Indian universe of *dharma* may have been conceived partly as a response to foreign influences from outside that universe, i.e., in Islamic astronomical texts and also the adoption of Islamic branches of astrology/divination such as *ramala* and *tājika*. However, a far more powerful force for the perpetuation of classical *jyotiṣa* in its traditional form was a decidedly human *siddhānta*, the magisterial *Siddhāntaśiromaṇi* of Bhāskara II composed around 1150.[32]

[30] See Pingree (1981, pp. 41–46) and Plofker (2009, pp. 274–277).

[31] The use of *bīja*-corrections in Sanskrit astronomy is surveyed in Pingree (1996a), Hayashi (2008a) and Rao (2000, pp. 281–285). Such *bījas* are found in, for instance, the *Grahajñāna* of Āśādhara, the *Mahādevī* of Mahādeva, works of Dinakara, and the *Brahmatulyasāraṇī*.

[32] Although it has been suggested in, e.g., Pingree (1978a, pp. 585–586) that this treatise may show traces of Islamic influence in its trigonometric rules and tables, it seems more likely to represent a parallel evolution within classical *jyotiṣa*.

The role of the verse-only handbook in the period from approximately the early thirteenth century, by which time the canonical status of Bhāskara's manual *Karaṇakutūhala* seems to have been well established, to the early sixteenth when the *Grahalāghava* of Gaṇeśa (Section 5.6.1) became the next standard *karaṇa* work, also raises some intriguing questions. The surviving textual corpus suggests that in the meantime, *koṣṭhaka* table texts successfully competed for the attention of *jyotiṣīs* interested in practical computation. It may not be too far-fetched to view the *Karaṇakutūhala* itself as a deliberate "reclamation" by Bhāskara, the master of both synthesis and innovative development in traditional *jyotiṣa*, of the traditional *karaṇa* style consisting of verse algorithms without numeric-array tables. If true, this seems to imply an equally deliberate rejection of earlier "hybridized" works with their blend of familiar verse algorithms and unconventional (even partly foreign?) arrays of pre-computed data in tabular form.

Chapter 2
Content and classification of table texts

In this chapter we outline two different aspects of the genre of Sanskrit astronomical table texts: the mathematical models underlying their construction and the chief approaches to their taxonomy. We begin by explaining in more detail the algorithms and models that were briefly outlined in Section 1.2.2. The subsequent section broadly surveys the numbers and distribution of manuscripts containing tables in Sanskrit manuscript collections, and describes the conventional notations and layouts that they exhibit. We then discuss various criteria for a rough taxonomy of table texts.

2.1 Computing astronomical quantities

This section analyzes some fundamental elements of the quantitative models used to solve astronomical problems in Sanskrit *jyotiṣa*. The full range of model details and variants is far too large to cover comprehensively here, but the following discussions should enable the reader to understand the basic purpose and construction of a majority of standard *koṣṭhaka*/*sāraṇī* tables.

2.1.1 Mean motions

The mean longitude $\bar{\lambda}$ of a celestial body at a moment t time-units since the time of epoch is given by

$$\bar{\lambda} = \lambda_0 + \Delta\bar{\lambda} \cdot t$$

C. Montelle, K. Plofker, *Sanskrit Astronomical Tables*, Sources and Studies in the History of Mathematics and Physical Sciences,
https://doi.org/10.1007/978-3-319-97037-0_2

where $\bar{\lambda}_0$ is its epoch mean longitude and $\Delta\bar{\lambda}$ its mean motion or mean velocity. Conventionally the standard longitudes in Indian astronomy are sidereal, i.e., measured from a fixed starting-point among the stars. Astronomical treatises and handbooks generally prescribe procedures for finding t in units of days (the *ahargaṇa*) and multiplying it by a mean daily velocity value ultimately derived from period relations between integer numbers of years and revolutions in an eon. Table texts' versions of these procedures usually involve tabulating mean longitude increments for various multiples of different time-units.

2.1.2 True motions: *manda* longitude/velocity corrections

A planet's true longitude λ is obtained by applying to its mean longitude $\bar{\lambda}$ one or more correction terms (conventionally known as "equations") derived from the trigonometric relationships between the positions of the earth, the planet, and its orbital apogee or *ucca*. The corrections may be produced directly from the trigonometric formulas described below, or from various approximations to them discussed in the later sections.

The "equation of center" or "eccentric correction" of celestial bodies in geocentric spherical astronomy is the difference between their mean and true longitudes produced by what is termed the "eccentricity" of their orbits. Qualitatively, it accounts for the fact that an object on an elliptical orbit revolves about its focus rather than its center. In Indian astronomy, this is called the *manda* or "slow" correction, and is the only one required to find the true position of the sun or the moon. Its effect can be modeled, as shown in Figure 2.1, by a circular orbit whose center is displaced from the center of the earth or "eccentric" by the distance e (OC in the figure). Equivalently, it can be represented by a concentric circle of equal size bearing the center of a small circle or "epicycle" with radius $r_M = e$ ($\bar{P}P$ in the figure). In this model, the true planet moves uniformly on the epicycle with the same period as the epicycle center's motion in the opposite direction on the concentric circle. Finally, abandoning the uniform-motion assumption, one can instead think of the true planet schematically as moving on the concentric orbit with a point at the apogee longitude exerting some kind of force at a distance to speed or slow its progress.[1]

Due to this equivalence, the equation of center (denoted μ) or *manda*-equation, called *mandaphala* in Sanskrit, is mathematically the same in both epicyclic and eccentric models: an epicyclic version is shown in Figure 2.2. Its dependence upon the so-called mean anomaly angle (*mandakendra*, κ_M) between the body's mean longitude $\bar{\lambda}$ at $\bar{P}$ and the longitude λ_{A_M} of its apogee A is given by

$$\kappa_M = \bar{\lambda} - \lambda_{A_M}, \qquad \sin\mu = \frac{r_M \sin\kappa_M}{H_M}$$

[1] The eccentric/epicycle equivalence forms part of the conventional description of the *manda*-effect in, e.g., *Siddhāntaśiromaṇi* 2.27–29 of Bhāskara (Śāstrī 1989, p. 47). The non-uniform-motion interpretation is described in, e.g., *Sūryasiddhānta* 2.1–10 (Pāṇḍeya 1991, pp. 17–19).

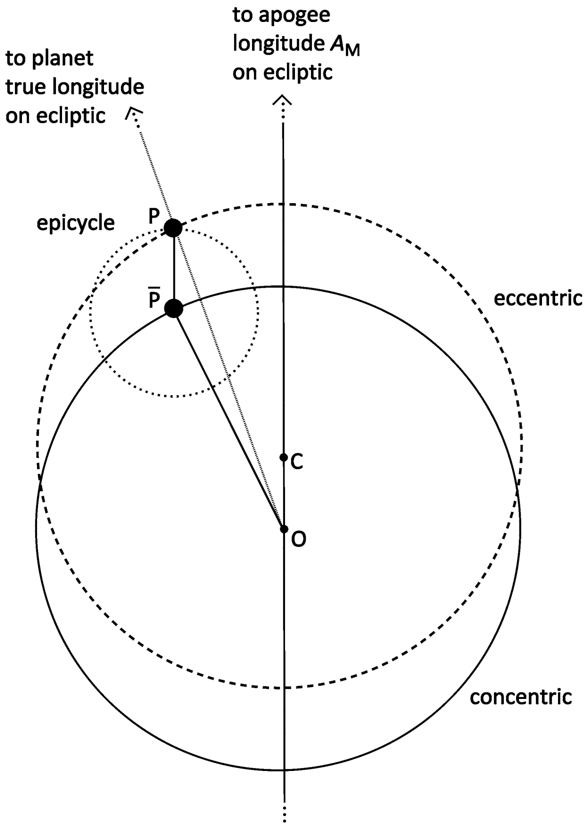

Fig. 2.1 Equivalence of eccentric and epicycle geocentric orbital models for an observer at O, the earth. Planet P may move with its mean velocity on an eccentric circle with eccentricity OC, from the apsidal line OCA_M through the angle $A_M CP = A_M O\bar{P}$. Or the center $\bar{P}$ of its epicycle may move on a concentric circle through angle $A_M O\bar{P}$ while the planet P moves on the epicycle with the same speed in the opposite direction. In both cases, the true planet will appear to be on the line OP instead of at its mean position $\bar{P}$.

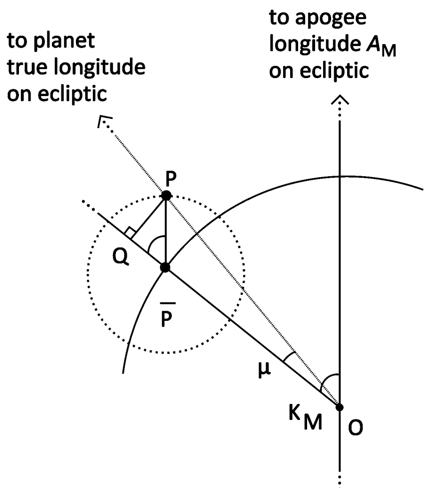

Fig. 2.2 Trigonometry of the *manda*-equation μ determined by the *manda*-anomaly angle κ_M, its scaled sine or *bhujaphala* PQ, and its scaled cosine or *koṭiphala* $\bar{P}Q$.

Table 2.1 Behavior of sequences of approximate values of the *manda*-equation $\mu \approx \arcsin(\sin\kappa_M \cdot r_M/R)$, and of the *manda*-derived velocity correction $\overset{M}{\Delta}_v$, depending on the value of *manda*-anomaly κ_M.

κ_M quadrant	Sequence of *manda*-equation values μ	Sequence of velocity-correction values $\overset{M}{\Delta}_v$
0–90°	Positive increasing $(0, \ldots, \mu_{\max})$	Negative increasing
90–180°	Positive decreasing $(\mu_{\max}, \ldots, 0)$	Positive increasing
180–270°	Negative decreasing $(0, \ldots, -\mu_{\max})$	Positive decreasing
270–360°	Negative increasing $(-\mu_{\max}, \ldots, 0)$	Negative decreasing

where r_M as before is the epicycle radius and H_M is the distance between the *manda*-corrected planet and the earth (OP in the figure). The equation μ is zero when the body is at its apogee or perigee, where the mean and true longitudes coincide.

The hypotenuse or *karṇa* H_M can be expressed in terms of κ_M, R, and r_M using the Pythagorean theorem to give the equivalent expression

$$\sin\mu = \frac{r_M \sin\kappa_M}{\sqrt{(r_M \sin\kappa_M)^2 + (R + r_M \cos\kappa_M)^2}}$$

where the right triangle sides $r_M \sin\kappa_M$ and $r_M \cos\kappa_M$ in the epicycle are sometimes called the *bhujaphala* and *koṭiphala* respectively. In practice, H_M is frequently assumed to be equal to R (Yano 1997). With this assumption, the above expression for $\sin\mu$ becomes merely a linear multiple of $\sin\kappa_M$, so the same values of *manda*-equation μ will occur symmetrically in the four quadrants of *manda*-anomaly as shown in Table 2.1. The argument range of most *manda*-equation tables consequently extends only up to 90°, where μ is at maximum.

Because a planet appears to move faster or slower depending on where it is on its orbit, the *manda*-equation adjustment in longitude involves a corresponding modification to the planet's mean velocity $\bar{v}$, which is termed the *manda-gatiphala* or angular velocity correction due to the *manda*-anomaly (here denoted $\overset{M}{\Delta}_v$). If we imagine the planet's true velocity v as approximately the finite difference between two successive true longitudes λ_1 and λ_2 with respective mean longitudes $\bar{\lambda}_1$ and $\bar{\lambda}_2$, we obtain $v = \lambda_2 - \lambda_1$ and

$$\overset{M}{\Delta}_v = \bar{v} - v = \left(\bar{\lambda}_2 - \bar{\lambda}_1\right) - (\lambda_2 - \lambda_1) = \left(\bar{\lambda}_2 - \lambda_2\right) - \left(\bar{\lambda}_1 - \lambda_1\right) = \mu_2 - \mu_1$$

Consequently, algorithms for the *manda-gatiphala* will depend in some way on change in the *manda*-equation. Bhāskara II's *Siddhāntaśiromaṇi* (2.36–38) prescribes for the *gatiphala* at a given moment a relation equivalent to the following formula:

$$\overset{M}{\Delta}_v = \left(\bar{v} - v_{A_M}\right) \cdot \cos\kappa_M \cdot (r_M/R)$$

where v_{A_M}, the velocity of the *manda*-apogee, is always taken as zero for all planets except the moon. This relation ingeniously takes the cosine of the *manda*-anomaly as the instantaneous change in its sine, which in turn is proportional to the change in $\sin \mu$.[2]

Qualitatively, the *gatiphala* correction $\overset{M}{\Delta}_v$ is at its minimum (most negative) value when the planet is moving most slowly, at apogee or 0° of anomaly. During the first quadrant, it increases to zero so that at 90° of anomaly, the *manda*-corrected velocity equals the mean velocity. The *gatiphala* continues to increase to its maximum value when the planet attains its fastest motion at perigee, and then symmetrically decreases through zero down to its minimum as the planet travels back towards the apogee. These values are to be applied with appropriate order and sign to the mean velocity, depending on the quadrant in which the value of the anomaly κ_M falls (see Table 2.1).

2.1.3 True motions: *śīghra* longitude/velocity corrections

In addition to the equation of center resulting from orbital eccentricity, the five star-planets also require a second correction known as the "equation of conjunction" or "synodic correction". Namely, from the vantage point of an observer on earth over weeks or months, the star-planets appear to periodically reverse direction while changing their speed. In a heliocentric model, this can be explained by the fact that the planets, in addition to the earth, are all revolving about the sun. The mechanism that Indian astronomers employed to account for this "synodic anomaly" is an epicyclic correction called *śīghra* or "fast" (since the velocity of a planet's *śīghra*-apogee is much faster than that of its *manda*-apogee).

This *śīghra*-equation σ is a function of the so-called *śīghrakendra* or *śīghra*-anomaly κ_S, measured between the mean longitude of the planet corrected by the *manda*-equation, $\bar{\lambda}_M$, and the longitude λ_{A_S} of the *śīghra*-apogee:

$$\kappa_S = \lambda_{A_S} - \bar{\lambda}_M$$

For superior planets, the *śīghra*-apogee is taken to be the mean sun, and in the case of inferior planets, the planet itself (see Figure 2.3). As explained in Section 2.1.4, because the *śīghra*-apogee A_S rotates faster than the mean planet about the earth, this model periodically produces apparent retrogradations of the planet when it is near opposition (or inferior conjunction for an inferior planet).

The trigonometric derivation of the *śīghra*-equation σ at any given moment is similar to that of the *manda*-equation in the epicyclic model (see Section 2.1.2):

$$\sin \sigma = \frac{r_S \sin \kappa_S}{H_S}$$

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[2] The "calculus" nature of such techniques has been frequently discussed, see, e.g., Arkasomayaji (1980, pp. 110–111) and Rao (2017).

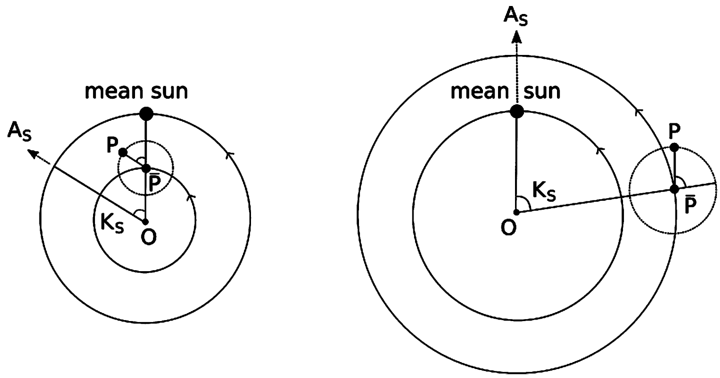

Fig. 2.3 Configuration of *śīghra*-anomaly κ_S for inferior planets (left) and superior planets (right). For Venus and Mercury the mean longitude of the sun coincides with that of the (*manda*-corrected) mean planet $\bar{P}$ itself, while for the other planets the mean sun is at the *śīghra*-apogee longitude A_S.

where r_S is the radius of the planet's *śīghra*-epicycle and its true distance H_S from the earth is given by

$$H_S = \sqrt{(r_S \sin \kappa_S)^2 + (R + r_S \cos \kappa_S)^2}.$$

But due to the much larger size of the planets' *śīghra*-epicycles as compared to their *manda*-inequalities, the substitution of R as an approximation to the true hypotenuse distance or *karṇa* H is not feasible in this case. Consequently, the maximum *śīghra*-equation value $\sigma_{\max}$ will occur not at the end of the first quadrant of *śīghra*-anomaly κ_S, but rather at the value of anomaly equivalent to

$$\kappa_S = \arccos\left(-\frac{r_S}{R}\right).$$

Depending on the *śīghra*-epicycle size for the planet in question, this anomaly value will fall somewhere between 90° and 135°. So tabulated *śīghra*-equation values will be symmetric only about the ends of the second and fourth quadrants of anomaly, not the first or third quadrant.[3]

The combination of *manda* and *śīghra* corrections is usually prescribed in treatises and handbooks as an abbreviated iteration process, sometimes with additional modifications specific to Mars.[4] Table texts that include these equation values may

[3]A modern analytical derivation of the $\sigma_{\max}$ result involves setting the derivative of the above expression for $\sin \sigma$ to zero and solving for the value of κ_S that satisfies it, see Montelle and Plofker (2015, pp. 18–20).

[4]The historical process of development and refinement of these iterated combinations of corrections is very obscure. Several versions used by Indian astronomers are described in Chatterjee

provide instructions explaining how to combine them, or may leave it up to the user to decide their application.

As in the case of the *manda*-correction μ discussed in Section 2.1.2, the *śīghra*-correction σ to a planet's longitude likewise entails a corresponding correction to its velocity due to the *śīghra*-anomaly, the *śīghra-gatiphala* or $\overset{S}{\Delta}_v$. Tabulated values of this quantity suggest that its computation frequently relies on approximating it by the product $\Delta\sigma \cdot v_{\kappa_S}$ of the *śīghra*-equation difference and the anomaly velocity. If we now call the mean velocity corrected by the *manda-gatiphala* the "*manda*-corrected" velocity $\bar{v}_M = \bar{v} + \overset{M}{\Delta}_v$, then the additive combination of this $\bar{v}_M$ with the *śīghra-gatiphala* produces the planet's fully corrected true velocity v:

$$v = \bar{v}_M + \overset{S}{\Delta}_v \approx \bar{v}_M + \Delta\sigma \cdot v_{\kappa_S}$$

To understand the structure of this *śīghra-gatiphala* in modern terms, we can analytically represent instantaneous true velocity v as the derivative of true longitude $\lambda = \bar{\lambda}_M + \sigma$[5]:

$$\begin{aligned} v = \frac{d}{dt}(\lambda) \quad &= \quad \frac{d}{dt}\left(\bar{\lambda}_M + \sigma\right) \quad = \quad \frac{d}{dt}\left(\bar{\lambda}_M\right) + \frac{d}{dt}(\sigma) \\ &= \bar{v}_M + \frac{d}{d\kappa_S}(\sigma) \cdot \frac{d}{dt}(\kappa_S) \\ &= \bar{v}_M + \frac{d}{d\kappa_S}(\sigma) \cdot v_{\kappa_S} \end{aligned}$$

where $v_{\kappa_S} = \dfrac{d}{dt}(\kappa_S)$ is the rate of change of the *śīghra*-anomaly $\kappa_S = \lambda_{A_S} - \bar{\lambda}_M$, or $v_{\kappa_S} = v_{A_S} - \bar{v}_M$. The other derivative $\dfrac{d}{d\kappa_S}(\sigma)$ can be approximated by the finite difference $\Delta\sigma$ between two successive values of the *śīghra*-equation.

2.1.4 Synodic phenomena

In modern heliocentric terms, the earth in its orbit periodically overtakes the slower-moving superior planets (Mars, Jupiter, and Saturn) and is overtaken by the faster inferior Venus and Mercury. This produces the recurring apparent halts, reversals of direction, disappearance in proximity to the sun, and reappearance that characterize the "wandering stars." From a geocentric point of view, the same phenomena are explained by models in which the average apparent eastward motion in longitude

(1981, vol. 2, pp. 51–53); a specially modified variant for Mars is given in *Siddhāntaśiromaṇi* 2.35 (Śāstrī 1989, p. 52).

[5] See the discussion in Montelle and Plofker (2015, pp. 23–25) for a full derivation of the following equations. Note in particular that the true velocity formula $v = \bar{v}_M + \dfrac{d}{d\kappa_S}(\sigma) \cdot v_{\kappa_S} \approx \bar{v}_M + \Delta\sigma \cdot v_{\kappa_S}$ is essentially equivalent to the *Siddhāntaśiromaṇi*'s rule in 2.39.

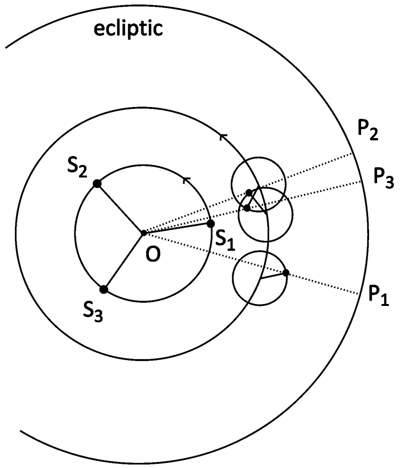

Fig. 2.4 Occurrence of retrogradation of a superior planet P as seen from the earth at O. The location of the mean sun (sampled for three moments at S_1, S_2, and S_3) determines the direction of the planet's *śīghra*-apogee, which causes the planet's perceived ecliptic longitude at P_3 to fall behind where it was previously at P_2.

of the inferior planets is faster than that of the sun, which in turn is faster than that of the superior planets. In these models, the *śīghra*-epicycles superimpose apparent smaller oscillations on the underlying net eastward motions. Figure 2.4 illustrates such an oscillation for a superior planet.

An inferior planet seems to zigzag back and forth in the sky like a racer on a distant oval track, never getting very far away from the sun. In heliocentric terms, this is because the planet's smaller orbit about the sun is confined within that of the earth. A superior planet, on the other hand, orbits beyond the earth, and appears to backtrack in the sky only when the earth passes it on the inside. All these motions appear slightly tilted with respect to the ecliptic due to the planets' small orbital inclinations, i.e., the nonzero angle between the plane of the ecliptic and that of the orbit. The following specific events produced by these motions are classified as synodic phenomena.

Superior planet:

First station (*vakra*, "reverse"). The planet appears to slow down and then stop in its eastward motion (as the faster-moving earth catches up to it).
Immediately after first station, the planet begins to "retrograde" or move backwards, i.e., westward, in the sky (as the earth passes by it).

Opposition. In the middle of the retrograde arc, the planet and the sun are 180° apart in longitude: the overtaking earth is on the straight line between them. The planet is visible in the night sky from sunset to sunrise.

Second station (*mārga*, "on course"). The retrograde motion appears to slow and then stop as the earth rounds the curve of its own smaller orbit and the planet is no longer falling behind in longitude.
After second station the planet's apparent motion among the stars is eastward once more, and will remain so until the next occurrence of first station. The faster-moving sun continues to approach the planet from the west, decreasing the nighttime interval in which the planet can be seen.

Acronycal setting/last visibility in west (*paścima-asta*, "west setting"). The planet is just far enough east of the sun to be briefly visible above the setting sun on the western horizon. On subsequent nights it will be too close to the sun to be seen.

Conjunction. The (invisible) planet has the same longitude as the sun, i.e., the positions of the planet, sun, and earth are again collinear, but this time with the sun in the middle.

Heliacal rising/first visibility in east (*pūrva-udaya*, "east rising"). As the faster-moving sun pulls away to the east, the planet rising on the eastern horizon will become visible just before sunrise. Until its next acronycal setting it will be visible for at least some part of every night.

Inferior planet:

First station (*vakra*, "reverse"). As the planet reaches its greatest eastern elongation from the sun (i.e., as it rounds the curve of its smaller orbit as seen from earth) its eastward motion appears to slow and then stop.
Immediately after first station, the planet appears to move westward or retrograde in the sky as its orbit takes it to the far side of the sun.

Acronycal setting/last visibility in west (*paścima-asta*, "west setting"). The westward-moving planet as it approaches the sun is visible just above the setting sun on the western horizon; this is the start of its first period of invisibility.

Superior conjunction. The planet has the same longitude as the sun, which lies directly between the planet and the earth and hides the planet from view.

Heliacal rising/first visibility in east (*pūrva-udaya*, "east rising"). The westward-moving planet passes the sun and becomes visible again on the eastern horizon just before sunrise.

Second station (*mārga*, "on course"). The westward motion slows down and stops as the planet reaches its greatest western elongation from the sun.
After rounding the western end of its orbit the planet moves east again on the near side of the sun.

Heliacal setting/last visibility in east (*pūrva-asta*, "east setting"). The eastward-moving planet, overtaking the sun, is visible for the last time on the eastern horizon before sunrise, then enters its second period of invisibility.

Inferior conjunction. The (invisible) planet once more has the same longitude as the sun, this time positioned between the sun and the earth.

Acronycal rising/first visibility in west (*paścima-udaya*, "west rising"). The faster eastward-moving planet pulls away from the sun and becomes visible on the western horizon after sunset.

The elongations for the following pairs of phenomena add up to 360°, as illustrated in Table 2.2: first and second station, east rising and west setting, west rising and east setting. This is because each of these pairs of phenomena is symmetric about opposition, superior conjunction, or inferior conjunction, respectively, where the synodic elongation is either 180° or 0°.

Table 2.2 Degrees of synodic elongation where the various synodic phenomena occur, as specified in, e.g., *Brāhmasphuṭasiddhānta* 2.48, 2.52–53, *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 2.41–44. Other authors record slightly different values for some of them (Shukla 1986, vol. 2, p. 221).

Planet	East rising	West setting	West rising	East setting	First station	Second station
Mars	28	332	–	–	163	197
Mercury	205	155	50	310	145	215
Jupiter	14	346	–	–	125	235
Venus	183	177	24	336	165	195
Saturn	17	343	–	–	113	247

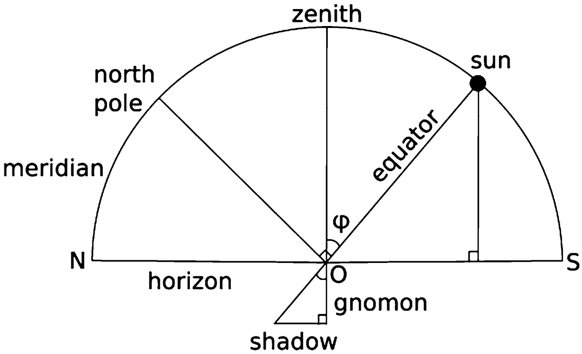

Fig. 2.5 Projection of the visible celestial hemisphere onto the plane of the local meridian at terrestrial latitude ϕ, showing the sun at true local noon on an equinox (i.e., at the intersection of the meridian and the celestial equator). A vertical gnomon whose tip is imagined at the observer's position O makes the angle ϕ with the ray of the equinoctial noon sun, casting a horizontal shadow.

2.1.5 Times, ascensions, and shadows

Terrestrial latitude and longitude. The timing and appearance of celestial phenomena at a particular location on the earth are dependent on the location's latitude and longitude. The terrestrial latitude ϕ is measured from zero at the earth's equator, conventionally designated Laṅkā. Latitude is frequently expressed in terms of the "noon equinoctial shadow" s_0, the shadow of a standard gnomon (usually 12 digits) when the sun is on the intersection of the local meridian and the equator, i.e., at noon on an equinoctial day. As shown in Figure 2.5, the angular distance from the local zenith to the noon equinoctial sun is equal to the latitude, so the relationship between ϕ and s_0 for a 12-digit gnomon can be expressed as follows:

$$\frac{\sin\phi}{\cos\phi} = \frac{s_0}{12}$$

Terrestrial longitude is measured to the east or west of the prime meridian, which in Sanskrit texts is generally represented by a list of localities including Laṅkā and

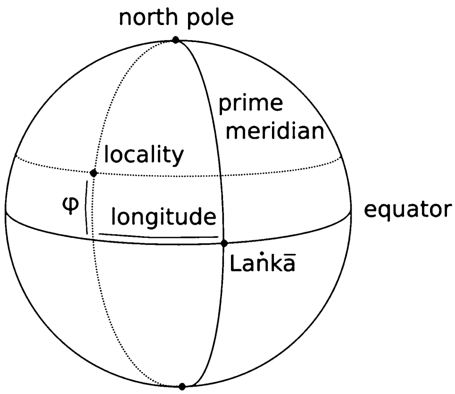

Fig. 2.6 The sphere of the earth, showing the latitude ϕ of a locality as the arc on the local meridian between its parallel of latitude and the equator, and the equatorial arc of longitude from the local meridian to Laṅkā on the prime meridian.

Ujjain. Geometrically, as illustrated in Figure 2.6, a nonzero longitude corresponds to the arc of the equator between the prime meridian and the local meridian. In practical terms, this arc can be found from the local time difference Δt in *ghaṭīs* between observations of the same celestial event at the specified locality and at a locality on the prime meridian (Sen 1975).

$$\frac{\text{longitude}}{360°} = \frac{\Delta t}{60}$$

Alternatively, if the earth's circumference C_E is known along with the locality's latitude ϕ and its distance d due east or west of the prime meridian, then the length of the equatorial arc between the prime meridian and the local meridian is equal to $d/\cos\phi$. So the local longitude in degrees can be determined from the following proportion:

$$\text{longitude} = \frac{d}{\cos\phi} \cdot \frac{360°}{C_E}$$

Since accurate values of longitude are in practice quite difficult to determine, Sanskrit texts may employ simple approximations for Δt based on estimates of d.

Ascensions. A fundamental concept in Indian astronomy is that of "rising" or "ascension." Qualitatively, mean time is related to the amount of arc of the celestial equator that rises above the observer's horizon in a given interval. As illustrated in Figure 2.7, ascension is the time interval or equatorial arc corresponding to the rising of a specified arc of the ecliptic above the horizon at a specified terrestrial latitude ϕ. Since the inclination of the ecliptic to the equator causes ascensions to differ for different arcs of the ecliptic, the longitudes used in computing them are tropical, i.e., measured eastward from the vernal equinox or ecliptic-equator intersection rather than from the ecliptic's sidereal zero-point.

Amounts of ascension at the terrestrial equator, where the celestial equator intersects the local horizon perpendicularly, are called *laṅkodaya* "rising at Laṅkā"

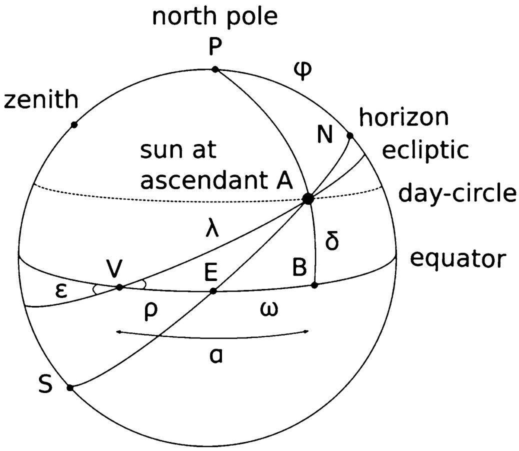

Fig. 2.7 The eastern half of the celestial sphere at sunrise on a summer day at northern latitude $\phi = \widehat{PN}$, showing the cardinal points S, E, and N on the local horizon and the vernal equinox V at the intersection (with obliquity ε) of the celestial equator and ecliptic. The rising sun at the ascendant point A has longitude $\lambda = \widehat{VA}$ and declination $\delta = \widehat{AB}$. The sun's right ascension $\alpha = \widehat{VB}$ is the sum of its oblique ascension $\rho = \widehat{VE}$ and the ascensional difference $\omega = \widehat{EB}$.

(i.e., where latitude is zero). In modern terminology they are known as "right" ascensions and denoted α. At localities with nonzero latitude, the celestial equator intersects the horizon at an angle equal to the complement $\bar{\phi}$ of the local latitude, and the corresponding "oblique" ascensions are more broadly called *udaya* "rising" (denoted ρ). The point of intersection between the ecliptic and the eastern horizon at any given moment is called the ascendant or horoscope-point (*lagna* "intersection"). Right or oblique ascension intervals themselves may be called *lagna*, as they represent the amount of time elapsed between two specified ascendants.

Right ascension depends trigonometrically on tropical longitude λ, the ecliptic obliquity ε, and the ecliptic declination δ as follows:

$$\sin\alpha = \frac{\sin\lambda\ \cos\varepsilon}{\cos\delta}$$

This declination (*krānti* "step," denoted δ) is the angular distance in degrees between the equator and a point of tropical longitude λ on the ecliptic, measured on a great circle passing through the poles of the equator. Its maximum value equals the ecliptic obliquity ε, which in Sanskrit *jyotiṣa* has the standard value 24°. A declination arc north of the equator is conventionally treated as positive, and a southern declination as negative. The trigonometric rules used to compute it are variants of the following equation:

$$\sin\delta = \sin\varepsilon \cdot \sin\lambda$$

The radius of a "day-circle" or small circle lying parallel to the equator between the equator and a tropic is proportional to $\cos\delta$.

Standard intervals of right ascension for the first three zodiacal signs are assigned the values 278, 299, and 323 *vighaṭīs* respectively. Note that they sum to 900 or equivalently 15 *ghaṭīs*, one quadrant of the celestial equator, for the entire first quadrant of the ecliptic.[6] By symmetry, the corresponding values of α for the three signs in the third quadrant are also {278, 299, 323}, while those for the signs in the second and fourth quadrants are {323, 299, 278}.

Corresponding intervals of oblique ascension for each zodiacal sign or some subdivision thereof are useful for telling the time at night at the observer's latitude, as well as for astrological procedures. For any given terrestrial latitude ϕ and solar declination δ, the difference between right and oblique ascension is known as *cara* "variation" or "ascensional difference," denoted ω. In Sanskrit texts the formulas for ω reduce to the following general equation or some approximation of it:

$$\sin\omega = \tan\phi\ \tan\delta$$

A desired oblique ascension corresponding to a given right ascension is easily computed as $\rho = \alpha \pm \omega$ (subtracted for signs in the first and fourth quadrants, added in the second and third). When converted to time-units the ascensional difference equals the difference between the length of half the current day and half an equinoctial day or fifteen *ghaṭikās* (6 hours). This property is reflected in the other common modern name of ω, "half-equation of daylight."

The standard ascensional differences for the first three zodiacal signs are known as their *carakhaṇḍas*, denoted ω_1, ω_2, ω_3. Since, as noted above, $\tan\phi$ is just the local noon equinoctial shadow s_0 divided by 12, each *carakhaṇḍa* can be approximated as the product of s_0 with some scaled coefficient representing $\tan\delta$ for the end of that sign.[7] Intermediate ascension values are generally computed by linear interpolation among the twelve symmetric values at the zodiacal sign boundaries.

Time corrections. As explained above, at localities with zero terrestrial latitude the celestial equator is perpendicular to the horizon, whereas at northern or southern latitudes these two great circles intersect at an oblique angle. This inclination produces the ascensional differences or discrepancies between the zero-latitude right ascension α and corresponding oblique ascension ρ, or local rising time, for a given arc of the ecliptic. The time interval between sunrise

[6]The three standard values of α, generally presented in versified form, appear as early as the sixth-century *Pañcasiddhāntikā* (4.23–25 and 29–31). The *Khaṇḍakhādyaka*'s twelfth-century commentator Āmarāja derives them using the following rule equivalent to the above equation for $\sin\alpha$ (Misra 1925, pp. 91–93):

$$\sin\alpha = \frac{\sqrt{\sin^2\lambda - \sin^2\delta}}{\cos\delta}$$

[7]*Karaṇakutūhala* 2.18 and *Grahalāghava* 2.5 use coefficients of 10, 8 and 10/3, respectively, while *Khaṇḍakhādyaka* 3.1 adjusts the first two to 159/16 and 65/8.

and sunset on a given day at a given northern or southern locality corresponds approximately to the total oblique ascension of the half-circle of the ecliptic beginning with the sun's current position. Consequently, this duration of daylight becomes longer or shorter than 12 h (or thirty *ghaṭīs*) by some amount depending on the time of year and the terrestrial latitude.

Half of the difference between the current day-length and thirty *ghaṭīs* is conventionally called the "half-equation of daylight" or ω, approximately equal to the total ascensional difference of the 90° arc of the ecliptic that began rising at sunrise on that day. (These equalities are only approximate because the sun moves about 1° on the ecliptic over the course of a day.)

The resulting change in the time of sunrise for a non-equinoctial day forms part of a set of required corrections which are frequently tabulated together.[8] These corrections are necessitated by the fact that celestial phenomena are computed with reference to mean solar time at zero terrestrial latitude and longitude, while local time is typically based on true solar position as viewed from a place with nonzero latitude and longitude. They consist of the following components:

deśāntara "place-difference," corresponding to the time-degrees between the zero-meridian of terrestrial longitude and the meridian of the given locality (see Section 4.4).

udayāntara "rising-difference," relating to the discrepancy between an arc of mean solar longitude on the ecliptic and its right ascension or actual time required for its rising.

bhujāntara " [positional] arc-difference," or right ascension of the solar equation, i.e., the longitudinal difference between the mean sun and the true sun.

cara "variable," the half-equation of daylight or time difference between sunrise on the equator and sunrise at the latitude of the given locality.

Each of these components generates a time interval during which the celestial bodies move from their computed positions. The amount of their motion must be calculated and applied to their computed positions to determine their actual positions at the true local time in question.

The combination of the *udayāntara* and the *bhujāntara* is sometimes called the *dvisaṃskṛti* or "two corrections," corresponding to the "equation of time" in western astronomy. The *dvisaṃskṛti* combined with the *cara* is the *traikya* or "sum of three"; the addition of the *deśāntara* produces the *catuṣṭayaikya* or "sum of four" (Pingree 1973, pp. 107, 111).

[8] Examples of such combined tables of corrections are described in Section 4.6; others appear in, e.g., the *Kheṭasiddhi*, the *Grahaprakāśa* of Devadatta, and the *Makaranda* (Pingree 1973, pp. 107, 147–148, 173).

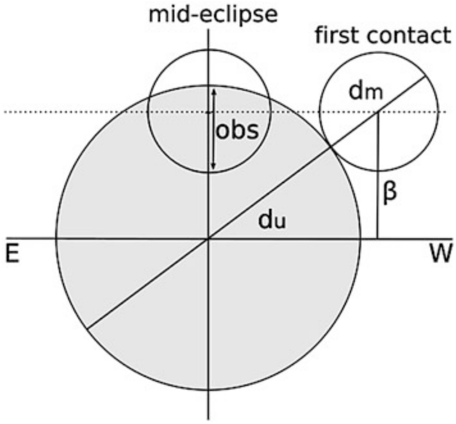

Fig. 2.8 The disk of the moon in a lunar eclipse moving west to east on its orbit (approximated by the dotted line parallel to the ecliptic) with approximately constant latitude β: at first contact with the earth's shadow (right), and at the eclipse midpoint (left). The amount of obscuration at mid-eclipse is $(d_m + d_u)/2 - \beta$.

2.1.6 Eclipses

The prediction and description of eclipses requires most of the previously discussed techniques for establishing true celestial longitudes and times, as well as the additional ones described below (refer to Figure 2.8).

Lunar latitude. Ecliptic latitude β is the angular distance from the ecliptic to a given point on the celestial sphere, measured on the great circle passing through that point and the poles of the ecliptic. The moon's position on its own orbit has a nonzero latitude everywhere except at the two points of intersection of the orbit and the ecliptic, i.e., the orbital nodes, called in *jyotiṣa* works Rāhu and Ketu. The trigonometric derivation of lunar latitude β is analogous to that for solar declination, producing the following formula:

$$\sin\beta = \sin\beta_{\text{max}} \cdot \sin(\Delta\lambda)$$

where $\Delta\lambda$ is the lunar-nodal elongation, i.e., the angular distance measured along the ecliptic from the moon's longitude to the nearest node. The maximum latitude βmax is the inclination of the lunar orbit to the ecliptic, commonly taken to be $4°;30$ or 90 digits (*aṅgulas*).[9]

Disk sizes and obscuration. The apparent sizes of the disks of the eclipsed and eclipsing bodies, determined by the sun's and moon's distances from the earth and consequently their apparent angular velocities v_s and v_m, are important because

[9]Latitude expressed in digits rather than degrees can be directly compared with apparent diameters to determine quantities such as the magnitude of an eclipse. There is not complete unanimity in the sources on digit/angle conversion ratios: one digit is equated to three arcminutes in, e.g., the *Karaṇakutūhala* of Bhāskara (Mishra 1991, pp. 51–52), but other authors vary the amount from $2.5'$ on the horizon to $3.5'$ on the meridian (Chatterjee 1981, vol. 2, pp. 127–128).

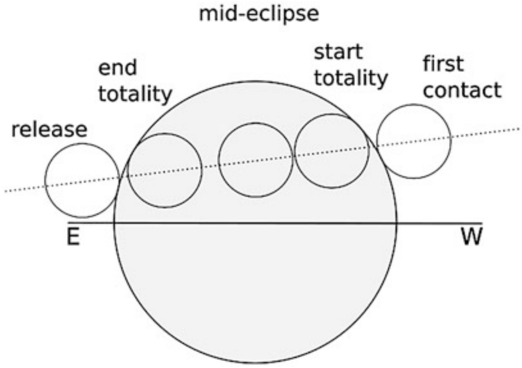

Fig. 2.9 Succession of phases (right to left) in a lunar eclipse.

they determine the magnitude or obscuration extent of an eclipse. A desired disk diameter d can be computed trigonometrically from the known orbital parameters and sizes of the sun and moon, but is usually more simply approximated by a scale factor applied to the value of true velocity. The following examples of such scale factors are taken from *Brāhmasphuṭasiddhānta* 4.6 (using the subscripts s, m, and u to denote sun, moon, and shadow, respectively):

$$d_s = \frac{11v_s}{20}, \qquad d_m = \frac{10v_m}{247}, \qquad d_u = \frac{8v_m - 25v_s}{60}$$

The mean angular diameters given by all such formulas are approximately $32'$ for the sun and moon and $81'$ for the earth's shadow. The amount of obscuration at mid-eclipse is computed by simple geometry (treating the disk diameters and latitude as line segments) as half the sum of the diameters of the eclipsing and eclipsed bodies, diminished by the lunar latitude.

Durations. In the course of an eclipse (see Figure 2.9), the half-duration (*sthityardha*, commonly abbreviated to *sthiti*) is the time elapsed from the moment of first contact between the disks of the eclipsed and eclipsing bodies to mid-eclipse, or from mid-eclipse to release. The half-duration of totality (*marda*) is the time elapsed from the moment of totality to mid-eclipse, or from mid-eclipse to the end of totality. Both of these quantities are likewise found from simple linear geometry models applied to the eclipse configuration. If $r_{\bullet}$ and $r_{\bigcirc}$ denote the radii of the disks of the eclipsing and eclipsed bodies, respectively, and v_m and v_s are again the true lunar and solar velocities, then the half-durations are computed as follows:

$$sthiti = \frac{\sqrt{(r_{\bullet} + r_{\bigcirc})^2 - \beta^2}}{v_m - v_s}, \qquad marda = \frac{\sqrt{(r_{\bullet} - r_{\bigcirc})^2 - \beta^2}}{v_m - v_s}$$

The *valana*. An important feature in traditional eclipse description is the angle of deflection related to the impact direction at the various eclipse phases. This angle,

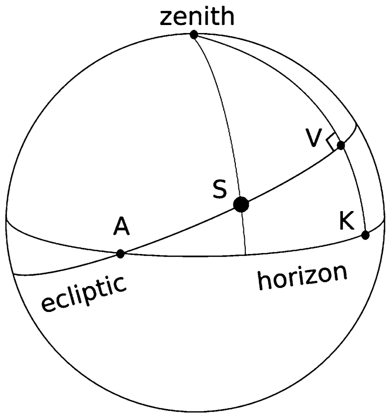

Fig. 2.10 The celestial sphere and the local horizon, showing the ascendant point A, the nonagesimal V, and the sun S at the approximate conjunction point where the moon's parallax must be calculated. The elongation $\Delta\lambda = \widehat{SV}$ determines the amount of longitudinal parallax *lambana*, and the nonagesimal altitude $\widehat{VK}$ (or some combination of declination δ and latitude ϕ approximating it) affects the latitudinal parallax component *nati*.

called *valana* "turning, deflection," is divided into two components: the *akṣavalana*, deflection due to local latitude, and the *ayanavalana*, deflection due to tropical solar longitude. The individual components are computed by some form of the following algorithms:

$$\sin(\textit{akṣavalana}) = \sin(90^\circ - \zeta) \cdot \sin\phi$$

where ζ is the zenith distance and ϕ is the terrestrial latitude, and

$$\sin(\textit{ayanavalana}) = \cos\lambda \cdot \sin 24^\circ$$

where λ is tropical solar longitude. The algebraic sum of the two components then becomes the total deflection. This angle is important for constructing diagrams (*parilekha*) of the predicted appearance of eclipses, something like Figure 2.9.

Parallax. The calculation of solar eclipses requires the additional task of finding the moon's parallax, which determines the eclipse's local visibility. Lunar parallax is treated in many astronomical traditions as a function of the moon's altitude: it becomes zero with the moon at the zenith and maximum when on the horizon. But in the Sanskrit literature it is typically separated into two individually (although not entirely independently) computed components, which are conventionally called longitudinal and latitudinal (see Figure 2.10).

Longitudinal parallax (*lambana*) is generally treated as a correction to the time of the eclipse rather than its celestial coordinates, so it is typically expressed in units of *ghaṭīs* with a standard maximum of 4 *ghaṭīs*, the time it takes the moon at mean velocity to travel about $53'$. The *lambana* depends upon the ecliptic arc of elongation $\Delta\lambda$ between the sun and the nonagesimal, the point on the ecliptic 90° west of the ascendant. (Although it is the moon rather than the sun whose parallax is sought, the slower-moving sun is used to identify the celestial position where the eclipsing conjunction will take place. Rules for solar parallax too may be prescribed in eclipse procedures, but will have negligible effects on the result.)

The *lambana* varies sinusoidally from zero to its 4-*ghaṭī* maximum as follows:

$$lambana = 4 \cdot \sin(\Delta\lambda)$$

This quantity may be adjusted by a scale factor corresponding to the altitude of the nonagesimal itself. That is, when the ecliptic hangs low in the southern sky and the nonagesimal altitude is small, the amount of sun-nonagesimal elongation $\Delta\lambda$ will not greatly affect the amount of parallax, whereas when the nonagesimal is at or near the zenith, changing $\Delta\lambda$ changes the parallax considerably.

The latitudinal parallax (*nati*), on the other hand, accounts directly for the effects of terrestrial latitude and declination on the altitude of the nonagesimal point and whether it falls south or north of the zenith. Sometimes the nonagesimal δ and ϕ are just combined arithmetically in *nati* formulas, and sometimes the zenith distance of the nonagesimal, which combines them in a more trigonometrically exact way, is used instead. The *nati* is usually expressed in digits and applied as a correction to the lunar latitude β.[10]

The computation of the position and time of an eclipse and their modification by parallax is typically prescribed as an iterated process. Table texts may include a brief summary of its steps, see, e.g., Montelle and Plofker (2013, pp. 43–48).

2.1.7 The *mahāpātas* or luni-solar symmetries

The ominous events sometimes called "parallel aspects" of the luminaries (*pātas* or *mahāpātas*) depend upon certain configurations of their tropical longitudes and declinations. In their simplest form, these events require the tropical longitudes of the moon and sun to add up to either 180° or 360°. This means that the bodies will be symmetrically placed equidistant from a solstice (with ecliptic declinations equal in magnitude and sign: *vyatipāta*) or from an equinox (with ecliptic declinations of equal magnitude but opposite sign: *vaidhṛti*), respectively, see Figure 2.11.

In this form, the difference between a given time and the occurrence of the nearest *mahāpāta* is quite simple to compute: it is the difference between 180° or 360° and the sum of the luminaries' tropical longitudes, divided by the sum of their

[10] Computation techniques for the parallax components are discussed in, e.g., Rao and Uma (2007, S205–S213), Śāstrī (1989, pp. 126–134, verses 5.1–14), Montelle and Plofker (2013, pp. 39–45), and Misra et al. (2016, pp. 28–30).

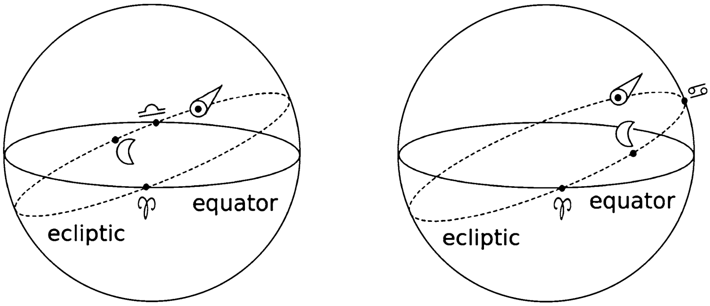

Fig. 2.11 Configuration of the "mean" *mahāpātas* where the luminaries' ecliptic declinations have equal magnitude. Left: An occurrence of *vaidhṛti* with the sun and moon symmetrically placed about the autumnal equinox, so that their tropical longitudes sum to 360°. Right: An occurrence of *vyatipāta* with the sun and moon located at equal distances from the summer solstice, with longitudes summing to 180°.

velocities. This will be the time interval required for their combined motion to bring their combined longitudes up to that ominous round number. Then the predicted individual longitude of each body at the moment of *pāta* is found by multiplying the time interval by its own velocity and adding the resulting increment to its own current longitude.

The computation can be further simplified by employing the *yoga* time-unit, which represents a longitudinal increment of 13° 20′ in combined luni-solar motion (see Section 1.4.3). The tropical longitudes of sun and moon add up to zero at the beginning of the *yoga*-cycle, and add up to 180° or 360° at the desired *vyatipāta* or *vaidhṛti* respectively. Since a body's tropical longitude is merely its sidereal longitude λ plus degrees of accumulated precession x, the conditions at the *pātas* can be described by

$$180^\circ = \lambda_\odot + \lambda_☾ + 2x \quad (\textit{vyatipāta}), \qquad 360^\circ = \lambda_\odot + \lambda_☾ + 2x \quad (\textit{vaidhṛti}).$$

The number n of *yogas* elapsed at this moment is computed for the case of *vyatipāta* as shown:

$$n = \frac{\lambda_\odot + \lambda_☾}{13^\circ 20'} = \frac{180^\circ - 2x}{13^\circ 20'} = 13;30 - 0;9x.$$

The corresponding computation for *vaidhṛti* gives $n = 27 - 0;9x$. In either case, when the fractional excess over integer *yogas* in n is converted to a time interval in *ghaṭīs*, it is added to the time of the beginning of the current *yoga* to indicate the moment of *mahāpāta*.

In most texts, the *pātas* are restricted by the more complicated requirement of equality between the sun's ecliptic declination and the moon's "true" declination, generally approximated by the algebraic sum of its ecliptic declination and its latitude. As explained in Section 2.1.6, this nonzero latitude is produced by the

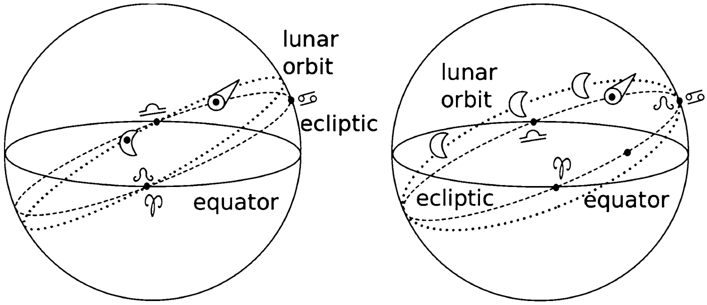

Fig. 2.12 Configuration of the "true" *mahāpātas* where the luminaries' true declinations have equal magnitude. Left: An occurrence of *vaidhṛti* asymmetrically configured about the autumnal equinox with the ascending node ☊ of the lunar orbit at the vernal equinox: the true declination of the moon on its orbit has equal magnitude with that of the sun on the ecliptic, but their longitudes do not add up to 360°. Right: A configuration with the ascending node at the summer solstice, where a "true" *vaidhṛti* about the autumnal equinox is impossible no matter where in the surrounding two quadrants the moon is located.

moon's motion upon its own orbit inclined about 4°;30 to the ecliptic, whose nodes revolve upon the ecliptic at a rate of about 18° per year. The luminaries' positions at the *pāta* so defined will have a much less straightforward relationship to the sum of their longitudes, see the left-hand diagram in Figure 2.12 for an illustration of this.

It appears that starting very early in the development of the *mahāpāta* concept, the difference between the time t_1 when the ecliptic declinations are of equal magnitude and the moment of true-declination equality t was determined by a *regula falsi* iterative computation. (The procedure outlined here is specified for *vyatipāta*, but is exactly analogous to its counterpart for *vaidhṛti*.)

Namely, after calculating the initial declinations δ_{s_1} and δ_{m_1} of the sun and moon, respectively for time t_1, the user determines from comparing their magnitudes whether the true-declination *mahāpāta* is past or yet to occur. Depending on the answer, some arbitrary time interval Δt_2 is chosen before or after t_1, respectively, and new positions of the sun and moon and their corresponding true declinations δ_{s_2} and δ_{m_2} are calculated for time $t_2 = t_1 \mp \Delta t_2$. Then the next approximation Δt_3 to the required time interval is given by the following ratio:

$$\Delta t_3 = \frac{\Delta t_2 \cdot (\delta_{m_1} - \delta_{s_1})}{(\delta_{m_2} - \delta_{s_2}) \pm (\delta_{m_1} - \delta_{s_1})}.$$

(The sign in the denominator is positive or negative depending on whether the *mahāpāta* occurs within the assumed time interval Δt_2 or outside it.) The procedure is repeated until Δt is fixed, identifying the moment of the true-declination *mahāpāta*.

There are some circumstances, however, in which such an event is astronomically impossible. As illustrated in the right-hand diagram in Figure 2.12, if the nodes of the lunar orbit are placed so that throughout two successive quadrants the moon's true declination is always greater in magnitude than the sun's, the luminaries will never attain a *mahāpāta* about the solstitial or equinoctial colure between them.

2.1.8 Trigonometry

As briefly noted in Section 1.2.2, *jyotiṣa* treatises almost always include compact versified tables of trigonometric function values at the beginning of their chapters on computing true positions and velocities of the planets. The primary trigonometric function is the sine scaled to some non-unity canonical radius (Radius or R), denoted here by $R\sin$ and generally called *jyā* in Sanskrit. Trigonometric calculations also may involve the corresponding scaled cosine $R\cos$, generally just treated as the $R\sin$ of the complement, and versine $R\,\text{vers}$ (*utkramajyā*) defined for some angle θ by $R\,\text{vers}\,\theta = R - R\cos\theta$. Versified tables most frequently list values of $R\sin$ for θ in the first quadrant or else the differences between successive $R\sin$ values, which are smaller and thus easier to encode verbally as well as being directly applicable to interpolation procedures. Occasionally a table of $R\,\text{vers}$ or its differences (which are just the $R\sin$ differences in reverse order) will also be included.

These tables employ various values of the trigonometric Radius and function step size depending on computational convenience. The most nearly "standard" Radius value is 3438, which approximately equals 21,600 (the number of arcminutes in the circumference) divided by π, and thus equates linear measure and arclength in much the same way as modern radians. Likewise, it is more or less "standard" to divide the quadrant of the argument θ into 24 multiples of 3°;45. The numerous exceptions to these conventions include the idiosyncratic *laghu* "light" or "simplified" $R\sin$ tables, especially popular in *karaṇa* texts, which use only a few values of θ and a smaller round number as Radius.[11]

The construction of $R\sin$ tables was based on geometry of triangles and rectangles, and in later works on an accumulated set of trigonometric identity formulas. Numerous approximation methods were also employed to compute tabulated values, to interpolate between them, or to avoid using them altogether. By the early seventh century Indian astronomers had already developed a highly accurate algebraic approximation to the $R\sin$, and approximate versions of many trigonometric

[11] The first securely dated surviving Sanskrit trigonometric table, a list of 24 $R\sin$-differences with $R = 3438$, appears in the *Āryabhaṭīya* circa 500 CE (Shukla 1976, p. 41, *daśagītikā*, verse 12). However, an $R\sin$ table in the sixth-century *Pañcasiddhāntikā* with $R = 120$ likely originated in an earlier Sanskrit work, and may even have been ultimately inspired by a Greek table of chords with $R = 60$ (Neugebauer and Pingree 1970–1971, part II, pp. 52–55, verses 4.6–12). The *Brāhmasphuṭasiddhānta* of 628 tabulates both $R\sin$ and $R\,\text{vers}$ values scaled to $R = 3270$, a number chosen for reasons still not clearly understood (Dvivedī 1901–1902, pp. 23–24, verses 2.2–9); the same author's *karaṇa* text *Khaṇḍakhādyaka* employs a *laghu* $R\sin$ table with $R = 150$ and step size 15° (Chatterjee 1970, vol. 1, p. 59, verse 3.6). The twelfth-century *Siddhāntaśiromaṇi* of Bhāskara II contains "standard" tables of $R\sin$ and $R\,\text{vers}$ with $R = 3438$ as well as a *laghu* table of $R\sin$-differences, repeated in Bhāskara's handbook *Karaṇakutūhala*, using $R = 120$ and step size 10° (Śāstrī 1989, pp. 39–41, verses 2.2–9; Mishra 1991, p. 21, verses 2.6–7). Perhaps the most extreme example of a *laghu* table is the set of three R sines corresponding to the angles 30°, 60° and 90° in the tenth-century *Laghumānasa* of Mañjula/Muñjala, with some prescribed interpolation modifications to increase the accuracy of intervening values (Shukla 1990, p. 65, verse 3.2).

rules for specific astronomical quantities accumulated over subsequent centuries in astronomers' computational toolkit. The 904 treatise of Vaṭeśvara emphasizes the desirability of being able to solve various astronomical trigonometry problems with or without tabulated function values (Shukla 1986, vol. II, pp. 272–273, verses 2.3–4,10). Gaṇeśa's boast some six centuries later (quoted in Section 5.6.1), about his having entirely eliminated the arc and sine in favor of non-trigonometric formulas in his handbook *Grahalāghava*, indicates how burdensome many *jyotiṣīs* considered these trig-table calculations to be.[12]

This multi-pronged approach to trigonometric problems may partly explain why Sanskrit table texts, unlike those in Greek and Islamic astronomy, typically do not contain tables of basic trigonometric functions. The Indian works tabulate instead more complicated specialized functions for particular astronomical quantities, in which the necessary trigonometry or equivalent approximations are already incorporated. However, there do exist a handful of trigonometric tables in the table-text corpus, primarily in works with explicit dependence on Islamic sources. Their values are usually more numerous (generally stated for every 1° of argument) and more precise than those in the traditional verbal R sin or R vers tables which have to be composed in metrical verse by the author and then, ideally, memorized by the learner. See the discussion and examples in Section 4.12.

Most such numeric-array trigonometric tables contain values of R sin, occasionally of R vers. The tangent is not treated as a basic trigonometric function in classical Indian *jyotiṣa*, although the so-called shadow relation (*chāyā*; see Section 2.1.5), in the form of the noon equinoctial shadow equivalent to the tangent of the terrestrial latitude, is sometimes tabulated in geographical lists.[13]

2.2 Approximation of numerical values in table entries

Once a table-text author has chosen some algorithm to generate a table of some astronomical quantity, there remains the issue of how to select, denote, and use the

[12] Geometric construction of R sin values is discussed in detail in Bhāskara I's seventh-century commentary on the *Āryabhaṭīya* (Shukla 1976, pp. 77–85). Recursive procedures for producing them without geometry are also known (though not always clearly stated) (Plofker 2009, pp. 79–80, 128; Hayashi 1997a). For the widespread algebraic R sin approximation also ascribed to Bhāskara I and for various higher-order interpolation rules, see Plofker (2009, pp. 80–84), Hayashi (1991), and Gupta (1969). The use of second-order interpolation in finding R sin values is illustrated in, e.g., Shukla (1986, vol. 2, pp. 170–172, verse 2.1.63). Some complicated approximations tailored to a specific astronomical quantity in the work of Bhāskara II are discussed in Plofker (2016).

[13] The Indian "shadow" in fact appears to have been the inspiration for, and to have lent its name to, the generalized tangent function in Islamic trigonometry; see Chalhoub and Rosenfeld (1997). Such general functions for tangent and cotangent (under the names "shadow" and "reversed shadow," derived respectively from the computations for a horizontal and a vertical gnomon) are occasionally found in Sanskrit trigonometric tables influenced by Islamic sources; see, e.g., Pingree (2003, p. 102).

particular values that will appear in the table. These questions generally involve techniques for interpolating between values of a nonlinear function or the precision with which the values are recorded.

2.2.1 Interpolation procedures

Authors of both versified and numeric-array tables often accompany them by instructions for linearly interpolating between their entries. Technical terms for the elements in the interpolation formula include *gata*, *bhukta* "past, previous" for the table entry preceding the desired value, and *eṣya*, *gamya*, *bhogya* "future, next" for the entry following it. The standard linear interpolation procedure is equivalent to the following relation, given two successive tabulated argument values A_{bhukta} and A_{bhogya} and a desired argument A falling between them:

$$\text{desired amount} = bhukta \pm \frac{bhogya - bhukta}{A_{bhogya} - A_{bhukta}} \cdot (A - A_{bhukta})$$

The following diverse examples from both handbooks and table texts illustrate how such instructions are typically presented.[14]

***Khaṇḍakhādyaka* of Brahmagupta.** In a table of R sin-differences for 15° intervals of arc with trigonometric radius $R = 150$ (Misra 1925, p. 98, verse 3.7):

> *triṃśatsanavarasendujinatithiviṣayā gṛhārdhacāpānāṃ*
> *ardhajyākhaṇḍāni jyā bhuktaikyaṃ sabhogyaphalam* || 7 ||

> Thirty increased by nine, six, [and] one [respectively], twenty-four, fifteen, five are the R sin-differences of half-sign (i.e., 15°) arcs [namely, 39, 36, 31, 24, 15, 5]. The sum of the past (*bhukta*) R sin[-differences] plus the proportional part (*phala*) of the next (*bhogya*) [is the desired] R sin.

Tabulating the successive differences of the function values instead of the values themselves not only reduces the length of the number-words spelled out in the verse, but also saves the user the trouble of performing the subtraction of the *bhukta* from the *bhogya*.

***Brahmatulyasāraṇī*.** In a table of orbital equations with successive degrees of anomaly (*kendra*) as the argument (Montelle and Plofker 2015, pp. 10–11, verse 4):

> *kendrasya doraṃśamitiś ca koṣṭe*
> *bhuktaṃ tadagraṃ parabhogyakaṃ ca* ||
> *kalādikaṃ tadvivarāhataṃ tu*
> *ṣaṣṭyuddhṛtaṃ bhuktakamānakena* || 4 ||

[14] Although the higher-order interpolation rules mentioned in footnote 12 are well attested in *jyotiṣa* treatises, we have never seen a nonlinear interpolation method prescribed in the instructions for using a table text.

> And the amount of degrees of the [given] arc of anomaly [is] the *bhukta* in the table cell before that [arc], and the *bhogya* [is] next. The [fractional part of the given anomaly in] minutes etc. is multiplied by their difference, divided by sixty, [and then increased] by the amount of the *bhukta*.

This formulation, taking $A - A_{bhukta}$ as just the fractional degrees of the given argument A, is specific to tables with argument given for successive integer degrees.

***Candrārkī* of Dinakara.** In a table of lunar equations and accompanying differences between successive table entries, with argument successive degrees of anomaly. This rule takes into account the possibility that the sequence of tabulated values may be decreasing instead of increasing (Kolachana et al. 2018, p. 334, verse 16):

> *kendrarāśyaṃśamānena phalaṃ grahyaṃ ca sāntaram* ||
> *antaraghnaṃ kalādyaṃ ca ṣaṣṭyāptena yutonitam* || 16 ||

> The equation [whose argument is commensurate] with the amount of signs and degrees of the [given] anomaly is to be selected [from the table] along with [its] difference. The [fraction of the given anomaly in] minutes etc., multiplied by the difference and divided by 60, is added to or subtracted from [the *bhukta*-value as the successive entries are increasing or decreasing, respectively].

***Rājamṛgāṅka* of Bhojarāja.** A general interpolation rule is given for tables with entries for every 2° of argument rather than for every successive degree (Pingree 1987b, p. 8, verses 2.29–30ab):

> *bhuktaśuddhāṃśakoṣṭhas tu tadgataiṣyāntarāhatam*||
> *kalādyaṃ vikalpaṃ bhaktaṃ dvighnaṣaṣṭyā phalānvitam* || 29 ||
> *gatakoṣṭhaphalaṃ bhogye vṛddhe hīne tadūnitam* ||

> The [given] table-cell [argument has] the degrees of the *bhukta* [argument] subtracted. [This] difference in arcminutes etc. is multiplied by the difference of [the table entries for the arguments] preceding and following that, [and] divided by twice sixty. The previous table entry is increased by the result when the following [table entry] is greater [than itself], decreased by it when less.

This applies to tables of solar declination, planetary hypotenuse, and *śīghra*-equation all tabulated in the *Rājamṛgāṅka* with 2° argument intervals.

Linear interpolation was employed not only by table users to obtain intermediate results within tables, but also sometimes by their compilers for convenience in generating some of the table entries. Figure 2.13 illustrates this practice in a table for Mars's *manda*-equation with argument 1–90° of anomaly from a version of the *Grahalāghavasāriṇī* (see Section 5.6.2). The table entries at every integer 15° of argument are taken from the trigonometrically correct values of this nonlinear function given in the *Grahalāghava* of Gaṇeśa (Jośī 1994, p. 79, verses 3.7–8). But the entries in between these six key values have been produced by linear interpolation, with constant differences between the successive entries in each 15° block.[15]

[15]Other instances seen in our sample tables of using linear interpolation to generate nonlinear function values include the orbital equations in Figures 4.9, 4.15, and 5.6, the oblique ascensions

Fig. 2.13 Excerpt from a manuscript of the *Grahalāghavasāriṇī* (Plofker 24 ff. 24r–25r) showing part of the table of the nonlinear *manda*-equation function for Mars, with most of the entries computed by linear interpolation.

2.2.2 Precision of data values

The precision of numerical values in versified tables composed as part of Sanskrit verse texts is obviously fixed by the author's choice of meter and words. When it comes to compiling or copying numeric-array written tables, on the other hand, much more variability is possible. The issue of precision in computing and recording results is not generally discussed in detail in *siddhāntas* and *karaṇas*, although a

in Figures 4.23 and 4.27, the rising-difference corrections in Figure 4.28, the declinations and lunar latitudes in Figures 4.34 and 4.47, and the *valana* in Figures 4.40, 4.41, and 4.42.

basic understanding of the concept is clearly marked by the contrast of terms such as *sthūla* "rough" for an imprecise value with, e.g., *sūkṣma* "fine, exact" for a precise one.[16]

An example of computation techniques directly concerned with precision is seen in the solar daily mean longitude increment formulas of Bhāskara II's *Karaṇakutūhala*. The first is stated in the following verse (Mishra 1991, p. 5, verse 1.7ab):

ahargaṇo viśvaguṇas trikhāṅkair
bhaktaḥ phalono dyugaṇo lavādyāḥ

The accumulated civil days are multiplied by 13 and divided by 903. [Then] the accumulated civil days decreased by [that] result produces the [solar mean daily motion] in degrees and so on.

In other words, if the accumulated days or *ahargaṇa* is represented by a, the rule is equivalent to

$$a - \frac{13 \cdot a}{903} = \frac{890}{903} \cdot a \qquad \text{where} \qquad \frac{890}{903} = 0;59,8,10,21,55,\ldots$$

There is no specification of the precision to be used when converting that ratio into a sexagesimal fraction approximation in "degrees and so on," but several verses later the same parameter is stated more directly with precision to arcseconds, as follows (Mishra 1991, p. 11, verse 1.13a):

nandākṣā bhujagā raveḥ ...

[The mean daily velocity] of the sun is 59;8 [minutes]...

For planets besides the sun, the *Karaṇakutūhala* gives some additional correction terms in units of arcseconds, which conforms to the common practice of treating arcseconds as a useful precision limit.[17]

The *Karaṇakutūhala*'s treatment of mean longitude increments has been variously interpreted by the compilers of table texts derived from it (see Section 5.3.1). Figure 2.14 shows mean longitude increment values of the sun as presented in a version of the *Laghukhecarasiddhi*. The daily value of the increment in the fifth column is precise to arcseconds ($0^s 0°;59,8$). But reconstructing the computation indicates that the number actually used to produce these values must have been $0^s 0°;59,8,10,22$, presumably derived from the first *Karaṇakutūhala* rule rounded at the fourth sexagesimal place.

[16]Note that such terminology can also indicate a distinction between different levels of accuracy rather than precision. Examples of the terms *sthūla* and *sūkṣma* used to differentiate between approximate and geometrically exact formulas are seen in, e.g., the *Brāhmasphuṭasiddhānta* (Dvivedī 1901–1902, p. 189, verse 12.21) and the *Līlāvatī* (Āpaṭe 1937, pp. 197, 203, verses 199, 203).

[17]See Plofker (Forthcoming). Gaṇeśa in the *Tithicintāmaṇi* follows a similar practice of breaking out such corrections into separately (and perhaps optionally) applied components, in which the smaller correction term fine-tunes the result of the larger (Ikeyama and Plofker 2001). This "modularization" of successively finer adjustments to a computed result in Sanskrit astronomy deserves further study.

Fig. 2.14 Mean longitudinal displacement tables from the *Laghukhecarasiddhi* (IO 2408b f. 5v, Sun).

Fig. 2.15 Excerpts from the solar mean daily longitude increment table in manuscripts of the *Brahmatulyasāraṇī*. Left: Argument 0–8 days (Smith Indic 45 f. 2r). Right: Argument 1–10 days (Smith Indic MB LVIII f. 1r).

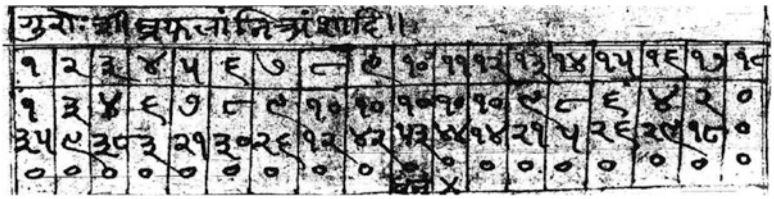

Fig. 2.16 *Śīghra*-equation table from the *Laghukhecarasiddhi* (IO 2930 f. 7v, Jupiter) with an apparent precision to arcthirds that is actually spurious since the final digits are all zero.

Figure 2.15 shows two versions of the solar mean longitude increment table from different manuscripts of the *Brahmatulyasāraṇī*, differing slightly in their precision. Reconstruction of the tabulated values shows that the scribe of the first version, rounding to thirds of arc, appears to have used a number very close to $0°;59,8,10,8$ while the scribe of the second version has used $0°;59,8,10$ and has rounded to the second sexagesimal place.

Not every sexagesimal digit appearing in a table can always be taken for a reliable measure of computational precision, however. For instance, Figure 2.16 shows a table of the *śīghra*-equation of Jupiter from the *Laghukhecarasiddhi*,

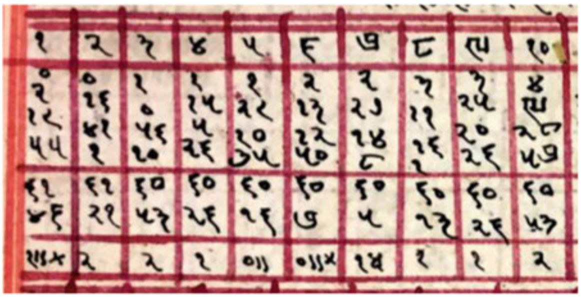

Fig. 2.17 Table of nodal-solar elongation per *avadhi* from a manuscript of the *Karaṇakesarī* (Smith Indic MB XIV f. 4r). The entries in the bottom row approximately equal $1/14$ of the differences between successive entries in the middle row, and use *daṇḍa*-quarters to denote their fractional part in quarter-units.

superficially precise to arcthirds but with all digits in the thirds place recorded (mostly inaccurately) as zero. (See Section 5.3.3 as well as Figure 5.9, which contains the *śīghra*-equations of Venus and Saturn for the same phenomenon.) Perhaps the scribe only intended values precise to two sexagesimal places and simply added a row of zeros for completeness.

The '*daṇḍa*-quarters'. The table reproduced in Figure 2.17 illustrates the scribal convention of strokes we have called "*daṇḍa*-quarters" used as a non-sexagesimal notation for one-fourths of a unit. (Another version of the same table from a different manuscript of the text is shown in Figure 3.5, and many other examples of this notation are enumerated in Section 3.2.1.) This simple "tally-style" notation represents each quarter-unit from 0 to 3 by a single vertical stroke: consequently the numbers in the bottom row of the table are to be read with their sign-change symbols (see Section 3.2.1) as 1;30 $\times$, 2, 2, 1, 0;30, 0;30 $\times$, 1 *dha*, 1, 1, 2. (This reading is also consistent with the apparent generation of the numbers by dividing by 14 the successive differences between the entries in the second row.)

The *daṇḍa*-quarters thus constitute a compromise between precision and convenience, enabling the scribe or compiler to avoid writing out the digits of *nāgarī* sixtieths in a taller column while still not throwing away too much information about the table entry value.[18] This compromise is clearly on view in Figure 2.18 which compares the same table from two different manuscripts of the *Makaranda*, one using *daṇḍa*-quarters in the entries in the second and fourth row and the

[18] The origin and history of this simple notation using vertical strokes to represent multiples of one-quarter-unit appear to be somewhat obscure. There is an obvious resemblance to the now-standard practice of writing a single *daṇḍa* after a quarter-verse and a double *daṇḍa* after a half-verse, but we have not successfully traced the development of that practice from the more irregular use of *daṇḍa*-marks occurring in early inscriptions (Bühler 1904, pp. 88–90). At present, the paleographic study of Sanskrit manuscripts concerning prosody, metrology, and possibly music is our best guess for pursuing the history of this notation.

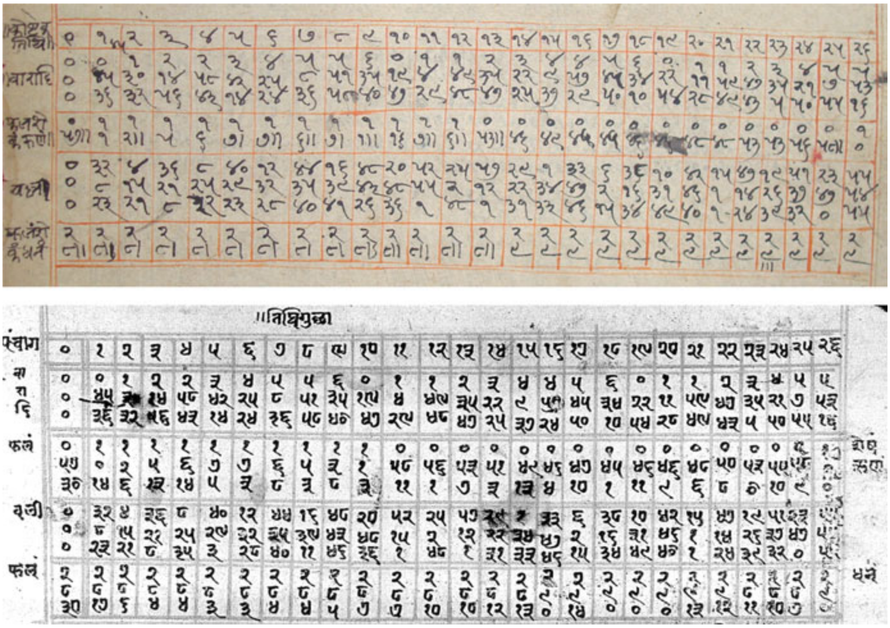

Fig. 2.18 *Tithi*-weekday/time tables for the beginning of each *pakṣa* from two manuscripts of the *Makaranda*. Top: Rows 2 and 4 using *daṇḍa*-quarters (Baroda 3225 f. 2r). Bottom: Rows 2 and 4 without the use of *daṇḍa*-quarters (BORI 546 f. 2v).

other instead writing out the digits of the corresponding sexagesimal place. This difference strongly suggests that *daṇḍa*-quarters were employed at the discretion of the scribe, who of course in many cases was also the user.

2.3 Classification schemata for table texts

After broadly surveying in the previous section the primary content of Sanskrit astronomical tables presented in numeric-array format, we now attempt to clarify their taxonomy. Three table-text classification systems were suggested and employed by David Pingree: in Pingree (1970) he categorized such works by their parameter sets or *pakṣas* and also by their structural "arrangements" (discussed in Section 2.3.2); in Pingree (1981) he also drew distinctions between the "classes" of planetary tables, tables for computing *tithis*, *nakṣatras* and *yogas*, and tables for computing eclipses. Since the purposes and forms of these works are so diverse, and in many cases still so poorly understood, none of these basic classification schemes so far proposed for the genre is entirely successful in categorizing all the texts. Nonetheless, we will describe and to varying extents rely on all of them for making sense of this vast genre of technical literature.

Table 2.3 The major named astronomical table texts belonging to the various schools or *pakṣas*.

Āryapakṣa	(no known major table texts; probably best represented in *vākya* works)
Ārdharātrikapakṣa	(no surviving complete table texts known)
Brāhmapakṣa	*Mahādevī* of Mahādeva (1316, Mahārāṣṭra?); various works of Dinakara (ca. 1580, Bareja, Gujarāt); *Jagadbhūṣaṇa* of Haridatta (1638, Mewar, Rājasthān)
Saurapakṣa	*Makaranda* of Makaranda (1478, Kāśī); *Rāmavinoda* of Rāma (1590, Delhi)
Gaṇeśapakṣa	Various works of Gaṇeśa (ca. 1525–1550, Nandigrāma, Gujarāt); *Grahalāghavasāriṇī* (various versions, based on Gaṇeśa's *Grahalāghava*)

2.3.1 Classification by table type or *pakṣa*

Since table texts are generally compiled based on the algorithms prescribed in some book belonging to one of the major parameter schools or *pakṣas*, it is not surprising that they tend to exhibit the same sort of *pakṣa* affiliation, see Table 2.3 for a list of some prominent exponents of the various *pakṣas* in this genre, and Appendix A for the *pakṣa* identifications of all such works known to us. Caution is warranted, however, since some compilers did not adhere exclusively to any one school. They might combine in their works algorithms taken from books adhering to different *pakṣas*, or even supply multiple tables of the same astronomical function computed for different *pakṣas*. Moreover, there is at least one table text known under the same name in two different recensions conforming to two different *pakṣas*.[19]

The distinction between what we here term "calendric tables" (for computing *pañcāṅga* elements such as *tithis*, *nakṣatras* and *yogas*) and "planetary tables" is also a broadly useful though not consistently reliable one. Although several table texts do contain both calendric and planetary tables, there are enough such works specializing in either one or the other type to indicate that many *jyotiṣīs* found it convenient to deal with calendar production and planetary forecasting in separate "volumes." The same table-text author might compose two separate works with different titles to treat these two topics (see the *Candrārkī* and *Kheṭakasiddhi*, respectively, of Dinakara in Appendix A).

The category of "eclipse tables" is also validated by the eclipse-specific content of several known table texts, as discussed in Section 4.8. But the list of specialized subjects does not end there. For example, there is at least one existing set of tables devoted exclusively to computing the *mahāpātas* (Section 4.11). Individual labelled tables of geographical data, of ascensions, and related functions crop up as separate items in many manuscript collections, inviting the question of what criteria might consistently distinguish a particular "table" from a "table text." The designation

[19]Namely, the *Candrārkī* of Acalajit appears in both Brāhmapakṣa and Saurapakṣa versions. Illustrating the other practices mentioned here are, e.g., the *Śīghrasiddhi* of Lakṣmīdhara which includes both Āryapakṣa and Brāhmapakṣa versions of certain tables, and the *Karaṇakesarī* (Montelle and Plofker 2013) which combines algorithms from works in different *pakṣas*. See Appendix A for details on all these works.

patra/*pattra* "leaf, page" in the name of such an item might be taken to indicate a special textual category of individual tables; but then there are several works consisting of deliberately compiled and arranged sets of tables that also use *pattra* in their titles.

Also of interest is the distinct dearth of specialized astronomical tables for computing many additional phenomena (elevation of lunar cusps, planet-star conjunctions, etc.) that are explained in most treatises and handbooks, but which we have not seen represented in table texts. The flourishing growth of the *sāriṇī*/*koṣṭhaka* genre seems to have been limited to the subset of astronomical topics that were both frequently required and fairly straightforward to compute. But we should not forget, even though they are only touched upon in this study, the prodigious variety of types of specifically astrological tables which make up such a large part of the total *jyotiṣa* corpus.

2.3.2 Classification by computational structure

Pingree distinguished three basic "arrangements" used by *jyotiṣīs* in constructing planetary tables, calling these "mean linear'' (for which we prefer the more descriptive "mean with equation"), "true linear" (or "mean to true" in our discussions), and "cyclic." The following overview explains their main characteristics.

Mean with equation. This template, described by Pingree as "mean linear," somewhat resembles the standard structure of the astronomical tables of Ptolemy and his successors in the Greco-Islamic tradition. It combines tables of increments in mean longitude, produced by a planet's mean motion over time periods of varying length, with tables of equations or corrections for adjusting a given mean longitude to the appropriate true longitude.

In the mean-with-equation scheme, all computations begin from the planet's specified epoch mean longitude, i.e., its mean position at the date and time designated as the epoch or starting-point for that set of tables. A mean longitude for a desired date is obtained by adding to the planet's epoch mean position all the mean longitude increments accumulated in the intervening time. Entering a table of equation with the longitudinal anomaly corresponding to that mean longitude, the user looks up the appropriate equation value and corrects the mean longitude with it. For planets that have more than one orbital inequality, Ptolemaic-type Greek and Islamic astronomical tables yield two equation components applied simultaneously; in the Indian tradition, the eccentric anomaly (Sanskrit *manda*) and the synodic anomaly (*śīghra*) produce separate corrections that are tabulated and applied sequentially.

Mean to true. Pingree denoted this type of table "true linear." Like the preceding mean-with-equation, this structure also relies on tables of the increments to a planet's mean longitude produced by its mean motion over various time intervals. But it also tabulates pre-computed values of true longitude and velocity produced by specified combinations of mean longitudes of the planet and the sun. Since the eccentric or *manda* anomaly depends only on the planet's mean longitude and the longitude of the orbital apogee which is considered fixed, and the synodic or *śīghra* anomaly depends only on the relative positions of the mean planet and mean sun, these data are all that is required to find the true longitude. Thus once the desired mean longitude is known, the user can just look up the corresponding true longitude instead of calculating it by applying equations.

The mean-to-true template appears to be an Indian innovation, but the actual operation of Indian mean-to-true tables is somewhat more involved than the above brief description suggests. For one thing, the mean longitude increments are recorded not in standard degrees of arc but in coarser arc-units consisting of some number n of degrees which may or may not be an integer: these n-degree arc-units (denoted "n° arc-units"), like regular degrees, are divided sexagesimally. Each planet has one true longitude table for each successive n° arc-unit of its mean longitude, or $360/n$ such tables per planet. And each true longitude table has for its argument the *nakṣatra* or $13^\circ;20$ arc of the ecliptic occupied by the mean sun, running from 1 to 27 (since $27 \cdot 13;20 = 360$). However, these argument values are generally interpreted not as longitudinal arcs but as corresponding time-periods known as *avadhis*: one *avadhi* is the time required for the mean sun to traverse one $13^\circ;20$ *nakṣatra* in longitude, or a little less than 14 days, i.e., $1/27$ of a year. The table entries contain the planet's true longitude and velocity at the start of each *avadhi*.

To illustrate the process, let us suppose that a user wants to find the true longitude of a given planet at a time equal to some fractional *avadhi* a after the lapse of A integer *avadhis* in a particular year. The user determines from the mean longitude increment tables that at the beginning of this year the mean planet has traversed P integer arc-units plus some fractional arc-unit p. Consequently, the planet's Pth and $(P+1)$th true longitude tables must be consulted. The user first interpolates with the fraction p between the true longitude entries for *avadhi* A in tables P and $P+1$, and similarly between the entries for *avadhi* $A+1$ in the same two tables. Then interpolating with the fraction a between the two intertabular values thus found for *avadhi* A and *avadhi* $A+1$ gives the desired true longitude at the given time.

One final caveat on mean-to-true tables: Since true longitude increments and velocities, unlike mean ones, occasionally change sign due to retrogradation, straightforward linear interpolation between table entries for successive *avadhis* will not always produce the correct value. Therefore the true longitude table entries are marked in their margins with the times and locations of any synodic phenomena (i.e., start and end of retrogradation, heliacal rising and setting) that will occur during that *avadhi*, so the user can adjust the interpolation accordingly.

Cyclic. This is a variant of the mean-to-true template which adjusts the chronological extent of the tables for each planet to cover one full true-longitude "cycle" or period for that planet. The "cyclic" structure likewise employs tables of a planet's true longitudes and velocities as the mean sun progresses from *avadhi* 1 to *avadhi* 27. But the number of such true longitude tables for each planet is not some common constant $360/n$, but rather a number specific to that planet, equal to the number of integer years over which the planet returns to nearly the same true longitude at the start of the year, i.e., the length of its true-longitude "cycle."

Thus, in a sense, these cyclic tables are perpetual. This arrangement is similar to those underlying the Babylonian "Goal-Year" periods and Ptolemy's cyclic schemes, and may perhaps have been directly inspired by works composed according to this arrangement that were circulating in the second millennium, such as that by al-Zarqālī (Montelle 2014).

The cyclic table-text arrangement did not appear on the scene until the mid-seventeenth century. Despite their single-lookup feature and their perpetual scope, such table texts seem never to have been as popular as their counterparts in the mean-with-equation and mean-to-true categories.

Chapter 3
Table-text manuscripts

Hitherto we have discussed the content and structure of Sanskrit astronomical tables abstracted from their physical embodiment in manuscripts. In this chapter we consider the manuscripts themselves: their overall position within the corpus of Sanskrit *jyotiṣa* and the scribal conventions that characterize them.

3.1 Table texts in Sanskrit scientific manuscript collections

Compared to other forms of Sanskrit scientific texts in which authorial voice and expository form are more strongly marked, table texts can be difficult to distinguish and identify. Their modular structure as a compilation of multiple separate tables means that they can be expanded, truncated, recompiled, or otherwise modified for different users' convenience. Their individual differences are often masked by their superficially similar (and very dry) presentation as sequences of numerical grids, generally without much accompanying explanatory text. It is not surprising that early Western catalogues of Sanskrit scientific manuscripts, even some compiled by highly learned Indologists, identified many such works merely as "tables" with no attempt to analyze their contents in detail.[1]

[1]Examples include MS 984 in Aufrecht's Leipzig catalogue, a 55-folio item listed simply as *Sāraṇī* with the description "tabellarische Berechnungen zum Behuf der Anfertigung von Kalendern" (Aufrecht 1901, p. 304); several items in Eggeling's India Office Library catalogue (e.g., MSS 2049c, 1051f (Eggeling 1896, pp. 1053–1054)); and the items described in H. Poleman's North American *Census* (Poleman 1938, p. 246) as "[a] collection of several hundred miscellaneous folios, mostly tables not important enough and not bearing sufficient information to identify at all" (an assessment contradicted by various interesting discoveries within this bundle, a number of which are sampled in Chapter 4).

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Table 3.1 Topics in (non-astrological) *jyotiṣa* as represented in the Sanskrit manuscript holdings of Columbia University (CU); the India Office Library's Gaekwad Collection (G); the Wellcome Library (W); and the Bodleian Library's Chandra Shum Shere Collection (CSS).

Topic	CU	G	W	CSS
Vedic				2
siddhānta	8	5	7	24
karaṇa	15	10	39	25
koṣṭhaka	150	15	48	28
pañcāṅga	3		19	
Eclipses			4	
Star charts				
Geography				
Instrumentation	6	2	5	8
Miscellaneous			4	9
Encyclopedias		1	9	
Translations				
Lexica		2		

In the present state of our knowledge, around 50–60 distinct Sanskrit table texts can be identified (see Appendix A).[2] The ones whose location of composition can be (even tentatively) identified are predominantly from the north and northwestern parts of India. Although the full corpus of such works has not been definitively quantified even to within an order of magnitude, the available evidence suggests that it is very extensive. The total number of Indic manuscripts in the broadest sense, both within and outside India, was estimated by David Pingree at (very approximately) 30 million (Pingree 1988, p. 638; Wujastyk 2014, p. 160). More conservative estimates for manuscript holdings in India itself have been placed around five million as of 2007 (Goswamy 2007, p. 17) and seven million as of 2014; a recent and by no means exhaustive documentation project has recorded at least two million items (Wujastyk 2014, p. 160). In the entire universe of surviving Sanskrit manuscripts Pingree put the probable number of surviving *jyotiṣa* works of all types, including astrological genres, at around 10,000, and the number of extant manuscripts containing them at perhaps 100,000 (Pingree 1978b, p. 364).

A rough assessment of the prevalence of *koṣṭhakas* among *jyotiṣa* works can be obtained from comparing the numbers of manuscripts treating *koṣṭhaka* relative to those addressing other astronomical topics in Sanskrit manuscript collections. A sampling of such comparisons is displayed in Table 3.1. (Note that the topics listed therein are restricted to those including aspects of technical astronomy or *gaṇita* rather than purely astrological ones.)

These preliminary tallies suggest that a conservative estimate for the percentage of table texts among all non-astrological *jyotiṣa* material may be anywhere from one-quarter to one-half. If these figures are accurate, then the manuscripts devoted to *koṣṭhaka*/*sāraṇī* works number at least in the thousands if not tens of thousands.

[2]This list neglects works labelled "Anonymous" or otherwise lacking indication of their title or author, which cannot be reliably assigned as variants of a named table text.

This section attempts to probe the data further for additional details about the representation and distribution of the table-text genre in diverse collections of manuscripts.

3.1.1 General surveys in *SATIUS* and *SATE*

Systematic efforts to analyze the corpus of Sanskrit numerical tables commenced with David Pingree's surveys of collections at Columbia University, Harvard University, and the University of Pennsylvania. Struck by the sheer magnitude of the tables corpus, Pingree undertook to provide a guide for scholars engaged in similar cataloguing efforts so that they would be better equipped identify the table texts they encountered. The resulting catalogue including table descriptions and preliminary analyses was published in 1968 as *Sanskrit Astronomical Tables in the United States* (*SATIUS*) (Pingree 1968). *SATIUS* updates and expands the relevant material from Poleman's *Census* (Poleman 1938) of North American Sanskrit manuscripts, retaining Poleman's numbering. The Columbia collections' *jyotiṣa* holdings that yielded the astronomical tables analyzed by Pingree comprise 194 individual manuscripts, of which 66 are known to contain table texts and 5 *pañcāṅgas*, as well as the so-called Miscellaneous Bundle of 105 complete or fragmentary table texts in the Smith Indic collection (all apparently from Gujarāt, Rājasthān, and possibly Uttar Pradesh). The 217 *jyotiṣa* manuscripts at the University of Pennsylvania, among which Pingree identified at least 18 items relating to tables, likewise originated in western, northern, and central India, as did the Harvard manuscripts used in *SATIUS*.

Following an introductory overview, *SATIUS* divides its survey into two parts: a traditional catalogue of the manuscripts of table-text items including title, author, scribe, date, and folio-by-folio content for each item, and a technical analysis of each table-text represented, accompanied by background information about the author (where available), important dates relating to the work, and a list of its known manuscripts worldwide. This corpus contains 34 table texts, 19 of which are identified by title; various similarities between the remaining unidentified texts and known works are noted. *SATIUS* closes with an index of significant numerical parameters from the astronomical systems used in the tables.

Soon after the appearance of *SATIUS*, the results of similar cataloguing efforts in repositories in the UK were published in Pingree's 1973 *Sanskrit Astronomical Tables in England* (*SATE*) (Pingree 1973), drawing on various Sanskrit manuscript collections in the British Museum, Oxford and Cambridge Universities, the India Office Library (now incorporated into the British Library), the Bodleian Library, and the Royal Asiatic Society. They include manuscripts from Bengal, Benares, Kashmir, and south India in addition to many sources from western India. The structure of *SATE* and many of the 34 table texts that it describes (including its only anonymous work, Anonymous of 1638) are familiar from *SATIUS*. The chronological range of these works extends from Āśādhara's *Grahajñāna* (1132 CE) to the *Pattraprakāśa* of Viśrāmaśukla (1777 CE). The majority of the manuscripts were copied in the seventeenth, eighteenth, and nineteenth centuries.

More than four decades later, *SATIUS* and *SATE* remain the only two manuscript catalogues dedicated exclusively to astronomical numerical tables in Sanskrit sources. Their descriptions of manuscripts, identifications of (some of) the authors who composed the texts and the scribes who copied them, information about dates and geographical locations, and technical analyses supply crucial information concerning the practice, methodologies, and evolution of South Asian astral sciences. Table 3.2 lists 39 such works described in *SATE* and/or *SATIUS*, along with the combined numbers of their North American and UK manuscripts recorded in those surveys. Two separate columns list the numbers of their manuscripts in two individual collections discussed below. (Note that this list is not exhaustive: there are a dozen or so other items included in Pingree's lists that are sufficiently fragmentary or derivative to suggest that they may represent recensions of other known works or standalone reference tables not incorporated into a full *koṣṭhaka* text. Where possible, we have assigned these items and their recorded manuscripts as part of the textual tradition of the known work that they seem to belong to, as in the case of the work "Anonymous of 1520," which we list with the *Grahalāghavasāriṇī*. Where we have not felt confident in making such an assignment, we have simply omitted the fragments from this table.)

In addition to these specialized surveys, a number of more general manuscript catalogues and other bibliographic studies, especially ones by Pingree, include sections on Sanskrit table texts.[3] The abundance of the textual material fully supports Pingree's conviction that such catalogues and surveys are indispensable to gaining a clearer understanding of the technical aims and methods of Indian astronomers in the second millennium.

3.1.2 Smith Indic and Smith Sanskrit Collections, Columbia University

The renowned historian of mathematics and pioneer of mathematics education David Eugene Smith (1860–1944) acquired a substantial collection of nearly 350 Sanskrit manuscripts, as well as some in other South and Southeast Asian languages. These materials constitute the Smith Indic and Smith Sanskrit collections in the Rare Book Library of Smith's home institution Columbia University, which also holds a few Sanskrit manuscripts from other sources. Since Smith was primarily

[3] E.g., Pingree (1970–94), Raghavan et al. (1968–2007), Pingree (2004), Pingree (2003, pp. 44–102), Bhandarkar Oriental Research Institute (1990–1991, vols. 30–32), Rajasthan Oriental Research Institute (1963–2007), and Sarma and Sastry (2002); still other Sanskrit manuscript catalogues that pertain to astronomical tables are listed in our bibliography. A section in Pingree (1981, pp. 41–46) lists author, title, and the identification of key features (such as epoch, table type, and relationships to other tables) for 35 known *koṣṭhaka/sāraṇī* works.

Table 3.2 Sampled manuscripts of major table texts: in collections analyzed in *SATIUS* and *SATE* (S & S); in the India Office Library Gaekwad Collection (G); and in the Wellcome Library (W).

Work	Epoch	S & S	G	W
Anantasudhārasasāriṇī of Ananta	ca. 1525			1
Kalpalatā	1359	1	1	1
Kāmadhenu of Mahādeva	1357	1		
<Khaṇḍakhādyakasāriṇī>		1		
Khecaradīpikā of Kalyāṇa	1649	1		
Kheṭasiddhi of Dinakara	1578	1	1	
Gaṇitamakaranda of Rāmadāsa Dave	1618			1
Gaṇitarāja of Kevalarāma Pañcānana	1728	1		
Grahakaumudī of Nṛsiṃha	1603	1	2	
Grahajñāna of Āśādhara	1132	2	2	
Grahaprakāśa of Devadatta	1662	1		
Grahaprabodhasāriṇi of Yādava	1663	3	1	
Grahamañjarī		2		
Grahalāghavasāriṇī I	1520	13	1	
Grahalāghavasāriṇī of Prema	1656			2
Grahalāghavasāriṇī	1754	1		
<Grahasāraṇī>		1		
Grahasāraṇī of Gaṅgādhara	1630	2		
Grahāgama of Govindasūnu	1773	2		
Candrārkī of Dinakara	1578	23	1	4
Jagadbhūṣaṇa of Haridatta	1638	6	1	
Jayavinodasāraṇī	1735	1		
Tithikalpadruma of Kalyāṇa	1605	1	1	
(*Laghu*)*Tithicintāmaṇi* of Gaṇeśa	1525	15		1
Tithidarpaṇa of Murāri	ca. 1665	1		
Tithisāraṇī of Dinakara	1583	2		
Tithyādicintāmaṇi of Dinakara	1569	1		
Pañcāṅgavidyādharī of Vidyādhara	1643	2		
Pañcāṅganayanasāraṇī	1718	1		
Pattraprakāśa of Viśrāmaśukla	1777	1		
Bṛhattithicintāmaṇi of Gaṇeśa	1552	6		
Brahmatulyasāraṇī		4		3
Bhāgapañcāṅga		1		
Makaranda of Makaranda	1478	24		13
Mahādevī of Mahādeva	1316	32		1
Ravisiddhāntamañjarī of Mathurānātha Śarman	1609	3		1
Rāmavinoda of Rāmacandra	1590	3		3
Laghukhecarasiddhi of Śrīdhara	1227	1	1	
Laghutithidarpaṇa	1665	1		
Śīghrasiddhi of Lakṣmīdhara	1278	2		

(continued)

Table 3.2 (continued)

Work	Epoch	S & S	G	W
Anonymous	1594	3		
Anonymous	1638	2		
Anonymous	1704	12		
Anonymous	1741	4		
Unidentified tables			1	

seeking documentation of the evolution of Indian place-value decimal numerals for his research (Smith and Karpinsky 1911), his Sanskrit acquisitions were heavily weighted towards mathematical and astronomical *jyotiṣa* topics (around 318 of the approximately 350 manuscripts collected). These *jyotiṣa* items include over 180 treating various aspects of astronomy, of which nearly 150 represent numeral-rich *koṣṭhaka/sāraṇī* works. Almost all seem to have been originally copied in the western and northern regions of India.

3.1.3 Gaekwad Collection, India Office Library

A more organically developed Indic manuscript library is that of the Gaekwad Mahārāja of Baroda in Gujarāt, Anandrao/Ānandarāja, who in 1809 donated to the East India Company 507 manuscripts (primarily in Sanskrit) dating from the late fourteenth to the late eighteenth century (Pingree 1976, p. 1). Now part of the collections of the India Office Library, they include 95 items containing texts on *jyotiṣa* (Pingree 1976, Appendix A), which primarily represent the Brāhmapakṣa and the Gaṇeśapakṣa. Nearly two-thirds of these 95 manuscripts concern astrological topics, while approximately thirteen of the 35 non-astrological items can be confidently identified as table texts mostly well-known works: they are enumerated in column G in Table 3.2. (Note that most of these manuscripts are counted also in column S&S of the same table, as they were previously described in more detail in Pingree (1973).)

As the following descriptions confirm, a distribution of this sort, in which *koṣṭhaka/sāraṇī* material makes up about one-third of all non-astrological *jyotiṣa* manuscripts which in their turn constitute about one-third of general *jyotiṣa* holdings, appears to be much more representative of actual Indian technical libraries in Sanskrit than the overwhelmingly tables-focused Smith collections at Columbia.

3.1.4 Wellcome Library

The Sanskrit holdings of the Wellcome Collection form part of the extensive acquisitions by the pharmaceutical entrepreneur Sir Henry Wellcome (1853–1936) of artifacts, books, manuscripts, and art relating to the development of medicine worldwide. In keeping with the main theme of the collection, most of the several thousand Sanskrit manuscripts are medical in nature; but approximately one thousand of them, representing around five hundred distinct texts, have been identified as pertaining to astronomy, mathematics, astrology, and divination. Over the course of about 20 years David Pingree and Dominik Wujastyk rationalized and catalogued these materials (Pingree 2004).

Pingree notes (Pingree 2004, p. xviii) that the collection is comprised of many rare works, including a number of table texts. The *Tithikalpalatā*, the *Anantasudhārasasāraṇī* of Ananta, the *Gaṇitamakaranda* of Rāmadāsa Dave, and the *Devīvilāsasāriṇī* are among some of the manuscripts he highlights.

3.1.5 Chandra Shum Shere Collection, Bodleian Library

This is one of the largest Sanskrit manuscript collections ever brought to England, containing no fewer than 6330 manuscripts, of which 575 treat *jyotiṣa* topics. It was acquired in the early twentieth century through the efforts of the chancellor of Oxford University, Lord Curzon, and Mahārāja Chandra Shum Shere, then prime minister of Nepal, who purchased the collection from an anonymous seller in India and donated it to Oxford in 1909.

Pingree's 1984 catalogue (Pingree 1984) of the *jyotiṣa* manuscripts in this collection, based on the initial lists compiled by T. Gambier Parry and E.H. Johnston who accessioned it at the Bodleian, also includes some non-*jyotiṣa* manuscripts bound together with *jyotiṣa* texts. The *jyotiṣa* holdings were found to be for the most part seventeenth- through nineteenth-century copies of standard works, with a sprinkling of texts previously unknown to Indologists. Their contents range over the genres of astronomy, mathematics, divination, astrology (genethlialogy, *muhūrta* and *praśna*), reference/lexical works (encyclopedias and *kośa*), and rites (*pūjā*, *śānti* and magic). The chief strength of this collection, according to Pingree, is its documentation of Indian reactions to Islamic science in the Mughal period (particularly in astronomical literature and the new astrological disciplines of *ramala* and *tājika*), as well as the spread of *muhūrta* or catarchic astrology in northern India.

The 96 manuscripts in the astronomy genre are divided into various subgenres, shown with their corresponding numbers of manuscripts in Table 3.2. The *koṣṭhaka* subgenre comprises the most manuscripts, accounting for around 30% of the total; the works they contain are enumerated in Table 3.3. Notable items in this category include the copy of the *Śīghrasiddhi* by Lakṣmīdhara accompanied by a unique exemplar of Janārdana's supplement, an autograph copy of Kṣemaṅkara's *Subodhikā*, the first known copy of Gaṇeśa's *Pañcāṅgasiddhi*, and the various tables of consecutive lunar and solar eclipses.

Table 3.3 Astronomical table texts in the Chandra Shum Shere collection.

Work	No. of MSS
Abhinavatāmarasa of Puruṣottama Bhaṭṭa, commentary on the *Makaranda*	1
Grahaṇamālā (consecutive lunar and solar eclipses from 1836 to 1945)	1
Grahaṇamālāsāraṇī (consecutive lunar and solar eclipses from 1843 to 1909)	1
Grahalāghavasāriṇī of Gaṇeśa	2
Candrārkī of Dinakara	1
Camatkārasiddhi of Vīrasiṃha	1
Tithicintāmaṇi of Gaṇeśa	4
Dinacandrikā of Rāghavaśarman	1
Pañcāṅgasiddhi of Gaṇeśa	1
Pañcāṅganayanasāraṇī (1718)	1
Pātasāraṇīvivaraṇa of Divākara, commentary on Gaṇeśa's *Pātasāraṇī*	1
Prakaṭagrahaṇamālā (consecutive lunar and solar eclipses for 1666–1704)	1
Brahmāryopakaraṇasiddhi of Janārdana, appendix to the *Śīghrasiddhi*	1
Bhāvapattra	1
Makaranda of Makaranda	1
Makarandodāharaṇa of Viśvanātha	2
Rāmavinoda of Rāma	1
Śīghrasiddhi of Lakṣmīdhara	1
Subodhikā of Kṣemaṅkara	1
Sūryasiddhāntarahasya of Rāghavaśarman	2
Consecutive lunar and solar eclipses 1779–1855	1
Various parts of *koṣṭhakas*	2
Unidentified tables	1

3.1.6 Mahārāja Man Singh II Museum, Jaipur

The library of the Mahārāja Sawai Man Singh II located in Jaipur contains three substantial manuscript collections: Khāsmohor, Puṇḍarīka, Pothikhānā, containing a total of about 12,500 manuscripts. Some of their especially notable items are on display in the so-called Museum collection. Of the approximately 276 manuscripts identified as pertaining to astral science, 12 are in the Museum collection, which also contains a further 14 astronomical works in Persian or Arabic. The astral-science manuscripts in these collections were catalogued and analyzed by a team of specialists led by Pingree, including S. Ikeyama, C. Minkowski, K. Plofker, S.R. Sarma, and G. Tubb in the early 1990s. A descriptive catalogue based on their notes was published as Pingree (2003).

As can be appreciated from Table 3.4, the *koṣṭhaka* genre contains the most manuscripts by far. Out of the 276 astronomical works catalogued, around 118 of these are table texts of some kind, or a little over 40%. Some of the especially important table texts included in this library are a unique manuscript of Moreśvara's *Makarandaṭippaṇa*, a unique manuscript of Harinātha's *tithi*, *nakṣatra*, and *yoga*

Table 3.4 Topics in (non-astrological) *jyotiṣa* as represented in the Sanskrit manuscript holdings of the Mahārāja Man Singh II Museum (MS); the Jaina Vidya Sansthan (JVS); and the Sri Ram Charan Museum of Indology (SRC), all in Jaipur; and in a set of manuscripts purchased in Jaipur (KP).

Topic	MS	JVS	SRC	KP
Vedic	2			
siddhānta	43	5	3	3
karaṇa	50	9	7	12
koṣṭhaka	113	13	16	28
pañcāṅga		1		
Eclipses	12	1	1	2
Star charts	5			
Geography	3		3	
Instrumentation	18	3	3	2
Miscellaneous	6	10	1	6
Encyclopedias				
Translations	24			
Lexica				

tables, a unique manuscript of Goparāja's *Khagataraṅgiṇī*, and two of the four known manuscripts of Kevalarāma's *Pañcāṅgasāriṇī*. In addition, the tabular work of Philippe De La Hire's tables and related Sanskrit translations of this work, and a table comparing Jayasiṃha's observed lunar positions with those computed using De La Hire's tables are contained in this library.

3.1.7 Some miscellaneous collections in north India

Several collections in Rajasthan and Bihar were examined by one of the authors (Plofker) in 2003–2004.

Jaina Vidya Sansthan, Jaipur

The exact sciences manuscripts of this Jaina *bhandar* or institutional library range in copying date from the sixteenth to the early twentieth century.[4] According to the register of around 5000 manuscripts total, among some 255 *jyotiṣa* items in Sanskrit, Hindi, Rajasthani, and other vernaculars, about 40 pertain to non-astrological topics; these are broken down by subject in Table 3.4. Unsurprisingly, some of its notable items include works by Jaina authors such as Mahendra Sūri and Dedacanda.

[4]For a general survey of Jaina manuscript libraries in India, see Cort (1995); for some details about their history and holdings including *jyotiṣa*, see Jain (1963) and Pingree (2001–2004, pp. 705–707).

Table 3.5 Table-text manuscripts at SRCMI, Jaipur.

Manuscript	Title/subject
1.20 i.63	*Mahādevī* commentary
1.20 i.94	*Mahādevī*
1.20 i.97	*Mahādevī*
1.20 i.115	*Grahalāghavasāriṇī*
1.20 i.145	*Candrārkī* ?
1.20 i.207	*sāraṇī*
1.20 i.211	*sāraṇī*
1.20 i.212	Makaranda commentary
1.20 i.226	*Grahalāghavasāriṇī*
1.20 i.228	*Mahādevī* commentary
1.20 i.250	*varṣa sāraṇī*
1.20 i.271	*koṣṭhaka*
1.20 i.287	*rāmabīja sāraṇī*
1.20 i.315	*Tithicintāmaṇi*
1.20 i.322	*Candrārkī* ?
1.20 i.343	*pañcāṅga sāraṇī*

Sri Ram Charan Museum of Indology, Jaipur

This private collection may have begun as a family library but seems to have been amassed largely by the efforts of the individual founder of the institution in the twentieth century. The catalogue volumes (Sharma 1986–1994) treat *gaṇita* and *jyotiṣa* as separate categories, and distinguish geography/cosmology (*bhūgola-khagola*) from both of them. Of the volumes that treat Sanskrit *jyotiṣa* works proper, volume 1 lists 46 such manuscripts of which none seem to relate to *koṣṭhaka*/*sāraṇī* works; volume 3 enumerates 329 *jyotiṣa* manuscripts, volume 5 two, and volume 6 three, of which the following sixteen are known or guessed to be tables-related (see Table 3.5).

Manuscripts in private commerce

An important though extremely irregular source of knowledge about tables as well as other aspects of Sanskrit science is the variety of manuscripts offered for sale by private owners and arts/handicrafts dealers. The provenance of these documents is generally obscure, but it is speculated that they primarily represent the private family libraries of hereditary practitioners of astral sciences, usually deaccessioned by their descendants. A sample summary of *jyotiṣa*-related manuscripts in one such dealer's collection seen in Jaipur in 2004 is shown in Table 3.6.

Many of these manuscripts, if not obtained by individual researchers or libraries, are doomed to dismemberment at the hands of art dealers (if they contain visually interesting diagrams or figures) or miniature painters who use their text as decorative

Table 3.6 Primary topics and works among manuscripts on *jyotiṣa* seen in a dealer's collection in Jaipur, 2004.

Topic	Number of manuscripts
siddhānta	5 (*Sūryasiddhānta*, *Siddhāntaśiromaṇi*)
karaṇa	12 (*Grahalāghava*)
koṣṭhaka	21 (*Grahalāghavasāriṇī*, *Makaranda*, *Candrārkī*)
Instrumentation	4
Mathematics	6 (*Līlāvatī*, *Bījagaṇita*)

background for a painting. Scrap-paper commerce absorbs many manuscript folia as well.[5]

3.1.8 Collections in Kerala and Tamil Nadu

The late K.V. Sarma published in 2002 an overview (Sarma and Sastry 2002) of some 12,244 manuscripts containing at least 3473 texts on Sanskrit sciences (of which about 9713 manuscripts and 2506 texts deal with astronomy, astrology, and mathematics) located in approximately 395 public and private collections in Tamil Nadu and Kerala. The category "I. Astronomy and mathematics" lists titles 1–889, plus 1–47 in a supplementary list, while "II. Astrology" includes titles 890–2420 as well as 1–43 in a supplement. The remaining material deals with other scientific disciplines: medicine, veterinary science, chemistry, music theory, botany, and architecture. Sarma notes (Sarma and Sastry 2002, p. 15) that less than 7% of the entire scientific corpus he surveyed, and about the same percentage of the specifically astral/mathematical material, has been published.

The survey covers 247 manuscript repositories in Kerala and 148 in Tamil Nadu, ranging from small personal libraries of no more than 4 or 5 manuscripts to collections in university libraries and research institutes including those of Calicut University, Kerala University, the Adyar Library, the Government Oriental Manuscripts Library at the University of Madras, and the Tanjore Mahārāja Serfoji's Sarasvati Mahal Library in Thanjavur, preserving tens of thousands of manuscripts in various languages and subject areas. We have not tried to determine what percentage of their total Sanskrit holdings deal with the exact sciences of mathematics, astronomy, and astrology (including divination). However, we have roughly computed the percentage of the exact-sciences works that seem likely to deal with tables, based on the occurrence of known *sāraṇī* and *koṣṭhaka* texts as

[5] It is extremely difficult to estimate the volume or nature of privately owned Sanskrit scientific manuscripts circulating in this sort of commerce within India, much less the ones discarded and destroyed. Some attempts have been made to study the documentation of such collections or to record the activities of current collectors; see, for instance, Zysk (2012) and Nagarajan (2016). A major project intended to create a database of all available manuscripts still maintained in existing collections was launched in 2004–05 by the National Mission for Manuscripts (https://namami.gov.in).

well as ones that include variants of the terms *koṣṭha*, *cakra*, and *sāraṇī* in their titles (the listed works are not described in detail, so in most cases the listed title is the only indication of the subject matter).

As the following list indicates, the types of table texts most familiar from northern collections, either well-known works such as the *Tithicintāmaṇi* or ones explicitly titled -*sāraṇī* or -*koṣṭha* or some variant thereof, make up only about 15 distinct works and 17 manuscripts in the total corpus. The survey contains numerous other works ending in "-*phalāni*" ("equations" or "results") and other terms that may well refer to tables of function values, which we have omitted from consideration on grounds of uncertainty. Our current best guess, then, is that at a very conservative estimate about one to two percent of these works reflect the row-and-column tabular genre popular in north India in the same period.

When we turn to the specifically south Indian genre of the *vākya*, however, we find 66 texts in this category (base-texts and commentaries, including ones on the seminal *vākya* text *Veṇvāroha* of Mādhava), in 268 manuscripts relating to both astronomical and astrological computations. There are an additional 63 items in 360 manuscripts entitled simply *pañcāṅga* or calendar, and 33 others (six of which overlap with titles in the *vākya* category and another with one in the *sāraṇī* category) in 74 manuscripts whose names suggest they pertain to the computation of *pañcāṅgas*. For lack of fuller information, we have simply lumped these into the same category with *koṣṭhaka*/*sāraṇī* works, although some of them are probably not in tabular form. A note by the author (Sarma and Sastry 2002, p. 21) identifies at least 13 additional surveyed items in 15 manuscripts consisting of trigonometric tables recorded in *kaṭapayādi* compositions. Excluding the presumably ephemeral *pañcāṅga* items themselves, this means that at a fairly conservative estimate, about 357 of 9713 manuscripts and 143 of 2506 distinct texts, or between 4 and 6% of all the listed astral/mathematical material, represent some kind of table text. Over half of those works (and well over three-quarters of their manuscripts) involve the verbal rather than graphical tabular format of *vākyas* and similar *kaṭapayādi* constructions.

Two features of this rough analysis stand out: the relatively low prevalence of table texts compared to their representation in the collections and catalogues described above, and the predominance within this group of the uniquely south Indian *kaṭapayādi* verbal-table structure. This is a salutary reminder that one cannot draw general conclusions about Sanskrit astronomical/mathematical tables as a textual genre without thoroughly investigating the ancient and prolific south Indian *vākya* tradition and related compositions using *kaṭapayādi* notation to construct mnemonic tables.

Categories of (known or presumed) table works in Sarma and Sastry (2002):

Title includes *koṣṭha*. 3 works: I.116, I.205, I.300
Title includes *sāraṇī*. 8 works: I.260–263, I.268 (of Mahādeva), I.384, I.541, II.1356

Title includes *vākya*. 62 works: I.29, I.92, I.103, I.113, I.250, I.264, I.273–274, I.283, I.301–304, I.311, I.357–358, I.419–420, I.429, I.528, I.534, I.549, I.625, I.638, I.699–700, I.702, I.710, I.717–719, I.721–733, I.745, I.797, I.sup.7, I.sup.9, I.sup.20–21, I.sup.39–41, II.934–935, II.1052, II.1098, II.1441, II.1642, II.1656, II.1696, II.1707

Title includes *pañcāṅga*. 96 works: I.457–541, I.726–728, I.744, II.1167, II.1490, II.1704–1708

Known *koṣṭhaka*/*sāraṇī* texts. 8 works: I.319 (*Candrārkī* of Dinakara), I.380 (*Tithicintāmaṇi* of Gaṇeśa), I.748–751 (*Veṇvāroha* of Mādhava), II.1499 (*Tithyādicintāmaṇi* of Gaṇeśa), II.1892 (*Bṛhattithicintāmaṇi* of Gaṇeśa)

Identified *kaṭapayādi* trigonometric tables. 14 works: I.20, I.25, I.119, I.321, I.335, I.337–341, I.401, I.551, I.610, I.611

3.2 Spatial and graphical characteristics of table-text manuscripts

Many conventions of Sanskrit scribal orthography—the pagination, titles and so forth—are consistent across many Sanskrit technical genres including table texts. Eventually, detailed comparison of *koṣṭhaka* layouts with astronomical table formats in non-Sanskrit scientific traditions may shed more light on possible connections between them. In the meantime, the discussion in this chapter will focus on some distinctive features of the way *sāraṇīs* and *koṣṭhakas* are written.

3.2.1 Notation and layout conventions

Argument values. Most Sanskrit numerical tables are oriented so that their argument runs horizontally left to right across the width of a page.[6] Throughout this book, we refer to this sequence as the "argument row," with no assigned row number. Figure 3.1 illustrates this layout with argument values from 0 to 29 along the top of the table and numerical entries placed in the corresponding rows underneath.

When a table has more argument values than the width of the page can accommodate, the scribe simply continues the linear sequence by wrapping the table back to the left edge of the page. A lengthy table can be wrapped several times over a single page or even over several pages. See, for instance, the top image in Figure 3.2, where the table is wrapped after argument value 60.

[6]The width of a Sanskrit manuscript page is typically greater than its height, in keeping with the traditional "landscape" or *pothi* format derived from ancient use of birch bark and palm leaves as a writing material (Goswamy 2007, pp. 20–23).

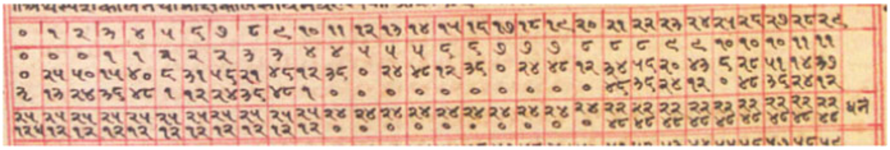

Fig. 3.1 Excerpt from a page of a manuscript of the *Karaṇakesarī* (RORI 12792 f. 4r) (Montelle and Plofker 2013; Misra et al. 2016), showing the arrangement of argument values and their corresponding entries in the first and second table rows with vertically stacked sexagesimal digits. The word *dhanaṃ* (positive) in the second row's right margin indicates that the differences in this row are to be applied positively to the entries in the first row.

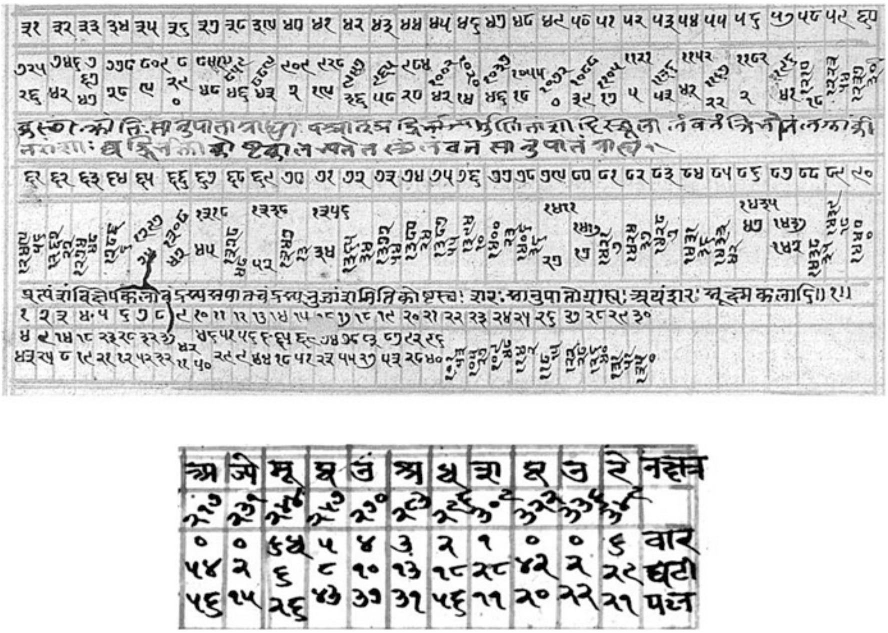

Fig. 3.2 Numerals in manuscript tables rotated sideways to fit their cells. Top: *Makaranda* (BORI 546 f. 12r). Bottom: Smith Indic MB C f. 1r.

A table's (single) argument may be split into two separate components laid out in different dimensions, such as an argument of celestial longitude represented by twelve rows of zodiacal signs and thirty columns of degrees within a sign, such as splitting argument values given in degrees into zodiacal signs and degrees. This commonly occurring layout appears in many examples within this book, including Figures 4.24, 4.27, 4.29, 4.53, and 4.55. Presumably it was chosen as a more efficient alternative to wrapping a table into multiple segments, each requiring a separate argument row.

Occasional exceptions to the conventional horizontal sequence of entries include the table shown in Figure 4.65, whose argument values run vertically down the page.

Such tables, usually directly influenced by Islamic or European sources, are found in manuscripts employing the (comparatively late and rare in the Sanskrit corpus) "portrait" format with the height of the page greater than its width.

True double-entry tables, as opposed to the split-single-argument tables described above, appear to be extremely rare in the Sanskrit corpus.[7] The only unambiguously double-entry table we have encountered therein occurs in manuscripts of Gaṇeśa's *Pātasāraṇī*, for the purpose of computing *mahāpātas* or parallel aspects of the sun and moon (Section 4.11); a copy is shown in Figure 4.61. Its two independent arguments are the zodiacal half-sign of nodal longitude (horizontal) and the current *nakṣatra* (vertical). A worked example in a commentary on the *Pātasāraṇī* by Dinakara (fl. 1812, MS UPenn 390 697 f. 3r–v) outlines procedures for double interpolation within it.

Decimal and sexagesimal numerals. Sanskrit numerical tables use standard Indian place-value base-ten numerals to write both decimal and sexagesimal (or quasi-sexagesimal) quantities.[8] The integer part of a sexagesimally divided number may be written in two places for zodiacal signs and degrees, as illustrated in row 1 of the table shown in Figure 3.1, or as a single decimal integer that may be greater than 60 (see Figures 3.5, 4.33 and 4.50).

Sexagesimal table data are usually arranged vertically in their cells with the most significant sexagesimal digit on top, as illustrated in Figure 3.1. Underneath the argument are two "rows" of entries in table cells delimited by red lines. The first row's entries show the degrees, minutes, and seconds places stacked vertically, while the entries in the second row, evidently in minutes and seconds alone, represent the differences between successive entries in the previous row.

The orientation of numerals may be tilted to fit them into a table cell, as illustrated by the examples in Figure 3.2 (see also Figure 4.48).

Digit separators or *daṇḍas* In ordinary text, vertical bars called *daṇḍas* are generally used as dividers between the sexagesimal places in a number. This is very rarely done within a table cell, perhaps because of the potential confusion generated by vertical lines in a ruled environment. Figure 3.3 shows the use of *daṇḍa* separators between sexagesimal digits representing zodiacal signs, degrees, minutes, and seconds, in a table title and within table cells.

Fraction approximation with *daṇḍa* strokes As noted in Section 2.2.2, table entries sometimes contain, in place of sexagesimal fractions, the approximations one-quarter, one-half, and three-quarters indicated by a corresponding number of

[7] Pingree (1968, p. 46 (item 52)) describes a table found in some manuscripts of the *Makaranda* as a "double-entry table of unknown purport" whose arguments run from 1 to 12 both horizontally and vertically. As this table is not yet fully analyzed, we cannot definitively confirm its double-entry structure.

[8] See Figure C.2. For more examples of scribal renditions and variant forms of the ten numeral glyphs in *nāgarī*, see www.hamsi.org.nz/p/how-to-read-devanagari-numerals.html.

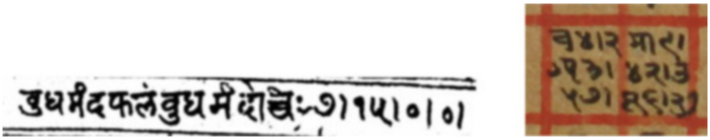

Fig. 3.3 Manuscript page excerpts showing sexagesimal number digits separated by *daṇḍas*. Left: *Karaṇakutūhalasāriṇī* (BORI 501 f. 16v). The text reads: *buddhamaṃdaphalaṃ buddhamaṃdoccaḥ* 7 | 15 | 0 | 0 | "The *manda*-equation of Mars: the *manda*-apogee is 7 [zodiacal signs] 15 [degrees] 0 [arcminutes and] 0 [seconds]". Right: *Jagadbhūṣaṇa* (RORI 10192 f. 14v). The cell entries read: *va*[*kra*] *4* | *2* '[*sta*]-*pa*[*ścima*] *3* | *57* | and *mā*[*rga*] *9* | *42* | *u*[*daya*]-*pū*[*rva*] *6* | *27*.

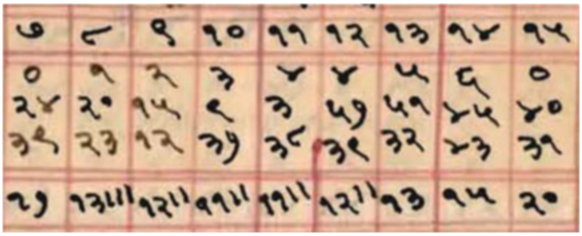

Fig. 3.4 Excerpt from a manuscript of the *Bṛhattithicintāmaṇi* (Smith Indic 151 f. 4r) showing a table with "*daṇḍa*-quarter" fractions in the entries in its second row.

Fig. 3.5 Above mentioned manuscript of the *Karaṇakesarī* (RORI 12792 f. 2r) illustrating the use of negative and positive symbols in table entries. The decrease or increase of solar velocity values in arcminutes and arcseconds for successive *avadhis* in row 2 is tracked by the sign changes of the daily velocity differences, in arcseconds with "*daṇḍa*-quarter" fractions, in row 3.

daṇḍas. This "*daṇḍa*-quarter" notation is demonstrated in Figures 3.4, 3.5, and 3.7; see also Figures 4.11, 4.51, 4.57, and 2.18.

Symbols for negative and positive numbers Negative and positive signs often appear in tabular data, sometimes accompanying zero quantities as well as nonzero ones. A cross sign ×, orthographical shorthand for the *nāgarī* character "*ṛ*" standing for *ṛṇaṃ* or "negative," would be written following the number it modified. So would the *nāgarī* character for *dha*, short for *dhanaṃ* or "positive" (see Figure 3.1).

The sign of a number in a table entry generally defaults to the last explicitly indicated sign symbol. Consider, for instance, the table excerpt in Figure 3.5, whose second row contains solar velocity values for successive *avadhis* or 14-day periods, while the third row shows their approximate daily change: i.e., 1/14 of the difference between consecutive velocity values, rounded to the nearest *daṇḍa*-quarter. The

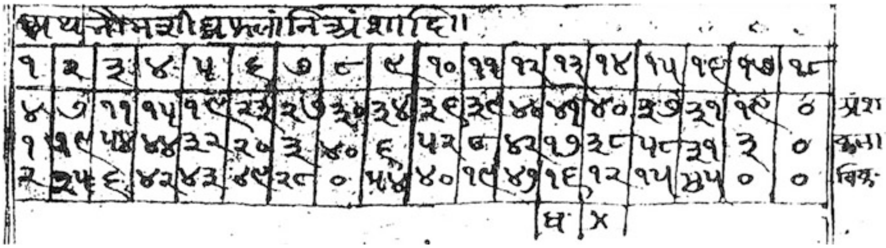

Fig. 3.6 Excerpt from a manuscript of the *Laghukhecarasiddhi* (IO 2408b f. 7r) showing the use of the symbols for indicating when a function is increasing and decreasing.

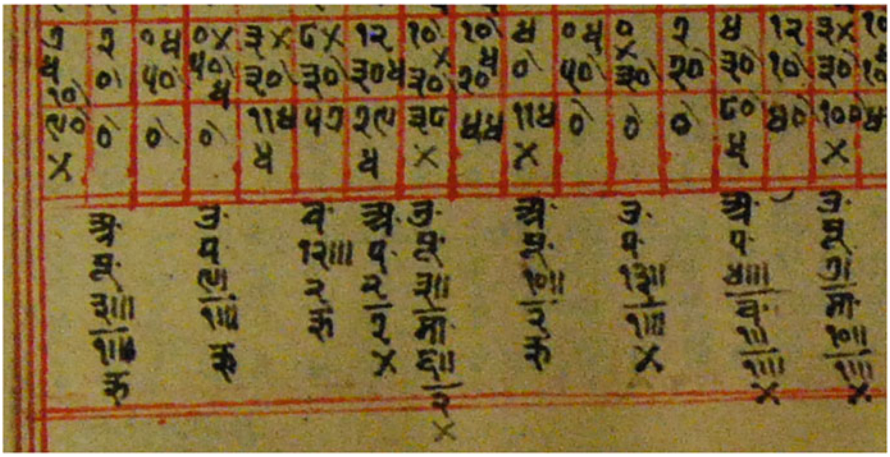

Fig. 3.7 Excerpt from a manuscript of the *Mahādevī* (RAS Tod 24 f. 27v) containing two different negative-sign symbols as well as the "*daṇḍa*-quarter" strokes for approximating fractions.

change in velocity from column 6 to column 7 is negative and thus marked with $\times$, while the subsequent velocity increases from column 7 to column 20 are flagged with a *dha* symbol, which is succeeded by another $\times$ as the values decrease again.

Figure 3.6, on the other hand, shows positive and negative sign symbols used to mark a transition (in columns 13 and 14) between increasing and decreasing table entries without explicitly recording their differences. Finally, the table in Figure 3.7 contains both the $\times$ and the *nāgarī* character *ṛ* for *ṛṇaṃ* to indicate negative quantities.

3.2.2 Scribal errors, emendations and markup

Simple uncorrected scribal errors, primarily writing a wrong number or set of numbers by mistake for the correct one, are ubiquitous in table manuscripts. In

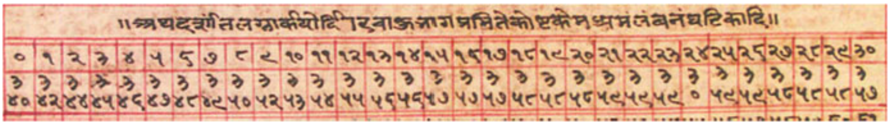

Fig. 3.8 Excerpt from *Karaṇakesarī* manuscript (RORI 12792 f. 7v) apparently showing the effects of scribal distraction, with the numeral 3 repeated in column 24 by mistake for the intended maximum value 4.

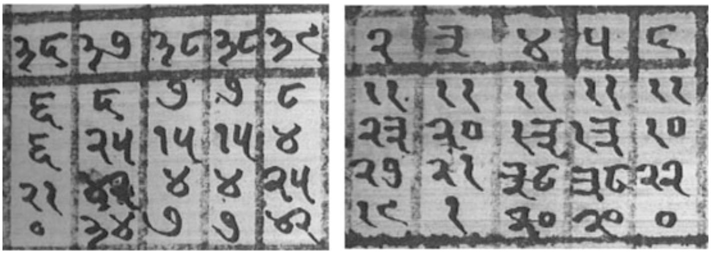

Fig. 3.9 Left: Excerpt from a different manuscript of the *Karaṇakesarī* (Smith Indic MB XXIV f. 3v) showing the scribe's uncorrected duplication of argument and entry in column 38. Right: The previous page (f. 3r) of this same manuscript shows a mistaken duplication in entry 4 which has apparently been noticed and flagged with light horizontal strikethroughs, but not corrected.

some cases, it is possible to infer what particular copying technique or feature of the content probably led the oblivious scribe to produce the given error. For example, Figure 3.8 shows an excerpt from a table whose values slowly increase from 3,40 to a maximum of 4,0 at argument 24 and then slowly decrease again. But the scribe has failed to notice the change from the long succession of 3's in the most significant sexagesimal digit to the single instance of 4, so he wrote 3,0 instead of 4,0.

In the manuscript table excerpt reproduced on the left of Figure 3.9, the scribe has duplicated an entire table column, both argument and entry. This redundancy does not affect the correctness of the table data, and no correction or erasure has been made. In the table excerpt shown beside it, on the other hand, the scribe has mistakenly duplicated the correct entry for argument 5 in the cell for argument 4. Subsequently the scribe (or possibly a later reader?) noticed the error and struck through the incorrect digits in column 4 with thin horizontal lines, but did not supply the correct values in their place.

When a copying error is noticed, it may be retrieved either by writing the correct figures more boldly over the mistaken ones, squeezing them into the table cell beside the stricken-out erroneous values, copying them separately into a page margin with insertion marks, or some combination of the above. Figures 3.9 and 3.10 display the overwriting technique, which often makes the corrected glyphs blotchy and difficult to read (see also Figure 3.16). The examples shown in Figure 3.11 illustrate the use of marginal corrections in conjunction with other methods.

Figure 3.12 illustrates the insertion of marginal corrections without attempting to change the incorrect data. The second row in the pictured table shows the

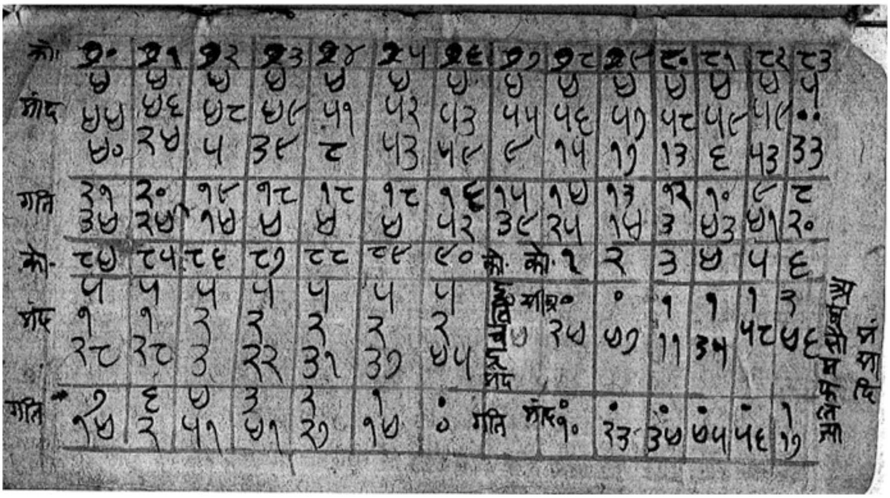

Fig. 3.10 A page from a manuscript of the *Makarandasāriṇī* (RORI 20231 p. 51) with overwritten argument values corrected from 7's to 8's.

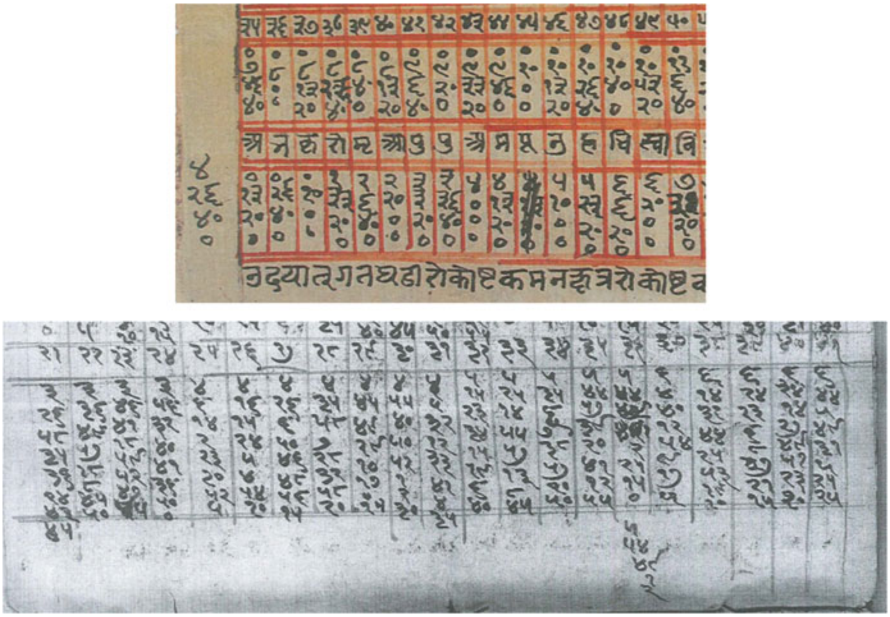

Fig. 3.11 Top: A manuscript of the *Candrārkī* (RORI 7752 f. 8r) showing an incorrect entry struck out and replaced with correct data in the left margin. Bottom: A manuscript of the *Makarandasāriṇī* (Smith Indic 185, unfoliated) showing an incorrect entry overwritten as well as replaced with correct data in the bottom margin.

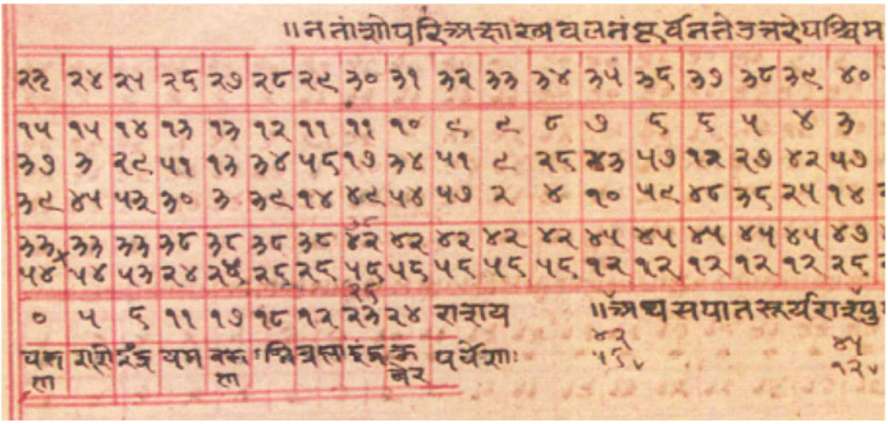

Fig. 3.12 Excerpt from *Karaṇakesarī* manuscript (RORI 12792 f. 3v). Instead of crossing out or overwriting the multiple mistakes in the second row, the scribe merely inserts the correct values beside or beneath them.

successive differences between the function values tabulated in the first row. Where the tabulated function is piecewise linear, the differences remain constant (although the constant value is not always precise to the nearest second). The scribe has incorrectly shifted his sequences of constant differences by one entry: i.e., the difference between entries 29 and 30 which appears under entry 30 should have had 38 in the minutes place, but 42 was written instead. As a result of this mistake, the subsequent jumps to 45 minutes in column 35 and to 47 minutes in column 40 were also premature. The scribe has detected these errors and written in the correct amounts of the differences, either in the same cell or as marginalia indicated with small inverted caret marks.

Overwriting incorrect entries was often preceded by, or omitted in favor of, painting them out with a colored pigment or paste, often made from turmeric.[9] The application of correction paste may sometimes reveal something about the way in which the scribe has originally copied the entries. For instance, the top image in Figure 3.13 shows a horizontal sequence of painted-out and overwritten errors. This configuration suggests that the scribe was copying the entries horizontally and propagated an initial error through several successive values.

In contrast, the two bottom images show sequences of painted-out errors running vertically down a column. The table on the left contains individual corrections to the second sexagesimal digit of successive entries in column 20, while the scribe of the table on the right has used paste to erase the entire contents of the column following column 22 before continuing with a new column 23. The inference is that in this

[9]"In a context, letter/s or word/s may have to be omitted [. . .] sometimes, yellow pigment or turmeric is smeared over such portion" (Murthy 1996, p. 109). See also Sircar (1965, p. 91) and Goswamy (2007, p. 38).

Fig. 3.13 Yellow turmeric paste used to paint over erroneous numbers. Top: *Karaṇakesarī* (RORI 12792 f. 5r). Horizontal sequence of erroneous final digits in entries in the third row, painted-out and then overwritten. Bottom left: *Karaṇakesarī* (RORI 12792 f. 8r). Painted-out and overwritten digits in column 20. Bottom right: *Jagadbhūṣaṇa* (RORI 10192 f. 54r). Entire column 23 painted-out and rewritten.

Fig. 3.14 Excerpt from a manuscript of the *Bṛhattithicintāmaṇi* (Smith Indic 151 f. 17v). Top: A single strip of paper originally pasted over a miscopied table row. Bottom: A table originally corrected with the above cut-and-pasted strip over its third row, which had initially been erroneously filled in with the table entries for the fourth row.

case, the scribe was copying the entries vertically and had corrupted part or all of a column before detecting his mistake. Other traces of correction paste elsewhere in these excerpts indicate rectification of isolated errors in separate entries.

For extensive corrections, a scribe might resort to cut-and-paste techniques. Figure 3.14 shows an example of such a correction applied to a miscopied row in a table. As the figure shows, the third and fourth rows of the table as initially written are identical, because the scribe too precipitately copied into row 3 the table-argument values intended for row 4. The error was remedied by copying the correct entries onto a separate small strip (shown in its present detached state in the figure) and sticking it over the miscopied row using some sort of adhesive, whose whitish traces are still evident in the image.

Besides sometimes writing numerical data incorrectly, scribes were prone to misjudge the layout and arrangement of their table structure. Examples of such errors are seen in the excerpts from three successive pages of a set of tables pictured in Figure 3.15: not only has the right edge of the table in the top left image spilled across the double-line marking the right margin, but entry 309 has been added on beyond the bottom right edge. The row containing the next entries beginning with 310, in the top right image, has clearly been ruled before attempting to write in the numbers: the scribe then found he did not have enough vertical room in the row, and drew in another horizontal line below the previously ruled one. On the next page, shown in the bottom image, the scribe evidently began to rule in more horizontal lines but gave up when he realized how few remaining table entries he had to fill them.

Figure 3.16 shows a similar miscalculation on the part of a tables scribe. He apparently pre-ruled the grids for two adjacent tables, the first with numerical argument values up through 67 and the second using syllabic abbreviations of zodiacal sign names as its arguments, and separated the two tables with a double vertical line. Then after writing entry 66 of the first table, he realized he had run out of columns and was forced to cross the double-line border to put entry 67 in the space reserved for the next table. In all these examples, even though the entries themselves are correct, the awkward layouts are potentially misleading and confusing for the user.

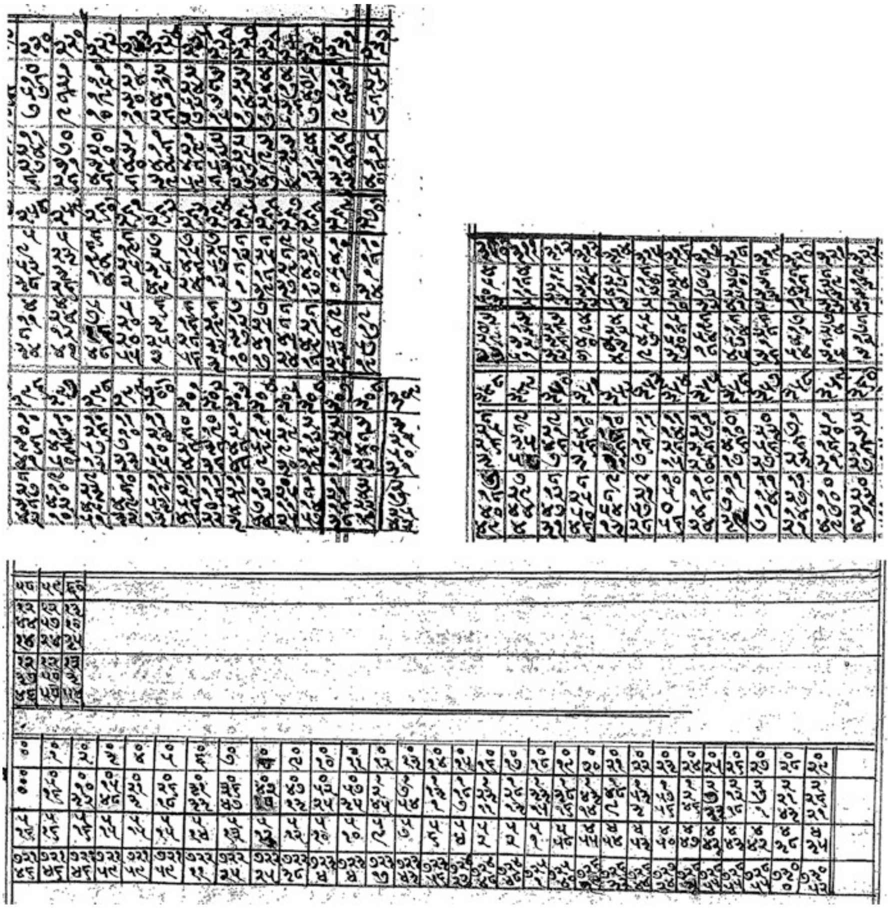

Fig. 3.15 A manuscript of the *Candrārkī* (Smith Indic 29 ff. 2r–3r, clockwise from top left) showing excerpts from images of three consecutive pages revealing table layout errors.

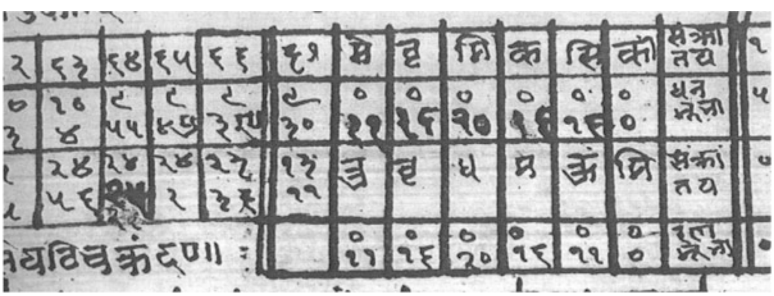

Fig. 3.16 A manuscript of the *Karaṇakesarī* (Smith Indic MB XXIV f. 4v) showing a table pre-ruled to end with argument 66 but actually extending up to argument 67, forcing the scribe to add the final entry in the next table's first column.

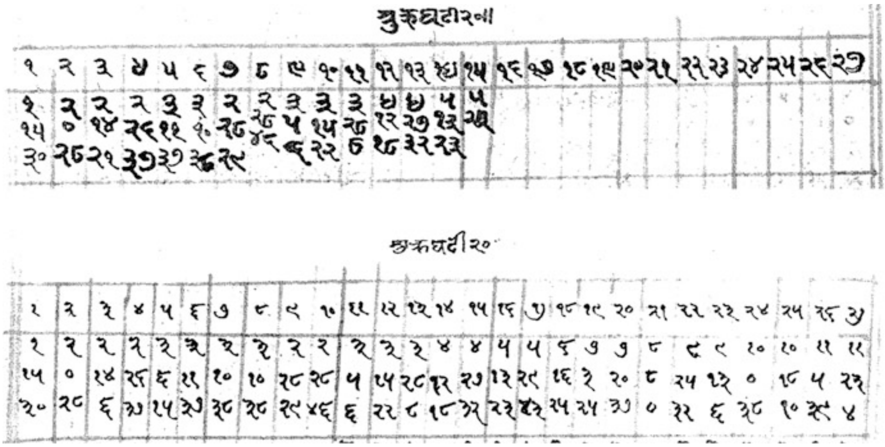

Fig. 3.17 A manuscript of the *Mahādevī* (Plofker 23 f. 102r–v). Top: The incomplete first row of a table that was pre-ruled but abandoned after being spoiled by the omission of an entry value. Bottom: The first row of the completed recopied version of the same table on the next page.

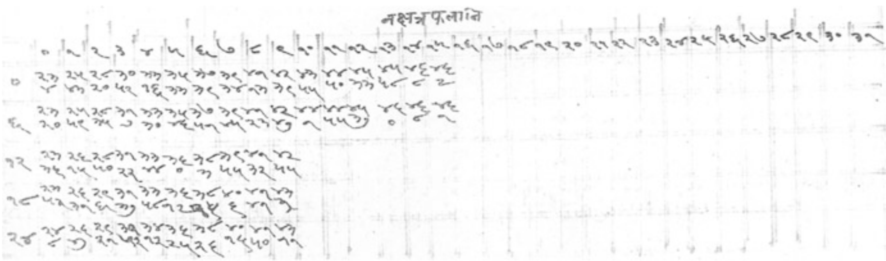

Fig. 3.18 A manuscript of the *Makaranda* (Smith Indic 185 f. 10r) with a table that was never completed.

Occasionally, individual tables are left incomplete after being deemed unsalvageable due to catastrophic copying errors. Figure 3.17 illustrates one such case, where a scribe presumably noticed (top image) in the middle of writing numbers in entry 15 that he had skipped the value proper to entry 5. He abandoned the table and (bottom image) recopied it on the next page, this time correctly including the value for entry 5 (although even in this recopied version he still omitted the correct value for entry 7, instead duplicating the value in entry 8).

Finally, Figure 3.18 shows a table simply abandoned by the scribe before completion, for no discernible reason. Such examples seem to be comparatively rare, presumably because unfinished tables would be of little use to practicing *jyotiṣīs*, and hence less likely to survive in manuscript form.

Chapter 4
Elements of table texts

The canonical structure and sequence of astronomical topics as represented in most major *siddhānta* and *karaṇa* works (see Section 1.4) are modified in the table-text genre in a number of different ways. As described in Section 2.3.1, a *sāraṇī* or *koṣṭhaka* is often dedicated to a particular astronomical task, such as finding planetary positions for horoscopes, computing eclipses, or constructing calendars. So individual tables containing values of specific astronomical functions are constructed, combined, and ordered differently in different table texts. For example, a work intended for making calendars or eclipse predictions will have tables for the motions of the sun and moon but not of the five star-planets, while a table text for planetary positions may omit tables for parallax, and so forth.

Because of this modular structure, we have undertaken in this chapter to describe a "standard" selection of individual tables as separate elements or components of diverse kinds of table texts. These tables represent the functions or algorithms most commonly employed in *koṣṭhaka*/*sāraṇī* works, although as the following examples indicate, their implementation and format vary widely depending on the intentions and skill of their compilers. The descriptions of many such works in Chapter 5 and Appendix A illustrate the variety of ways the individual tables can be combined to make up a *koṣṭhaka* customized for the compiler's purposes.

4.1 Mean longitudinal displacement

Since mean motions with their various corrections are the fundamental concept underlying most astronomical computations (see Section 2.1.1), almost all table texts address them. Typically, each celestial body is furnished with a set of tables of linear increments $\Delta\bar{\lambda}$ to mean longitude $\bar{\lambda}$, or displacements from its epoch

C. Montelle, K. Plofker, *Sanskrit Astronomical Tables*, Sources and Studies in the History of Mathematics and Physical Sciences,
https://doi.org/10.1007/978-3-319-97037-0_4

mean longitude, corresponding to successive integer multiples of various time-units. These units range from civil days (or even *ghaṭīs*) to "ideal months" of 30 days, "ideal years" of 360 days or actual (mean) years of 365+ days, and periods of 60, 100, or even thousands of years. Alternatively, longitudinal increments might be tabulated in multiples of integer days from 1 to 9, 10–90, 100–900, and so on for successive orders of magnitude.[1]

The user enters the mean longitude displacement tables for the desired planet with a given amount of time elapsed since epoch, typically the so-called *ahargaṇa* or accumulated civil days. This time interval is broken down into integer multiples of each of its component time-units t, such as multi-year periods and single years. The appropriate entry $\Delta\bar{\lambda}_t$ corresponding to each of these components is found in the tables, and all the entries are added up modulo 360° to produce the total displacement from the planet's epoch mean position at the desired date. The epoch mean position itself, generally called *kṣepaka* "additive [offset]" or *dhruva* "fixed [offset]," is frequently incorporated into one of these table entries. In such cases, the sum of the entries gives the planet's present mean longitude directly.

The tabulated mean longitudinal increments are usually computed from more precise mean-motion parameters, as we can infer from comparing their successive entries to corresponding integer multiples of their initial values. Nonetheless, such mean longitude tables are still too approximate to remain useful for any great period of time beyond their epoch date, even though their compact organization allows their compilers to nominally extend them far into the future over many centuries, millennia, or even more.[2]

Mean longitude tables: *Rājamṛgāṅka*

Figure 4.1 shows the mean longitudinal displacement tables for Jupiter from a manuscript of the *Rājamṛgāṅka* of Bhojarāja (see Section 5.1). The mean longitude increments or *bhogas* $\Delta\bar{\lambda}_t$ corresponding to successive multiples of each time-unit t are tabulated as shown in Table 4.1. Thirteen numbered subtables for each planet contain longitudinal increments for 1 to 9 days, 10 to 90 days, and so

[1] Such combinations of mean longitudinal increments for multiples of different periods had been employed in Greek and Islamic astronomical tables at least since the *Almagest* of Ptolemy, in which the increments are tabulated for 1–24 hours, 1–30 days, 1–12 30-day "months," 1–18 years, and 1–45 18-year periods. As the following examples show, Indian mean longitude tables employ very diverse combinations of time-units, usually in decimal multiples.

[2] See, for example, the multi-billion-year framework described in Table 4.1 in Section 4.1.

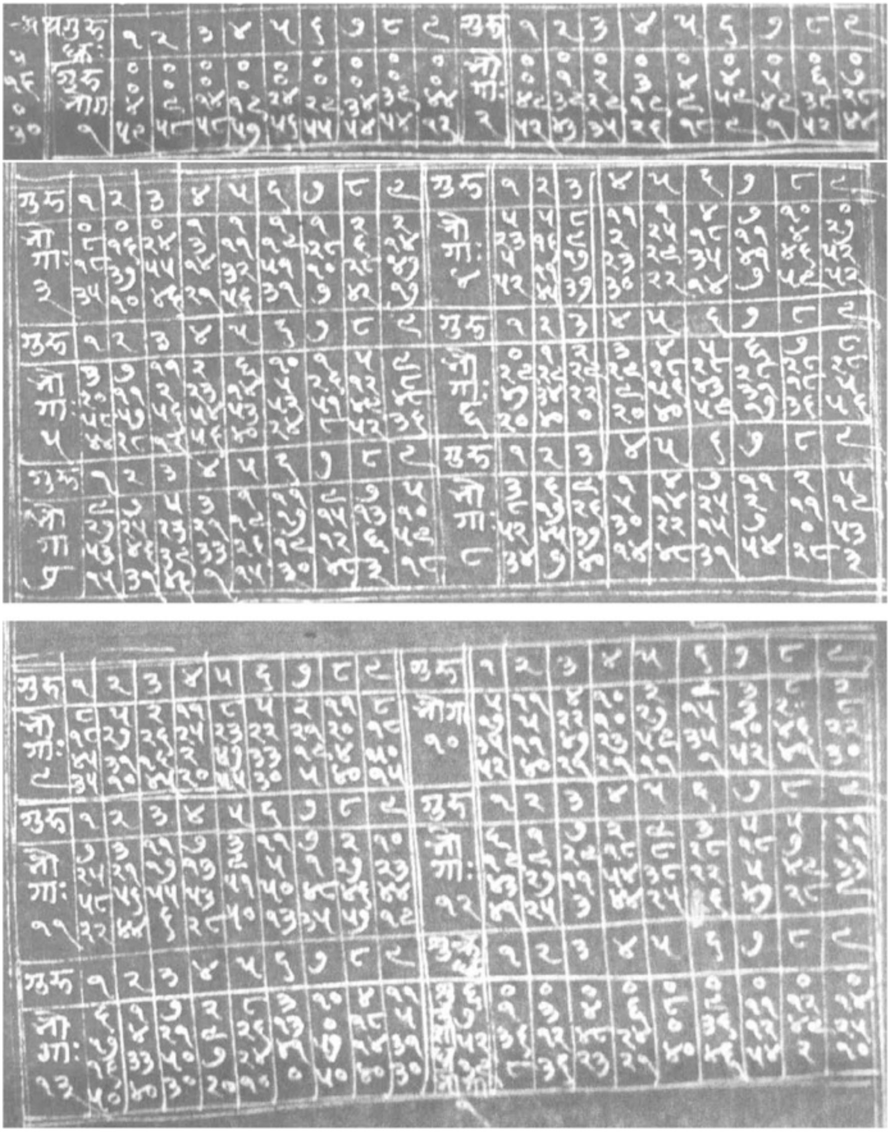

Fig. 4.1 Mean longitudinal displacement tables for Jupiter from the *Rājamṛgāṅka* (Baroda 9476 ff. 5r–6r).

on, up to 9,000,000,000,000 days or over 24 billion years (!). The mean daily longitudinal displacement used to generate these entries, given in a table earlier in the manuscript, is 0^s, $0°;4,59,9,8,37,23,45$.

Table 4.1 Characteristics of the mean longitudinal displacement tables for Jupiter in the *Rājamṛgāṅka* (Figure 4.1).

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ (to arcseconds)
1–9	Single days	0^s, 0°;4,59
	10 days	0^s, 0°;49,52
	100 days	0^s, 8°;18,35
	1000 days	5^s, 23°;5,52
	10,000 days	3^s, 20°;58,44
	10^5 days	0^s, 29°;47,20
	10^6 days	9^s, 27°;53,15
	10^7 days	3^s, 8°;52,34
	10^8 days	8^s, 28°;45,35
	10^9 days	5^s, 17°;35,52
	10^{10} days	7^s, 25°;58,22
	10^{11} days	6^s, 19°;43,41
	10^{12} days	6^s, 17°;16,50

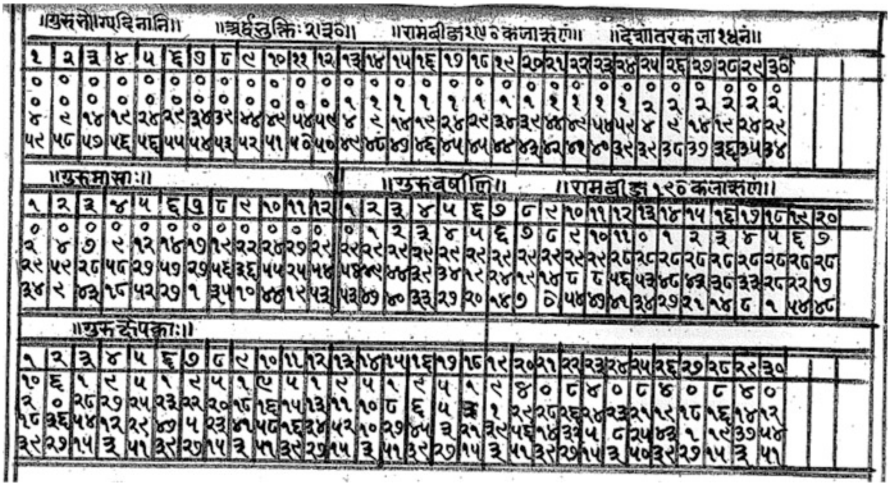

Fig. 4.2 Mean longitudinal displacement tables for Jupiter from the *Brahmatulyasāraṇī* (Smith Indic MB LVIII f. 4r). The table for single days is titled *gurubhogyadināni* “increments [for successive] days of Jupiter,” while that for 20-year periods is titled *gurukṣepakāḥ* “additive [offsets] of Jupiter”.

Epoch mean longitude: 3^s, 16°;0,30

Mean longitude tables: *Brahmatulyasāraṇī*

Figure 4.2 shows the mean longitudinal displacement table for Jupiter from a manuscript of the *Brahmatulyasāraṇī* (see Section 5.3), summarized in Table 4.2. Here the successive daily increments are called *bhogyas*.

Table 4.2 Characteristics of the mean longitudinal displacement tables for Jupiter in the *Brahmatulyasāraṇī* (Figure 4.2).

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ (to arcseconds)
1–30	Days	$0^s, 0°;4,59$
1–12	"Ideal months" of 30 days	$0^s, 2°;29,34$
1–20	"Ideal years" of 360 days	$0^s, 29°;54,53$
1–30	Periods of 20 "ideal years"	$7^s, 28°;17,48$

Epoch mean longitude: $2^s, 4°;0,51$ (from the *Karaṇakutūhala* (Mishra 1991, p. 4, verse 1.5)). This value is added as a one-time offset to $\Delta\lambda_t = 7^s, 28°;17,48$ in the first entry of the 20-year-period table. Consequently, mean longitude calculations for any date 20 years or more after the epoch date will automatically include the epoch mean longitude offset.

Paratext. The table headings include the following text:
ardhabhuktiḥ 2 | 30 "half-[daily] velocity 2;30 [minutes]."
rāmabīja 190 kalā ṛṇaṃ "*rāmabīja* 190 minutes, negative."
deśāntarakalā 1 dhanaṃ "minutes of longitudinal difference correction, 1, positive."

The procedure for computing a mean longitude is explained in the verse text which accompanies the tables (Montelle and Plofker 2015, pp. 7–8, verse 2):

> Firstly, computing the number of accumulated days as stated in the handbook, the learned who are cheerful in nature are to divide [it] by the amount 30; again this quotient should be divided by 12; one should divide the result by 20. Now, precisely these numbers called the four remainder-numbers, [entered into] their respective tables [with the corresponding entries] combined, are the [mean longitudes of the] planets at the city of Laṅkā [i.e., for zero degrees of terrestrial longitude].

Mean longitude tables: *Grahalāghavasāriṇī*

A version of the tables commonly called *Grahalāghavasāriṇī*, based on the *Grahalāghava* of Gaṇeśa (see Section 5.6.1), employs a somewhat similar mean longitude table structure illustrated in Figure 4.3 and summarized in Table 4.3, again using the example of Jupiter.

Epoch mean longitude: $7^s, 2°;16,0$ (from the *Grahalāghava* (Jośī 1994, p. 18, verses 1.6–8)). This value is added as a one-time offset to the first entry of the table of longitudinal increments for successive *cakras* or periods of 4016 days (approximately 11 years). Note that all the entries in this table are effectively given only to arcminutes, since the second sexagesimal place is constantly zero.

Paratext. The titles of the subtables in Figure 4.3, reading from bottom to top, form a condensed guide to the mean longitude computation:

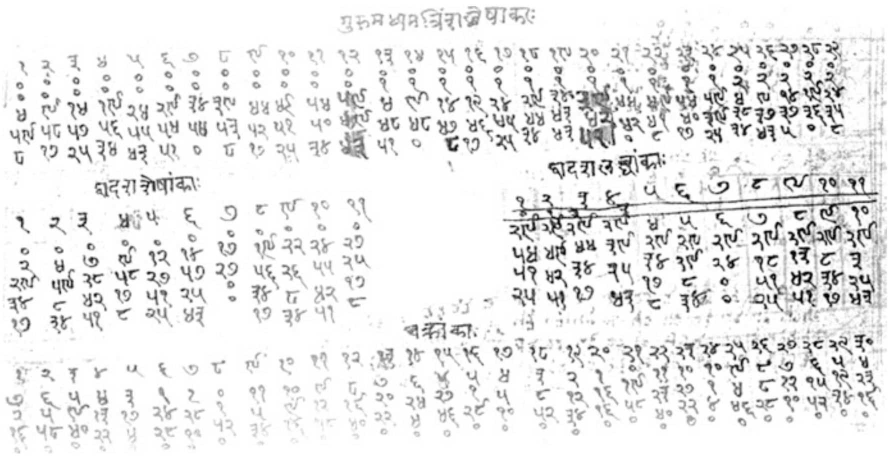

Fig. 4.3 Mean longitudinal displacement tables for Jupiter in the *Grahalāghavasāriṇī* (Plofker 26 f. 4r).

Table 4.3 Characteristics of the *Grahalāghavasāriṇī*'s mean longitudinal displacement tables for Jupiter (Figure 4.3).

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ (to arc-thirds)
1–29	Days	0^s, 0°;4,59,8
1–11	"Ideal months" of 30 days	0^s, 2°;29,34,17
1–11	"Ideal years" of 360 days	0^s, 29°;54,51,25
1–30	Periods of 4016 days (*cakras*)	11^s, 3°;42,0

"Cycle" table: *cakrāṃkāḥ* "numbers [pertaining to] the *cakras*," i.e., the periods of 4016 days. The first entry is the epoch mean longitude.

"Ideal year" table: *dvādaśalabdhāṃkāḥ* "numbers [pertaining to] the quotient [upon division] by 12," i.e., integer multiples of 12 30-day periods in the days left over from the integer number of 4016-day cycles.

"Ideal month" table: *dvādaśaśeṣāṃkāḥ* "numbers [pertaining to] the remainder [upon division] by 12," i.e., the remaining number of 30-day periods when integer multiples of 12 of them have been removed.

Day table: *gurumadhyamatriṃśaccheṣā[ṅ]kāḥ* "Jupiter mean [longitude] numbers [pertaining to] the remainder [upon division] by 30," i.e., the remainder in single days when integer multiples of 30 days have been removed.

Figure 4.4, summarized in Table 4.4, shows an additional table from the same manuscript containing Jupiter's mean longitudinal displacements for shorter time increments. A different recension of the *Grahalāghavasāriṇī*, in a manuscript shown in Figure 4.5, employs yet another set of time-units for its mean longitude tables.[3]

[3] See Table 5.11 for a summary of the varying systems of tabulating mean longitude increments in different recensions of the *Grahalāghavasāriṇī*. These recensions illustrate how variously table compilers might approach the task of recasting the same *karaṇa* in table-text form.

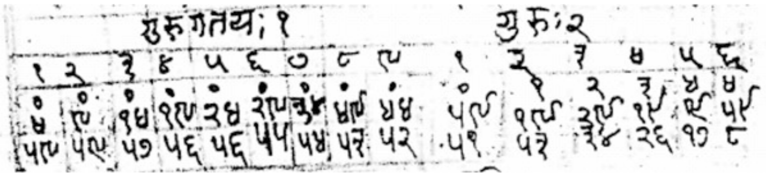

Fig. 4.4 Additional mean longitudinal displacement tables for Jupiter in the *Grahalāghavasāriṇī* (Plofker 26 f. 14r). Note that the scribe has miscopied the first two entries in the second table, so the initial entry appears different from the actual $\Delta\bar{\lambda}_t$ in Table 4.4.

Table 4.4 Characteristics of the *Grahalāghavasāriṇī*'s additional mean longitudinal displacement tables for Jupiter (Figure 4.4) .

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ (to arc-seconds)
1–9	Single *ghaṭīs*	0°;4,59
1–6	Intervals of 10 *ghaṭīs*	0°;49,51

Fig. 4.5 Excerpts from the mean longitudinal displacement tables for Jupiter from a second version of the *Grahalāghavasāriṇī* (Smith Indic 26). Left (f. 1r): Increments of 10 days and single days. Right (ff. 1v–2r): Decades beginning with Śaka 1676 and single years.

Mean longitude tables: *Makaranda*

A contrasting approach to arranging mean longitude displacement tables is used by Makaranda in the *Makaranda* (Section 5.5). Each planet has only one table containing sexagesimal entries called *vāṭikā*, precise to 8 places, which can be read in "floating-point" form to apply to any time-unit measured by successive powers of

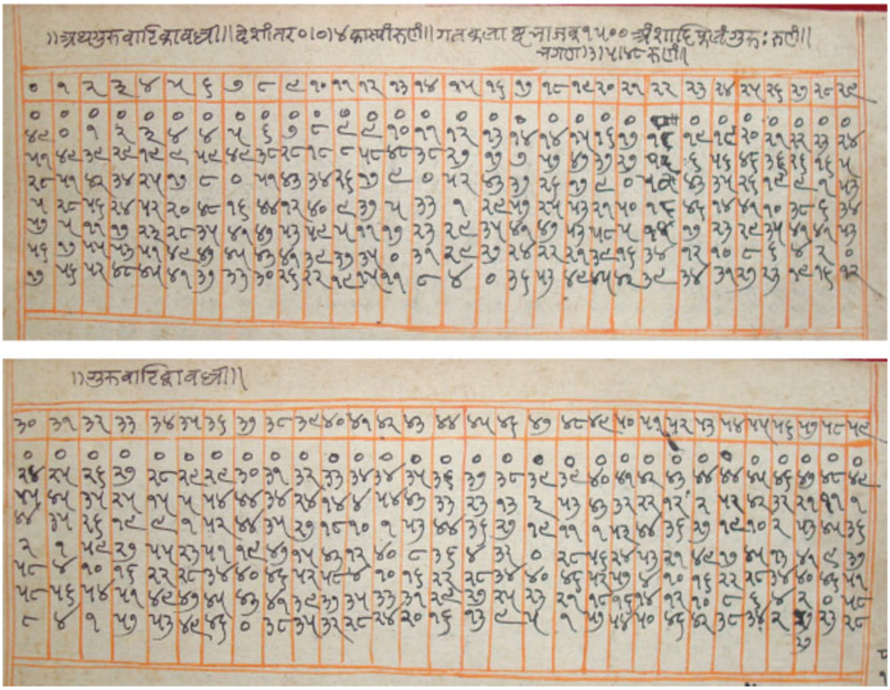

Fig. 4.6 Mean longitudinal displacement tables for Jupiter from the *Makaranda* of Makaranda (Baroda 3225 f. 18r–v) arranged according to the *vāṭikā* scheme.

Table 4.5 Characteristics of the mean longitudinal displacement tables for Jupiter in the *Makaranda* (Figure 4.6).

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ in "floating point form"
0–59	Any sexagesimal power of 10 *ghaṭīs*	0, 49, 51, 28, 5, 57, 56, 17

60, beginning with 10 *ghaṭīs*. Figure 4.6 shows the mean longitudinal displacement table for Jupiter from a manuscript of the *Makaranda*, whose numerical data is summarized in Table 4.5.

Under the argument value 0 appears the difference between successive entries, given as 0;49,51,28,5,57,56,17. The mean daily longitudinal displacement can be read off the sixth entry (= $10 \times 6 = 60$ *ghaṭīs* or 1 day): namely, 0°;4,59,8,48,35,47,37. With the appropriate adjustment in units, this column can also produce the mean longitudinal displacement for longer time intervals. For instance, the same cell, read as 4°;59,8,48,35,47,37, also represents the longitudinal displacement for $10 \times 6 \times 60 = 3600$ *ghaṭīs*, or 60 days. The table data is thus entirely sexagesimal.

Paratext. The table heading reads *guruvāṭikāvallī* || *deśāṃtara 0* | *0* | *4 kāsyāṃ ṛṇaṃ* || *gatakalābdabhājaka 1500 aṃśādiphalaṃ guruḥ ṛṇaṃ* || *bhagaṇa* | *3* | *5*

| *48 ṛṇaṃ* || "The *vāṭikā*-increments of Jupiter; [its] longitudinal difference 0, 0, 4 in Kāśī, negative. The divisor of elapsed Kali-years [reading *kalya* for *kalā*], 1500; correction [of] Jupiter in degrees etc., negative. The *bhagaṇa* [?] 3, 5, 48, negative."

Mean longitude tables: *Grahajñāna*

Figure 4.7 shows the mean longitudinal displacement table for Jupiter from a manuscript of the mean-to-true table text *Grahajñāna* by Āśādhara (see Section 5.2). The mean longitude increments corresponding to successive multiples of years are tabulated as shown in Table 4.6, where the mean longitude increment is given not in signs, degrees, minutes, and so on, but in units of *nakṣatras* (13°;20).

Epoch mean longitude: −3, 6;48,36,0 in 13°;20 arc-units.

Paratext. The table heading reads simply *bṛhaspati* "Jupiter," repeated in the table footer. The first table column, labelled *ko* (presumably *koṣṭhaka*) gives abbreviations for each of the units of the numerical data: *na· paṃ·*, *ghaṭī, pala, akṣa, vyakṣa*. These appear to denote *nakṣatra-paṅkti* "*nakṣatra*-arc-units" and the successive fractional

Fig. 4.7 Mean longitudinal displacement tables for Jupiter in the *Grahajñāna* (RAS Tod 36c f. 4r).

Table 4.6 Characteristics of the mean longitudinal displacement tables for Jupiter in the *Grahajñāna* (Figure 4.7) measured in 13°;20 arc-units.

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ (in 13°;20 arc-units)
1, 2, ..., 9	Years	2, 16, 34, 58, 18
10, 20, ..., 90	Years	22, 45, 49, 43, 0
100, 200, ..., 900	Years	11, 38, 17, 10, 0
1000, 2000, ..., 10000	Years	8, 22, 51, 40, 0

powers of the 13°;20 arc-unit. The following two columns are headed *kṣepa* "epoch longitude" and *bīja*.

Paratext running through the center of the table reads *atha śrī āśādhare brahmapakṣe ekā 'dau ayutapāryata kṣepakabījasaṃskāravinā gurukoṣṭā varṣabhogā dhanaṃ* || "Now [in the work of] Śrī Āśādhara in the Brāhmapakṣa: [from] one [year] in the beginning to ten thousand at the end, the annual-increment table entries for Jupiter without the epoch mean position and *bīja* correction, [applied] positively."

Mean longitude tables: *Mahādevī*

Figure 4.8 gives another example of a mean longitudinal displacement table in the mean-to-true format from a manuscript of the *Mahādevī* of Mahādeva (see Section 5.4). The mean longitude increments corresponding to successive multiples of each time-unit are shown in Table 4.7 where the mean longitude increment $\Delta\bar{\lambda}_t$ per given time-unit t is measured in 6° arc-units. Each subtable is titled *eka* "one," *daśaka* "ten," and *śata* "hundred," respectively.

Epoch mean longitude: 0, 23, 22, 20, 0, 56 in 6° arc-units. This value is added as a one-time offset in entry 0 of the table of single years. Consequently, mean longitude calculations for any date computing using single years will automatically include the epoch mean longitude offset. However, the 10-year period and 100-year periods do not have the epoch mean longitudinal offset included in them. If a date computation does not involve single years, the epoch mean longitude offset must be included in the calculation.

Fig. 4.8 Mean longitudinal displacement tables for Jupiter in the *Mahādevī* (BORI 497 f. 8v).

Table 4.7 Characteristics of the mean longitudinal displacement tables for Jupiter in the *Mahādevī* (Figure 4.8) measured in 6° arc-units.

Argument range	Time-unit t	$\Delta\bar{\lambda}_t$ (in 6° arc-units)
$1, 2, \ldots, 9$	Single years	5, 3, 31, 5, 7
$1, 2, \ldots, 9$	10-year periods	50, 35, 10, 51, 10
$1, 2, \ldots, 9$	100-year periods	25, 51, 48, 31, 40

Paratext. The table heading reads *guro koṣṭhakā sakṣepā sabījā* "Tables of Jupiter with the epoch mean longitude (*kṣepa*) and with the *bīja*-correction."

4.2 True longitudes and velocities

Almost all *koṣṭhaka* works include some tables for converting mean motions to true ones, for sun and moon alone in the case of the various luni-solar table texts, or else for all the planets. The form of such tables depends on the work's basic structure, as described in Section 2.3.2. "Mean-with-equation" tables provide additive corrections to mean positions and velocities based on the algorithms discussed in Sections 2.1.2 and 2.1.3.

4.2.1 Correcting longitude and velocity for *manda*-inequality

Manda-correction tables: *Brahmatulyasāraṇī*

Figure 4.9 shows the table of the sun's *manda*-equation and velocity correction or *gatiphala* from a manuscript of the *Brahmatulyasāraṇī*. See Montelle and Plofker (2015) for more details on the following description of its content.

Argument: 1 to 90° of *manda*-anomaly κ_M.

Table entries. The table contains the following three rows:

Row 1: Degrees, arcminutes and arcseconds of *manda*-equation.

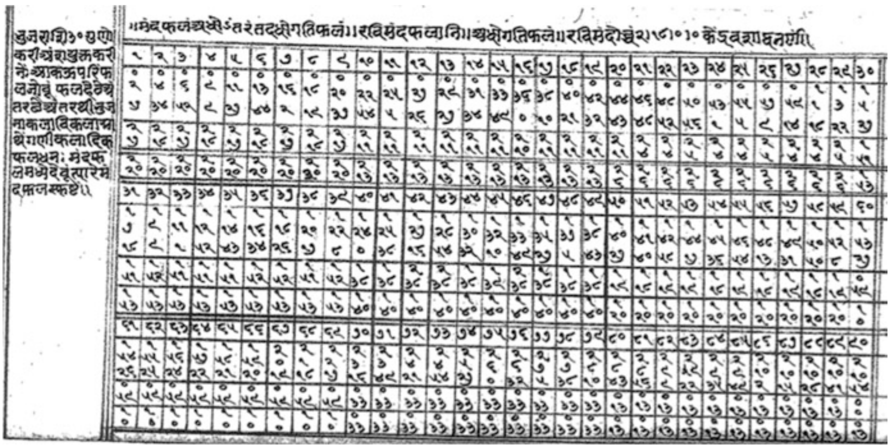

Fig. 4.9 Solar *manda*-equation table from the *Brahmatulyasāraṇī* (Smith Indic 45 f. 6v).

Row 2: Differences between successive entries in Row 1.
Row 3: Velocity correction due to the *manda*-equation, in arcminutes and arcseconds.

Maximum *manda*-equation: 2°;10,54.

Method of computation.

Row 1. The entries for the *manda*-equation are based on the *Karaṇakutūhala* approximation for μ discussed in Section 5.3.1. The data in the table in Figure 4.9 reveal that the table compiler has computed the solar *manda*-equation according to this algorithm for every integer multiple of 10° of argument. This mimics the structure of the *Karaṇakutūhala*'s R sin table (see Table 5.3), which similarly gives function values only for integer multiples of 10° of arc. Intermediate values have evidently been found by linear interpolation, as indicated by the entries in the differences row, which are constant for each 10° block.

Row 3. For the velocity correction due to the *manda*-anomaly or *gatiphala* $\overset{M}{\Delta} v$, the table compiler apparently employed the *Karaṇakutūhala*'s *gatiphala* formula likewise discussed in Section 5.3.1. The tabulated values in row 3 of the table in Figure 4.9 agree with the *gatiphala* numbers reconstructed in column 5 of Table 5.3, truncated at arcseconds.

Paratext. The table heading reads *mandaphalaṃ adho 'ntaraṃ tadadho gatiphalaṃ* || *ravimandaphalāni* || *adho gatiphalaṃ* || *ravimandoccaṃ 2* | *18* | *0* | *0 kendravaśād dhanarṇaṃ* || "The *manda*-equation, below [it] the difference, below it the velocity correction. The solar *manda*-equations. Below, the velocity correction. The solar apogee is $2^s, 18°; 0, 0$. According to the anomaly [it is to be applied] positively or negatively."

We have not translated the note in a Sanskrit-related vernacular in the left margin.

Manda-correction tables: *Grahalāghavasāriṇī*

Figure 4.10 shows a table of the sun's *manda*-equation and *gatiphala*, for each degree, from a manuscript of the *Grahalāghavasāriṇī* (compare the scaled values in Table 5.8).

Argument: 1 to 90° of *manda*-anomaly κ_M.

Table entries. The table contains the following two rows:

Row 1: Degrees, arcminutes and arcseconds of *manda*-equation.
Row 2: Velocity correction due to the *manda*-equation, in arcminutes and arcseconds.

Maximum *manda*-equation: 2°;10,45.

Paratext. The table heading reads *sūryamandaphalaṃ sūryamandoccaṃ 2* | *18* | *0* | *0* "*Manda*-correction of the sun. Solar *manda*-apogee [longitude] 2^s, 18°;0,0."

Figure 4.11 shows a differently constructed *manda*-correction table for the sun in another version of the *Grahalāghavasāriṇī*.

Fig. 4.10 Solar *manda*-correction table from the *Grahalāghavasāriṇī* (Plofker 26 f. 5v).

Fig. 4.11 Solar *manda*-correction table from another version of the *Grahalāghavasāriṇī* (Smith Indic 26 f. 3r). See Section 3.2.1 for a discussion of the vertical strokes used in some of the cells as rounded fractions.

Argument: 0 to 18 five-degree increments (corresponding to 0–90°) of *manda*-anomaly κ_M.

Table entries. The table contains the following five rows:

Row 1: Degrees, arcminutes and arcseconds of *manda*-equation.
Row 2: One-fifth of the difference between the corresponding and next entries in Row 1 (equivalent to the increment in *manda*-equation between successive individual degrees of anomaly).
Row 3: Velocity correction due to the *manda*-equation, in arcminutes and arcseconds.
Row 4: Approximately one-fifth of the difference between the corresponding and next entries in Row 3 (apparently intended to be equivalent to the increment in *manda* velocity-correction between successive individual degrees of anomaly; but the computations are inconsistent).
Row 5: Correction to the moon's position due to solar *bhujāntara* or equation of time component due to sun's *manda*-equation (see below).

Maximum *manda*-equation: 2°;10,45.[4]

Method of computation. The *manda*-equation values and the differences and velocity correction derived from them appear to be produced by the same algorithm used to generate the table in Figure 4.10. The table's last row, the so-called *bhujaphala* of the moon (not to be confused with the trigonometric *bhujaphala* described in Figure 2.2), is a correction to the moon's position based on the time discrepancy corresponding to the difference between the sun's mean and *manda*-corrected true positions. This difference is equivalent to the *bhujāntara* component of the equation of time (see Section 2.1.5). The table compiler seems to have employed a rough approximation to the algorithm for this quantity stated in *Grahalāghava* 2.7, where the lunar *bhujaphala* is the solar *mandaphala* divided by 27.

Paratext. The table heading reads *ravi caṃdra uca maṃdaphala gatiphala bhujaphala 'ṃtaraṃ* "Sun, Moon, apogee, *manda*-equation, velocity-correction, *bhujaphala* [i.e., *bhujāntara*, and] differences."

The header for Row 5 reads *bhujaphala caṃdrasya* "*bhujaphala* of the moon." The empty column at the end of the table contains in the argument row *la* (for *lavāḥ*, "degrees [of anomaly]"?). In the cells beneath it are *ravimaṃ u 2 | 18* "Sun's *manda*-apogee $2^s 18°$," *me dha* (presumably for *meṣādi dhana* "positive [*manda*-equation when anomaly is in the six signs] beginning with Aries"), *tu* × (presumably for *tulādi ṛṇa* "negative [*manda*-equation when anomaly is in the six signs] beginning with Libra").

Figure 4.12 shows an excerpt for Jupiter from the same *Grahalāghavasāriṇī* manuscript's table of *manda*-equation and *gatiphala*.

[4] Although this appears to read 2 14 45, an overwritten correction by the scribe suggests it should be read as 2 10 45.

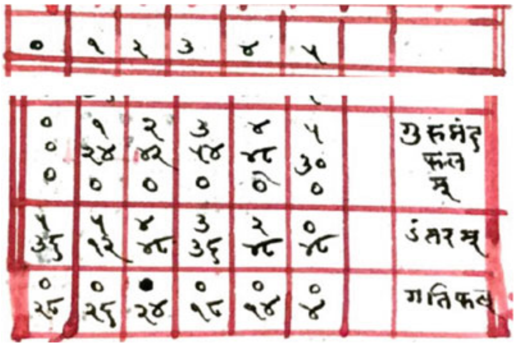

Fig. 4.12 Excerpt for Jupiter from the *manda*-correction table in the second version of the *Grahalāghavasāriṇī* (Smith Indic 26 f. 3v).

Argument: 0 to 5 fifteen-degree increments (corresponding to 0–75°) of *manda*-anomaly κ_M. (The scribe appears to have omitted the final column for the argument interval 75–90°.)

Table entries. The table contains the following three rows:

Row 1: Degrees, arcminutes and arcseconds of *manda*-equation.
Row 2: A fifteenth part of the difference between the corresponding and next entries in Row 1 (equivalent to the increment in *manda*-equation between successive individual degrees of anomaly).
Row 3: Velocity correction due to the *manda*-equation, in arcminutes and arcseconds.

Method of computation. The *manda*-corrections employed here are borrowed from the verse table of pre-computed scaled *manda*-equation values in the *Grahalāghava* (Jośī 1994, p. 79, verses 3.7–8) (see Table 5.8).

4.2.2 Correcting longitude and velocity for *śīghra*-inequality

Śīghra-correction tables: *Brahmatulyasāraṇī*

Figure 4.13 shows excerpts from the *śīghra*-equation tables from a manuscript of the *Brahmatulyasāraṇī*, for Jupiter and Mercury.

Argument: 1 to 180° of *śīghra*-anomaly κ_S.

Table entries. The tables each contain the following three rows:

Row 1: Degrees, arcminutes and arcseconds of *śīghra*-equation.
Row 2: Differences between successive entries in Row 1.
Row 3: *Śīghra*-hypotenuse H_S: decimal integer and sexagesimal fractional (to two places).

Maximum *śīghra*-equation.

Jupiter: 10°;59,1 at argument value 100°.
Mercury: 21°;36,58 at argument value 110°.

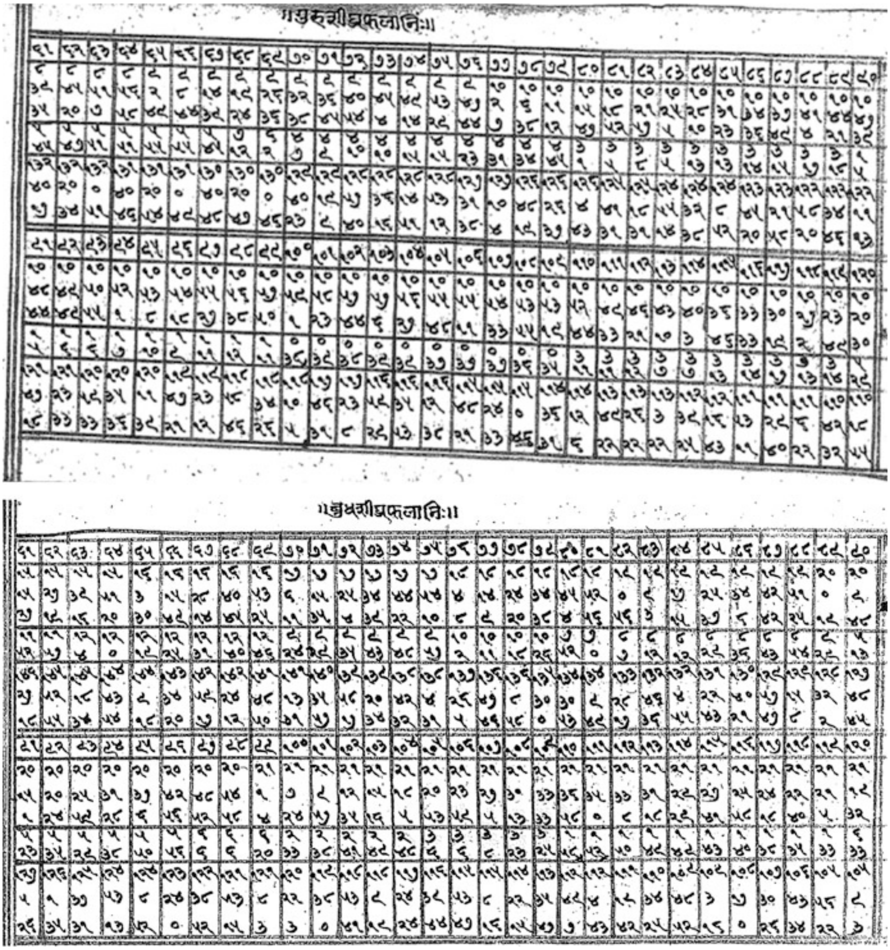

Fig. 4.13 Excerpt from *śīghra*-equation tables from the *Brahmatulyasāraṇī* (Smith Indic 45). Top: Jupiter, arguments 61–120 (f. 13r). Bottom: Mercury, arguments 61–120 (f. 11r).

Method of computation. Reconstruction of these tabulated values using the trigonometric formulas for H_S and σ described in Section 2.1.3, with the *Karaṇakutūhala*'s R sin table reproduced in Table 5.3, agrees with the attested entries to within a few arcseconds. So we infer that the table compiler used more or less the same procedure that we have described to determine the *śīghra*-equation.

Note that there is no table row for the *śīghra-gatiphala* or velocity correction due to the *śīghra*. Instead, the author of the *Brahmatulyasāraṇī* simply prescribes adding to the *manda*-corrected mean velocity $\bar{v}_M$ the product of the *śīghra*-anomaly velocity and the appropriate *śīghra*-equation difference from Row 2; see the discussion of this approximation in Section 2.1.3 and Montelle and Plofker (2015, p. 23).

Paratext. The table headings read as follows:

Jupiter: *guruśīghraphalāniḥ* "Jupiter's *śīghra*-equations."
Mercury: *budhaśīghraphalāniḥ* "Mercury's *śīghra*-equations."

Śīghra-correction tables: *Grahalāghavasāriṇī*

The *śīghra*-corrections employed by various versions of the *Grahalāghavasāriṇī* are borrowed from the verse table of pre-computed scaled *śīghra*-equation values in the *Grahalāghava*, see Section 5.6.1, particularly Tables 5.9 and 5.10.

Figure 4.14 shows an excerpt for Jupiter and Venus from the table of *śīghra*-equations and corresponding velocity corrections from a manuscript of the *Grahalāghavasāriṇī*.

Argument: 0 to 11 fifteen-degree increments (corresponding to 0–180°) of *śīghra*-anomaly κ_S.

Table entries. The following three rows are tabulated separately for Jupiter and for Venus:

Fig. 4.14 *Śīghra*-correction tables from the *Grahalāghavasāriṇī* (Smith Indic 26 f. 4v: Jupiter, top three rows, and Venus, bottom three rows).

Fig. 4.15 Excerpt from the *śīghra*-equation table from a second version of the *Grahalāghava-sāriṇī* (Plofker 26 f. 11v, Jupiter), for arguments 91–180.

Row 1: Degrees, arcminutes and arcseconds of *śīghra*-equation.
Row 2: One-fifteenth of the difference between the corresponding and next entries in Row 1 (equivalent to the increment in *śīghra*-equation between successive individual degrees of anomaly).
Row 3: Velocity correction due to the *śīghra*-equation, in arcminutes and arcseconds.

Maximum *śīghra*-equation.

Jupiter: 10°;48,0 at argument value 7 (= 105°).
Venus: 46°;6,0 at argument value 9 (= 135°).

Paratext. The table heading reads *śīghrakeṃdra da adhikaṃ tadā 12 śodhyam* “The *śīghra*-anomaly [? when] greater [than 180°], then subtracted from 12 [zodiacal signs, i.e., 360°].” The header of the argument row reads *rāśi* “quantity” (note the ambiguity of using this standard term for 30° zodiacal sign to mean a 15° arc).

Figure 4.15 illustrates a different form of *śīghra*-correction table for Jupiter from another version of the *Grahalāghavasāriṇī.*

Argument: 0 to 180° of *śīghra*-anomaly κ_S.

Table entries.

Row 1: Degrees, arcminutes and arcseconds of *śīghra*-equation.
Row 2: Velocity correction due to the *śīghra*-equation, in arcminutes and arcseconds.

Maximum *śīghra*-equation: 10°;48,0 at argument value 105°.

Method of computation: The *śīghra*-equation and *gatiphala* table entries for integer multiples of 15° of argument are borrowed from the *Grahalāghava*’s list. Intervening values are computed by linear interpolation in the case of the *śīghra*-equation and by simple repetition for the velocity correction.

Paratext. The table heading reads *guruśīghraphalaṃ* “Jupiter’s *śīghra*-equation.”

4.2.3 True longitude and velocity in mean-to-true table texts

As discussed in Section 2.3.2, table texts of the "mean-to-true" variety tabulate values of the desired true longitude and velocity directly, instead of supplying tables of *phalas* or additive corrections/equations to mean longitude and velocity. Figure 4.16 shows tables of true longitude and velocity for the first three 6° mean longitude intervals of Mars from a manuscript of the *Mahādevī*. The following description applies to each of these individual tables.

Argument: 1 to 27 *avadhis* of mean solar longitude.

Table entries. Each table contains the following five rows:

Row 1: 6° arc-units (sexagesimal integer and fractional to two places) of true longitude.
Row 2: Differences between entries in Row 1 and the corresponding entries in Row 1 of the next table.
Row 3: Arcminutes and arcseconds of true daily velocity.
Row 4: Differences between entries in Row 3 and the corresponding entries in Row 3 of the next table.
Row 5: Positive or negative integer "divisor" whose purpose is still unclear to us.

Occurrences of synodic phenomena (see Section 4.3) are noted beneath the table column of the *avadhi* in which they fall.

Paratext. Each table heading reads *bhaumaghaṭī* "Mars, mean longitude interval" followed by the table number. A note in the top margin reads *śrī gurur jayati* "The revered Guru is victorious."

4.2.4 True longitude and velocity in cyclic table texts

Like mean-to-true table texts, cyclic ones tabulate true longitude and velocity directly, assigning one table per year of the planet's cycle. Figure 4.17 shows tables of true longitude and velocity for the first three year-intervals of Mars from a manuscript of the *Jagadbhūṣaṇa*. The following description applies to each of these individual tables.

Argument: 1 to 27 *avadhis* of mean solar longitude.

Table entries: Each table contains the following two rows:

Row 1: Zodiacal signs, degrees, arcminutes and arcseconds of true longitude.
Row 2: Arcminutes and arcseconds of true daily velocity.

Occurrences of synodic phenomena (see Section 4.3) are noted in the right margin of the table in which they fall.

Paratext. The table heading reads *jagadbhūṣaṇe 1560 bhaumaḥ spaṣṭapaṃktayo likhyaṃte bhaume cakrabhrame kalā 35 dhanaṃ grahya* "In the *Jagadbhūṣaṇa*, [Śaka] 1560, Mars: the tables of true [longitude and velocity] are written. In the

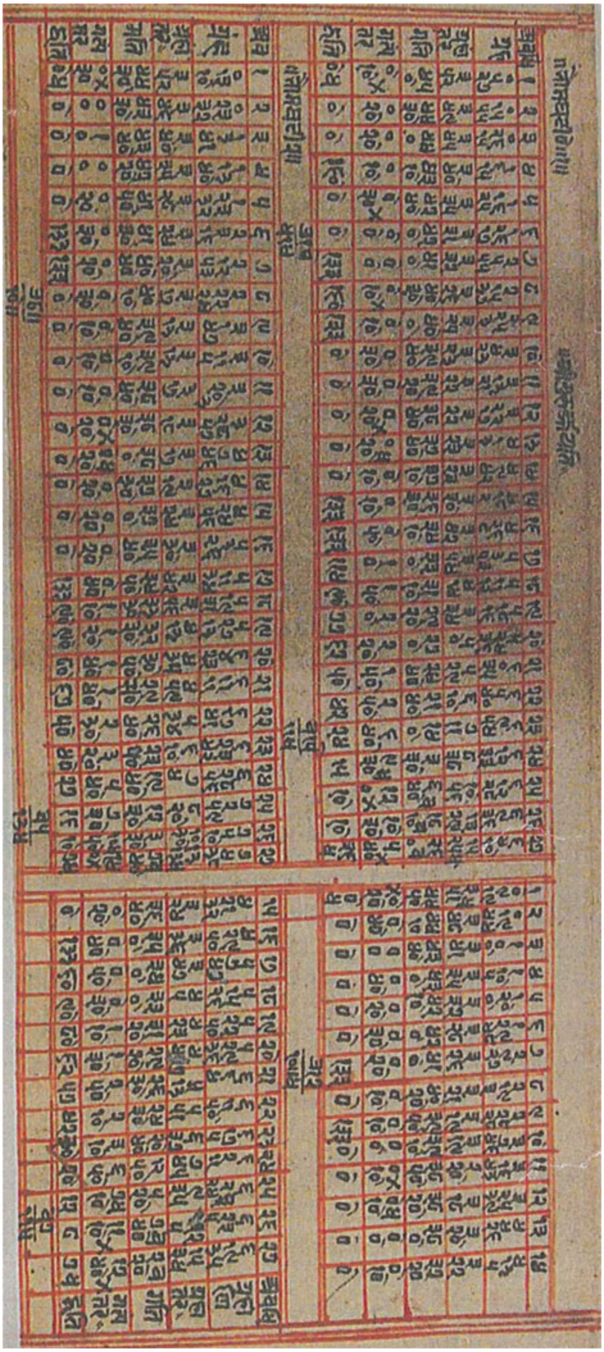

Fig. 4.16 Excerpt from the tables of true longitude and velocity for Mars from the *Mahādevī* (RAS Tod 24 f. 3v), for mean longitude intervals 0 (top left), 1 (right), and 2 (bottom left).

Fig. 4.17 Excerpt from the tables of true longitude and velocity for Mars from the *Jagadbhūṣaṇa* (RORI 20253 f. 1r), for years 0–2.

Table 4.8 Planetary cycle data from the *Jagadbhūṣaṇa*: periods of revolution in true longitude (years) and excess/deficiency in longitude over integer revolutions (arcminutes).

Planet	Cyclic period (years)	Remainder (′)
Mars	79	+35
Mercury	46	
Jupiter	83	−40
Venus	227	
Saturn	59	+40
Lunar node	93	−25;44

turning of a cycle in the case of Mars, 35 minutes [excess longitude], positive, [are] obtained.”

A note above the heading reads *hīnaḥ śākaiḥ* “diminished by the [accumulated] Śaka [years].” A list of planetary cycle values in the upper left (see Table 4.8) reads *maṃ 79 ka 35 dha bu 46 bṛ 83 ka 40 ṛ śu 227 śa 59 ka 40 dha rā 93 kalā 25 44* ×.

4.3 Synodic phenomena of planets

Mean-to-true table texts often include occurrences of retrogradation, heliacal rising, etc. (see Section 2.1.4), in tables of planetary true longitude, especially for the purpose of horoscope computations. Sometimes they merely tabulate the standard values of the *śīghra*-anomaly (i.e., the synodic elongation or distance between the planet and the sun, see Section 2.1.3) at which the various phenomena occur.

Synodic phenomena: *Laghukhecarasiddhi*

Figure 4.18 shows a table of the abovementioned standard synodic-elongation values (with occasional scribal errors) from a manuscript of the *Laghukhecarasiddhi* of Śrīdhara. The entries are in zodiacal signs and degrees with zero values for arcminutes and arcseconds.

Synodic phenomena: *Jagadbhūṣaṇa*

Figure 4.19 shows occurrences of synodic phenomena for Jupiter and Mercury in manuscripts of the *Jagadbhūṣaṇa* of Haridatta (see also Figure 4.17 and Table 5.15.) Each row lists true positions and velocities for *avadhis* 1–27 of the specified year or years.

The manuscript image on the left side shows the synodic phenomena for Jupiter at the end of the page, as the day and *ghaṭī* within the specified *avadhi* that the specified phenomenon occurs. Thus the entry column for the first occurrence reads

Fig. 4.18 Signs and degrees of synodic elongation for synodic phenomena of Mars, Mercury, Jupiter, and Venus (left), and of Saturn (right), from the *Laghukhecarasiddhi* of Śrīdhara (IO 2408b ff. 8v–9r).

' 8 [for *'vadhi*, i.e., *avadhi*, number 8] *' 9;36* [for *'sta* or *asta*, acronycal setting, day 9, *ghaṭī* 36]. The subsequent phenomena are denoted in succession by *u* (*udaya*, heliacal rising), *va* (*vakra*, first station), and *mā* (*mārga*, second station) before returning to *'* for *asta*.

In the manuscript on the right, the entry with the day and time for each occurrence for Mercury is inserted under the column for the appropriate *avadhi*. But the sequence of abbreviations is *' pa* (*asta-paścima*, last visibility in west), *mā* (*mārga*, second station), *u pū* (*udaya-pūrva*, first visibility in east), *' pū* (*asta-pūrva*, last visibility in east), *u pa* (*udaya-paścima*, first visibility in west), *va* (*vakra*, first station), and so on through the cycle again. We have not yet satisfactorily explained the divergence of this ordering from the expected chronological sequence of synodic phenomena described in Section 2.1.4.

4.4 Terrestrial latitude and longitude

Tables or lists of terrestrial longitude and/or latitude data for identified localities (see Section 2.1.5) are not included frequently or systematically in Sanskrit *koṣṭhakas*. They appear to have been constructed as a form of auxiliary reference materials whose corpus still remains largely unexplored (Pingree 1996b). The following examples illustrate some variants of the structure and appearance of latitude tables. Terrestrial longitude data, as far as we are currently aware, is confined to occasional lists of *deśāntara* values (see Section 2.1.5).

Terrestrial latitude tables: *Grahaṇasāraṇī*

Figure 4.20 shows a table from a manuscript of an anonymous *Grahaṇasāraṇī* relating digits of the noon equinoctial shadow s_0 cast by a 12-digit gnomon to degrees of terrestrial latitude ϕ.

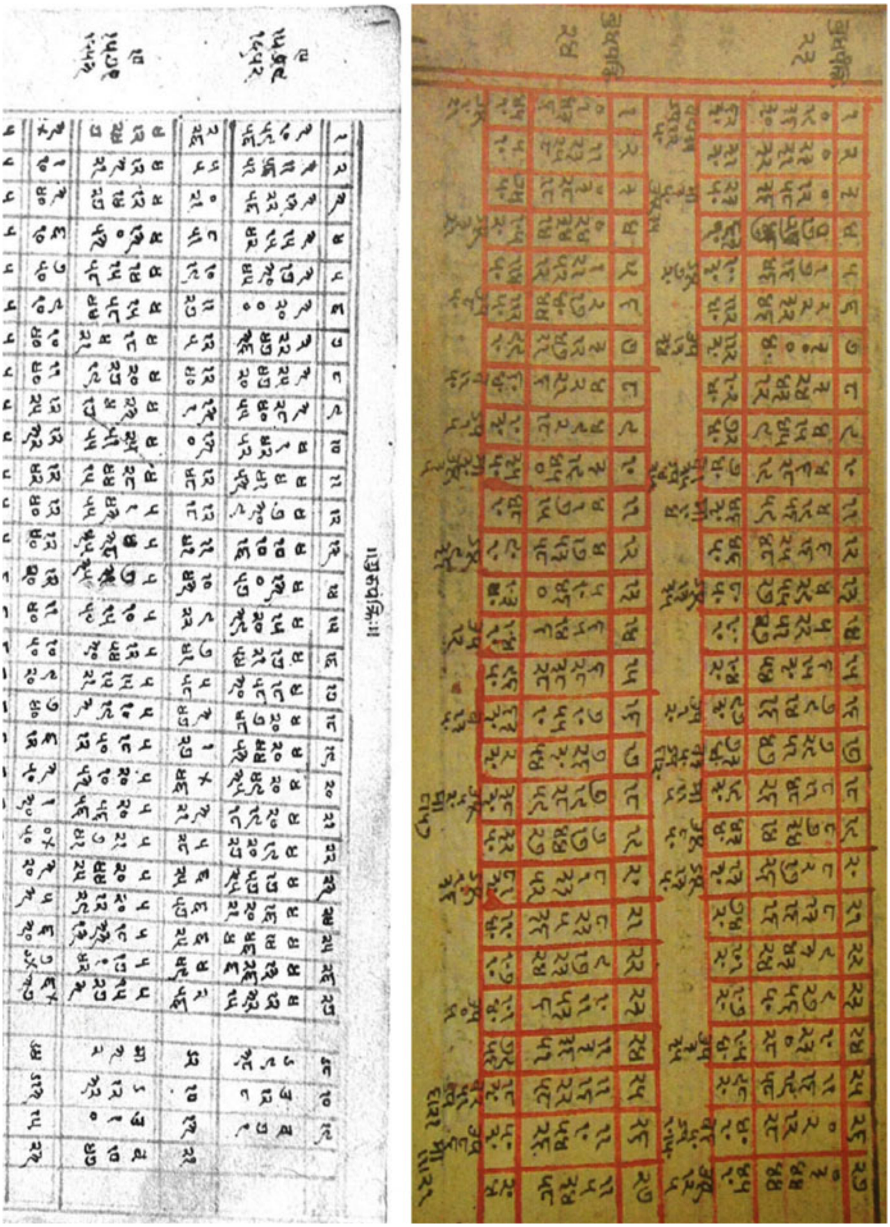

Fig. 4.19 Synodic phenomena occurrences in the *Jagadbhūṣaṇa* of Haridatta: for Jupiter (left, LDI 5545 f. 32v) at the right end of the page; for Mercury (bottom, RAS Tod 59 f. 27r) inserted beneath appropriate columns.

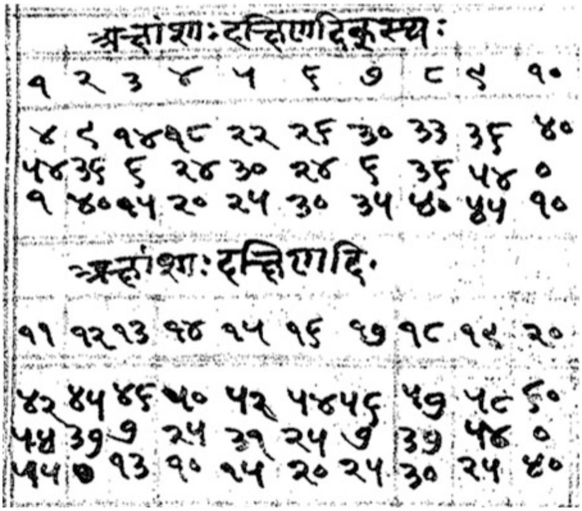

Fig. 4.20 Conversion table from digits of noon equinoctial shadow length to degrees of terrestrial latitude, from a *Grahaṇasāraṇī* (Plofker 58 f. 6r).

Argument: 1 to 20 digits of noon equinoctial shadow length.

Table entries. The entries in degrees, arcminutes and arcseconds increase monotonically from $\phi = 4°;54,1$ for $s_0 = 1$ digit to $\phi = 60°;0,40$ for $s_0 = 20$ digits. These values depend ultimately on the relation

$$s_0 = 12 \tan \phi$$

described in Section 2.1.5. Although the tabulated values approximately agree with reconstructed values computed using the modern arctangent function, it is not clear exactly what algorithm was used by the table compiler to produce them. For example, obviously the trigonometrically exact ϕ value for a 12-digit noon equinoctial shadow cast by a 12-digit gnomon ought to be $45°;0$, whereas the table lists $45°;37,0$.

Paratext. The table heading reads *akṣāṃśāḥ dakṣiṇadiksthaḥ* "Degrees of terrestrial latitude [with argument digits of noon equinoctial shadow] standing in the southern direction."

4.5 Ascensions and ascendant

Tables of ascension values (see Section 2.1.5) are a standard feature in the spherical astronomy sections of Islamic *zījes* (Kennedy 1956, p. 140), but not in Sanskrit *koṣṭhaka* works. Instead, the various functions that incorporate ascension are usually tabulated individually. When right and/or oblique ascension is given its own table, it is often for the purpose of calculating parallax (Section 4.8.6) or information about horoscopes.

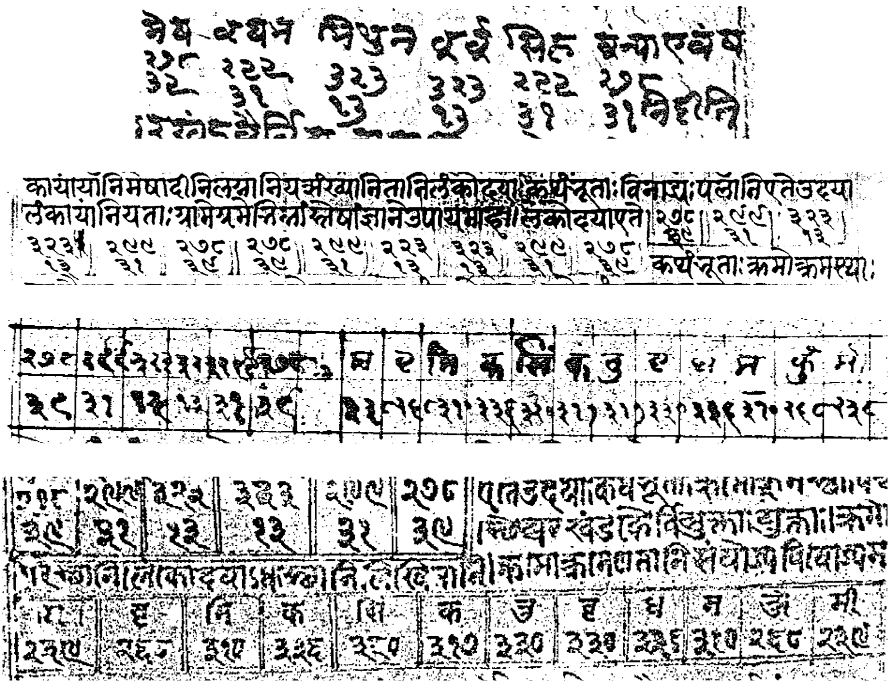

Fig. 4.21 Ascension values excerpted from manuscripts of a commentary on the *Karaṇakutūhala*, top to bottom: BORI 386 f. 14r, Jodhpur Fort 771 f. 15r, Leipzig 969 f. 12r, RORI 24027 f. 12r.

Ascension tables in *karaṇa* manuscripts: *Karaṇakutūhala*

The three standard values of right ascension for zodiacal signs (see Section 2.1.5) sometimes appear in a numerical grid inserted in the text in manuscripts of verse treatises or handbooks. Figure 4.21 shows a more precise form of these ascension values written out in tabular form in manuscripts of a fourteenth-century commentary on the *Karaṇakutūhala* (Mishra 1991). All the manuscripts include right ascensions and their successive differences for either six or twelve zodiacal signs; the last two images also contain a table of the corresponding oblique ascensions for the commentator's locality, a place called Mahāḍanagara (latitude 18°).

Ascension tables: *Grahalāghavasāriṇī*

Figure 4.22 shows a set of ascension tables stacked vertically in a manuscript of the *Grahalāghavasāriṇī*. The top row numbering the zodiacal signs from 0 to 11 is the argument for all the tables. Immediately beneath it is the table of right ascensions, and below that are tables of oblique ascensions for various localities. Samples of

Fig. 4.22 Ascension tables from the *Grahalāghavasāriṇī* (Smith Indic 26 f. 5r).

Table 4.9 Right ascensions and related quantities recomputed from the *Grahalāghava-sāriṇī* (Fig. 4.22). The columns in boldface reproduce the corresponding table rows in the manuscript.

Sign	$\alpha^{(gh)}$	$\alpha^{(vi)}$	$\Delta\alpha^{(gh)}$	$\Delta\alpha^{(vi)}$	$\Delta\alpha$ per $\circ\lambda$ (vi)	$(vi/60)$
0	**0;0**	0	4;38	278	**9;16**	**556**
1	**4;38**	278	4;59	299	**9;58**	**598**
2	**9;37**	577	5;23	323	**10;46**	**646**
3	**15;0**	900	5;23	323	**10;46**	**646**
4	**20;23**	1223	4;59	299	**9;58**	**598**
5	**25;22**	1522	4;38	278	**9;16**	**556**
6	**30;0**	1800	4;38	278	**9;16**	**556**
7	**34;38**	2078	4;59	299	**9;58**	**598**
8	**39;37**	2377	5;23	323	**10;46**	**646**
9	**45;0**	2700	5;23	323	**10;46**	**646**
10	**50;23**	3023	4;59	299	**9;58**	**598**
11	**55;22**	3322	4;38	278	**9;16**	**556**

Table 4.10 Oblique ascensions and related quantities for Gujarāt recomputed from the *Grahalāghavasāriṇī* (Fig. 4.22). The columns in boldface reproduce the corresponding table rows in the manuscript.

Sign	$\rho^{(gh)}$	$\rho^{(vi)}$	$\Delta\rho^{(gh)}$	$\Delta\rho^{(vi)}$	$\Delta\alpha^{(vi)}$	$\Delta\alpha - \Delta\rho^{(vi)}$	$\Delta\rho^{(vi/60)}$ per $\circ\lambda$ (vi)	$(vi/60)$
0	**0;0**	0	3;44	224	278	54	**7;28**	**448**
1	**3;44**	224	4;16	256	299	43	**8;32**	**512**
2	**8;0**	480	5;5	305	323	18	**10;10**	**610**
3	**13;5**	785	5;41	341	323	−18	**11;22**	**682**
4	**18;46**	1126	5;42	342	299	−43	**11;24**	**684**
5	**24;28**	1468	5;32	332	278	−54	**11;4**	**664**
6	**30;0**	1800	5;32	332	278	−54	**11;8**	**664**
7	**35;32**	2132	5;42	342	299	−43	**11;24**	**684**
8	**41;14**	2474	5;41	341	323	−18	**11;22**	**682**
9	**46;55**	2815	5;5	305	323	18	**10;10**	**610**
10	**52;0**	3120	4;16	256	299	43	**8;32**	**512**
11	**56;16**	3376	3;44	224	278	54	**7;28**	**448**

their reconstructed entries are shown in the bolded columns of Tables 4.9 and 4.10. The last table reorders the same right ascensions for what is called the "tenth ascendant" (*daśama lagna*) or midheaven, i.e., the equatorial rising times for the zodiacal signs to traverse midheaven, beginning with the tenth sign (when the first sign is crossing the eastern horizon).

Argument: 0 to 11 zodiacal signs of tropical longitude.

Table entries. Each table for a given locality contains the following three rows:

Row 1 (*dhruva* "fixed"): Cumulative ascension, in sexagesimal integer and fractional *ghaṭīs*, for the arc of the tropical ecliptic ending at the start of the corresponding zodiacal sign.
Row 2 (*gati* "motion"): Mean amount of ascension, in sexagesimal integer and fractional *vighaṭīs*, per degree of the corresponding zodiacal sign (or equivalently, differences between successive Row 1 entries divided by 30).
Row 3 (*bhājaka* "divisor"): Row 2 entries converted to decimal (i.e., mean amount of ascension, in integer sixtieths of a *vighaṭī*, per degree of the corresponding zodiacal sign).

The recomputed values in Table 4.10 indicate that the ascensional differences between right and oblique ascension were computed as usual by applying the prescribed scale factors of 10, 8, and 10/3 to the noon equinoctial shadow.

Localities. Laṅkā, Gūrjaradeśa (Gujarāt), Pāṭaṇapura (Pattan), Sthambhatīrtha (Khambat/Cambay), and Amadāvāda (Ahmadabad) (Pingree 1968, p. 70). These ascensions imply approximate latitudes for the last four localities of 24;14, 23;50, 21;48, and 23;2 degrees, respectively.

Paratext. The table heading reads *yasmin tasmin deśe daśamalagnam* "the tenth ascendant in whatever locality."

Ascension tables: *Karaṇakesarī*

Figure 4.23 shows excerpts from tables of oblique ascension at latitude approximately 22°;36 in a manuscript of the *Karaṇakesarī* (Montelle and Plofker 2013; Misra et al. 2016). Values of ρ are tabulated separately for integer degrees of tropical longitude and for integer minutes within a given degree.

Argument.

Degrees table: $0^s, 0°$ to $11^s, 29°$ of tropical ecliptic longitude. The argument is split between the vertical axis (zodiacal signs from 0/Aries to 11/Pisces) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).
Minutes tables: 1 to 60 minutes within a single degree of tropical ecliptic longitude.

Table entries.

Degrees table: The entries are sexagesimal integer and fractional (to two places) *ghaṭīs*. The values for the beginning of each tropical zodiacal sign (reproduced in boldface in Table 4.11) were apparently produced using ascensional-difference values computed by applying to a noon equinoctial shadow of 5 digits the scalar multipliers 10, 8, and 10/3 described in Section 2.1.5. The intermediate entries have been computed by linear interpolation (with common difference one-thirtieth of the rising time $\Delta\rho$ for the given zodiacal sign).
Minutes tables: There is a separate table for every pair of tropical zodiacal signs that share the same rising time $\Delta\rho$: e.g., Aries and Pisces, Gemini and Aquarius, etc. The entries are sexagesimal integer and fractional (to two places) *vighaṭīs*, with common difference equal to one-sixtieth of the ρ-increment for a single degree of the corresponding sign in the degrees table.

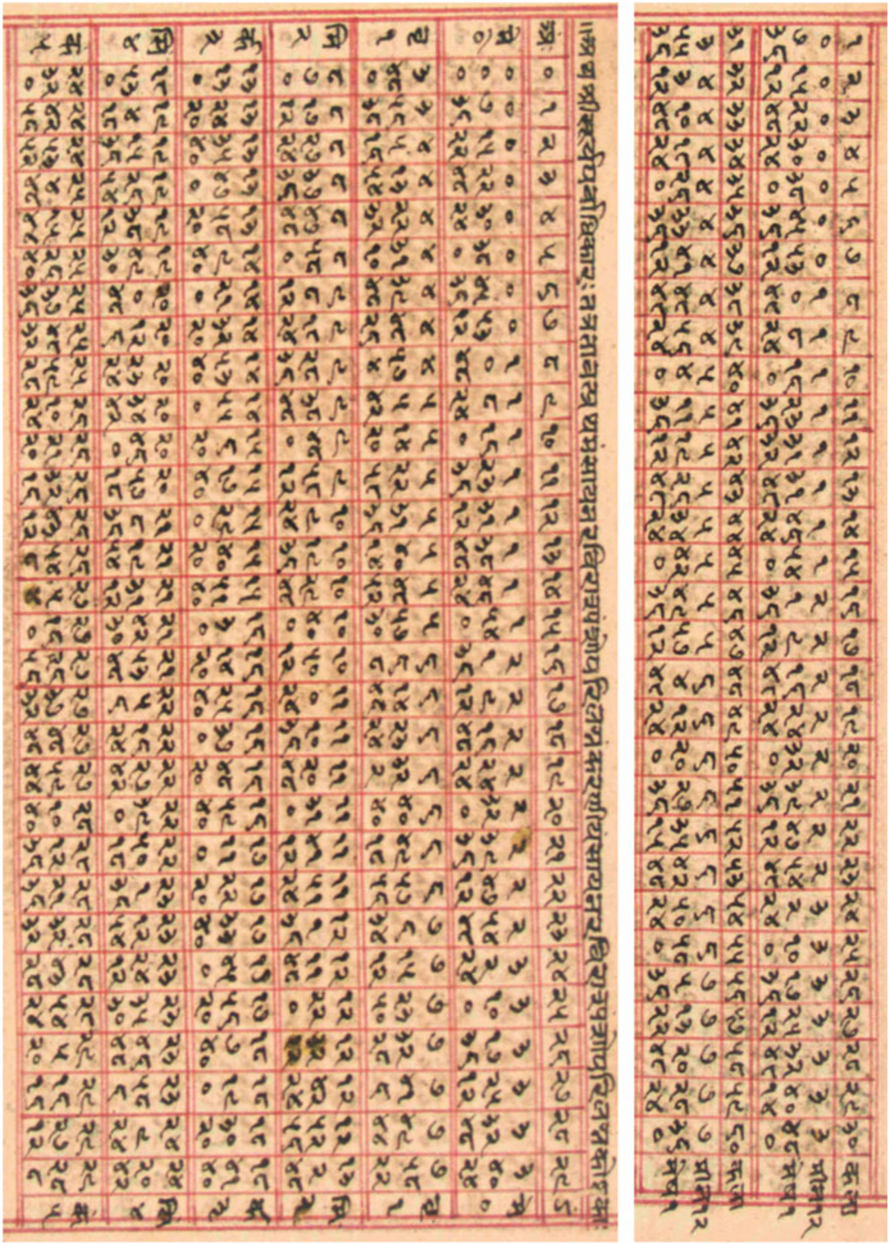

Fig. 4.23 Excerpts from tables of oblique ascensions in a manuscript of the *Karaṇakesarī* (RORI 12792). Left: Oblique ascensions for each degree of the first six tropical zodiacal signs (f. 5v). Right: Cumulative increments of oblique ascension for each successive minute of a degree in (tropical sign) Aries or Pisces (f. 6v).

Table 4.11 Oblique ascension values (in boldface) for tropical zodiacal signs from the *Karaṇakesarī* (RORI 12792 ff. 5v–6v), with their equivalents and successive differences in integer sixtieths.

Sign	$\rho^{(gh)}$	$\rho^{(vi)}$	$\Delta\rho^{(vi)}$
0	**0;0,0**	0	0
1	**3;48,0**	228	228
2	**8;7,0**	487	259
3	**13;13,0**	793	306
4	**18;53,0**	1133	340
5	**24;32,0**	1472	339
6	**30;0,0**	1800	328
7	**35;28,0**	2128	328
8	**41;7,0**	2467	339
9	**46;47,0**	2807	340
10	**51;53,0**	3113	306
11	**56;12,0**	3372	259

Locality. The values of ascensional difference used to construct the table imply that the latitude ϕ is approximately 22°;36, but no locality is explicitly identified.

Paratext. The table heading reads *tatra tāvat prathamaṃ sāyanaravirāśyaṃśoparilagnaṃ karaṇīyaṃ sāyanaravirāśyoparilagnakoṣṭhakāḥ* "Then because the first ascendant (*lagna*) [with its] above [argument] signs and degrees of precession-increased solar [longitude] is to be determined, [here are] the tabular values of ascension/ascendant (*lagna*) [with its] above [argument] signs [etc.] of precession-increased solar [longitude]."

Ascension tables: *Lagnasāraṇī*

Figure 4.24 shows a table of cumulative oblique ascensions from a manuscript of an otherwise unknown work entitled *Lagnasāraṇī*.

Argument: $0^s, 0°$ to $11^s, 29°$ of sidereal ecliptic longitude. The argument is split between the vertical axis (zodiacal signs from 0/Aries to 11/Pisces) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).

Table entries: Sexagesimal integer and fractional *ghaṭīs*. The zero value occurs at argument value 11/Pisces 9° rather than at 0/Aries 0°, suggesting that the argument is sidereal rather than tropical ecliptic longitude. This 21° difference between the vernal equinox and sidereal Aries 0° indicates a date of 1783 CE for the table's construction.[5] The ascensional differences of 61, 49, and 20 *vighaṭīs* evidently used to create the table seem more or less consistent with the results of multiplying the standard factors 10, 8, and 10/3 by a noon equinoctial shadow of approximately

[5]Using the standard Indian rule of one arcminute of precession per year, 21° corresponds to an interval of 1260 years since the canonical precession zero-point of 523 CE. See Pingree (1972) and compare the tables described in Pingree (1968, pp. 31–32).

Fig. 4.24 Oblique ascensions in a *Lagnasāraṇī* (Plofker 32 f. 1r).

6;6 digits. The individual tabulated values were possibly all computed trigonometrically, as they do not exhibit the constant differences associated with linear interpolation.

Locality. The reconstructed noon equinoctial shadow of 6;6 digits or thereabouts implies a latitude of approximately 27°, but no locality is explicitly identified.

Paratext. The part of the page containing the table heading is somewhat broken but the heading text can be approximately reconstructed as follows: [*laṁ*]*kodayāḥ* [*me*] *278* [*vṛ*] *299 mi 323 ka 323 si 299 ka 278 lagnaśāraṇ*[*ī x x*] *mabdam svadeśīyāḥ me 237 vṛ 250 mi 303 ka 343 si 348* [*ka*] *339* "Risings at Laṅkā: Aries 278, Taurus 299, Gemini 323, Cancer 323, Leo 299, Virgo 278. Ascension table [. . .] year [?] [ascensions at one's] own locality: Aries 237 [error for 217], Taurus 250, Gemini 303, Cancer 343, Leo 348, Virgo 339."

Ascension tables: *Bhaugolikajyotiṣa*

Figure 4.25 shows a set of tables of oblique ascensions in *vighaṭīs* for a number of Indian localities from a manuscript of an otherwise unknown work entitled *Bhaugolikajyotiṣa*.

Table entries. There are 24 simple tables or lists, each containing oblique ascension values in integer *vighaṭīs* for the first six zodiacal signs for the latitude of the associated locality/ies.

Localities. The named localities are reproduced in order with their modern identifications (where possible), ascensional differences ω_1, ω_2, ω_3 derived from the

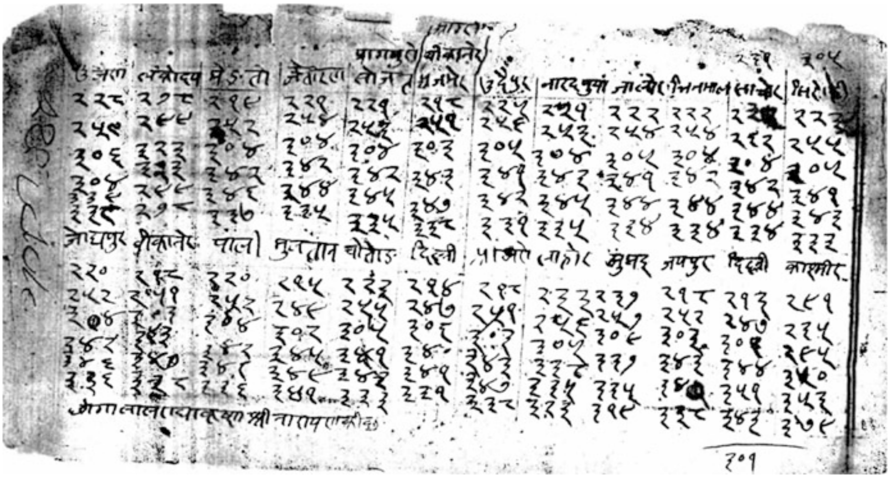

Fig. 4.25 Oblique ascensions for zodiacal signs at various localities, from a *Bhaugolikajyotiṣa* manuscript (SRCMI 120.i.335 f. 1r).

Table 4.12 Localities included in the oblique ascension tables of the *Bhaugolikajyotiṣa* (SRCMI 120.i.335 f. 1r) with tabulated ascensional differences ω_1, ω_2, ω_3 in *vighaṭīs* and reconstructed approximate latitudes ϕ.

Locality	$\omega_1, \omega_2, \omega_3$	ϕ (°)	Locality	$\omega_1, \omega_2, \omega_3$	ϕ (°)
Ujjain	50, 40, 17	23	Jodhpur	58, 47, 19	26
Laṅkā	0, 0, 0	0	Bikaner	60, 48, 20	26 ½
Merta (Meḍato)	59, 47, 19	26	Pali (Pālī)	58, 49, 19	25 ½
Jetpur (Jetārāṇa)	57, 45, 19	25	Multan	63, 50, 21	27 ½
Allahabad (Prāgpur), Sojata [?]	57, 46, 19	25 ½	Chittorgarh (Cītoḍa)	55, 44, 18	24 ½
Agra, Bikaner, Ajmer	60, 48, 20	26 ½	Delhi (Dillī)	64, 52, 17	28
Udaipur	53, 43, 18	24	Agra	60, 48, 20	26 ½
Nāradapumāṃ [?]	57, 46, 19	25 ½	Lahore	45, 36, 15	20
Jalore (Jālora)	56, 45, 18	25	Muṃmai [Mumbai?]	41, 36, 14	19
Bhinmal (Bhinamāla)	56, 45, 19	25	Jaipur	60, 47, 20	26 ½
Sanchore (Sācora)	56, 45, 19	25	Delhi (Dillī)	65, 52, 21	28
Sirohi	55, 44, 18	24 ½	Kashmir	87, 64, 28	34

tabulated ascensions, and reconstructed approximate latitudes ϕ (to the nearest half-degree) in Table 4.12.

The recomputed latitudes are only approximate because we do not know exactly what value of the noon equinoctial shadow length or what degree of precision the scribe employed for any locality to produce its tabulated ascension values. Nonetheless, this reconstruction allows us to detect not only occasional scribal errors in the numbers but also some more puzzling anomalies. For example, why is Delhi listed twice with two slightly different sets of ascensions, and why do the ascensions for Lahore, presumably meaning the city of that name in modern Pakistan, correspond to a latitude of only 20°?

Ascension tables: Works of Nityānanda

Siddhāntasindhu

The 1628 *Siddhāntasindhu* of Nityānanda (see Section 6.1.2) is a Sanskrit recension of the Indo-Islamic *Zīj-i Shāh Jahānī*, so its table structure is characteristic of Islamic *zījes*. Figure 4.26 shows an excerpt from a table of right ascensions in a manuscript of the *Siddhāntasindhu*.

Argument.

Degrees table: $0^s, 0°$ to $11^s, 29°$ of tropical ecliptic longitude. The argument is split between the vertical axis (zodiacal signs from 0/Aries to 11/Pisces) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).

Minutes table: 1 to 59 minutes within a single degree of tropical ecliptic longitude in each zodiacal sign. The argument is split between the vertical axis (1 to 59 arcminutes of a single degree) and the horizontal axis (degrees in each zodiacal sign from 0 to 29, read right to left for signs in a quadrant beginning with an equinox or left to right for signs in a quadrant beginning with a solstice).

Table entries.

Degrees tables.

Row 1: Time-degrees (rather than *ghaṭīs*) of cumulative right ascension: decimal integer and sexagesimal fractional (to three places).

Row 2: Differences between successive entries in Row 1.

Minutes tables. The entries are sexagesimal integer and fractional (to three places) time-degrees of cumulative right ascension for each arcminute of each single degree of a zodiacal sign.

An accompanying table contains the same right ascensions data beginning with the value for Capricorn 0° rather than Aries 0°. The *Siddhāntasindhu* also tabulates cumulative oblique ascension values and their successive differences for each degree of tropical longitude for the latitudes of Samarqand (ϕ = 39°;37,23), Argālapura (Agra, ϕ = 26°;43), and Lābhapura (Lahore, ϕ = 31°;50), as well as for each successive degree of terrestrial latitude from ϕ = 1° to 88°.

Paratext. The minutes-table heading shown in Figure 4.26 reads *laṃkodayasādhanārthaṃ śeṣakalātaḥ phalaṃ* "For the sake of determining right ascensions, result [with respect] to the remainder [in] arcminutes." The corresponding degrees-table heading (f. 54v) reads *laṃkāyāṃ meṣādisvodayāṃśāḥ* "Degrees of their own risings for Aries etc. at Laṅkā."[6]

[6]The structure and labelling of these tables are evidently strongly influenced by the corresponding tables in the work that served as the model for the *Siddhāntasindhu*, the *Zīj-i Shāh Jahānī*. Similarities between manuscripts of the two works that we have consulted (MSS British Library

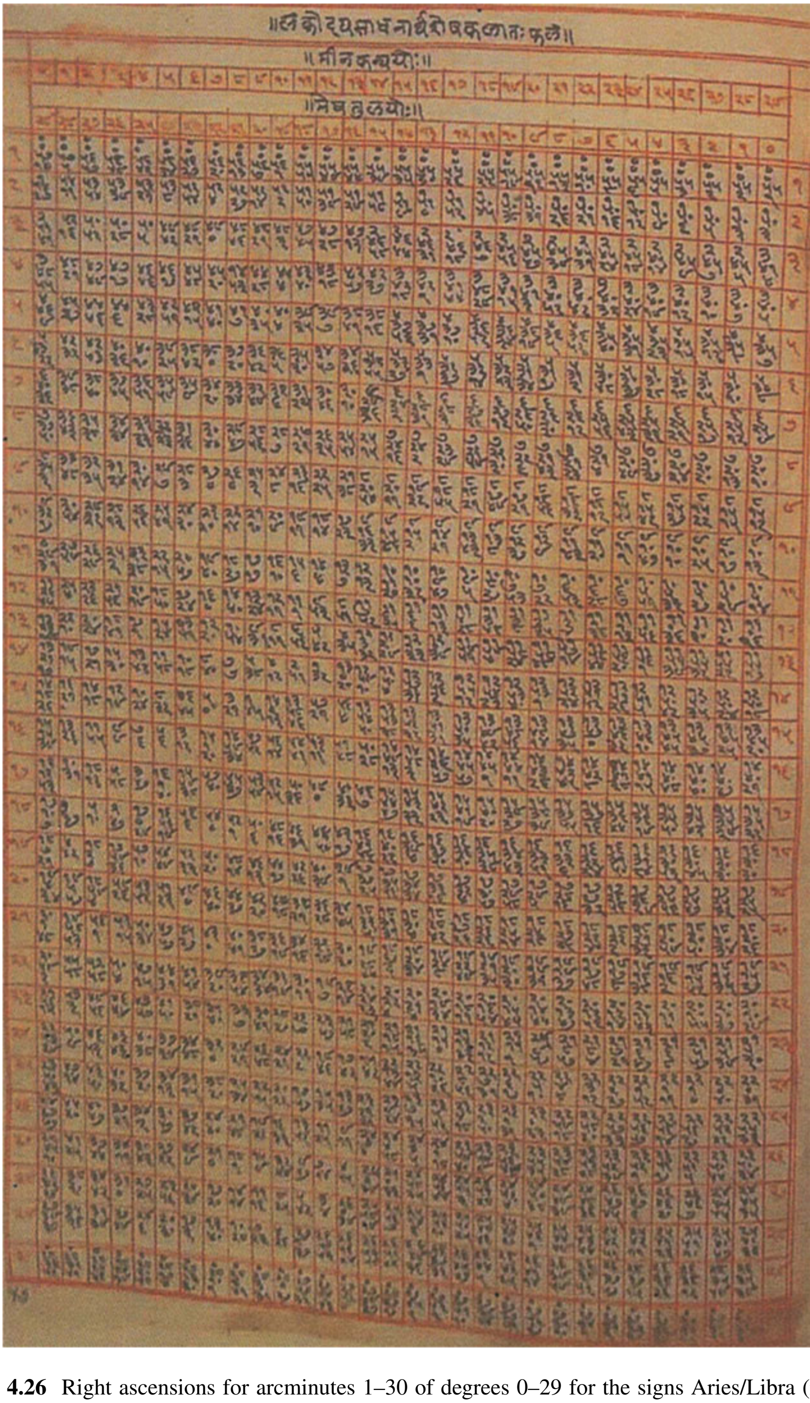

Fig. 4.26 Right ascensions for arcminutes 1–30 of degrees 0–29 for the signs Aries/Libra (read right to left) and Virgo/Pisces (read left to right), from the *Siddhāntasindhu* of Nityānanda (Jaipur Khasmohor 4962 f. 57v).

Fig. 4.27 Oblique ascensions for terrestrial latitude (approximately 28 ½ degrees) corresponding to a noon equinoctial shadow of 6;30 digits from the *Amṛtalahāri* of Nityānanda (Tokyo MF13 f. 44v).

Amṛtalahāri

Figure 4.27 shows an excerpt from a table of oblique ascensions from another work by Nityānanda, the *Amṛtalahāri* or *Kheṭakṛti*, which employs a more traditional calendric *koṣṭhaka* framework. This excerpt is part of a set of tables including right ascensions (counting the argument from Capricorn 0°) and oblique ascensions separately tabulated for values of noon equinoctial shadow length at half-digit intervals, from 4;30 to 8;30 digits or approximately $20\frac{1}{2}$ to 35 degrees of latitude (Pingree 2000, p. 217).

Argument: $0^s, 0°$ to $5^s, 29°$ of tropical ecliptic longitude. The argument is split between the vertical axis (zodiacal signs from 0/Aries to 5/Virgo, but also applicable in reverse from 11/Pisces to 6/Libra, which are not explicitly mentioned) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).

Table entries: Sexagesimal integer and fractional (to two places) *ghaṭīs*. The values for the beginning of each zodiacal sign were apparently produced by means of the standard ascensional-difference algorithm (i.e., noon equinoctial shadow multiplied by the factors 10, 8, and 10/3 to give ω_1, ω_2, and ω_3, respectively). Intermediate values were computed by linear interpolation.

Paratext. The table heading reads *1 akṣabhā 6 |30 cāpīkṛtacarakhaṇḍaiḥ svodayāḥ* "1. Latitude-shadow 6;30: [locality's] own risings with ascensional differences converted to arcs."

OR 372 and Jaipur Khasmohor 4962) include the use of time-degrees rather than *ghaṭīs*, the use of black ink for tabulated ascension values and red ink for their differences, and the ordering of pairs of zodiacal signs sharing the same table arguments.

4.6 Mean time correction functions

See Section 2.1.5 for a discussion of the various corrections to mean time corresponding to differences between mean and true positions of the luminaries.

Rising-difference tables: *Brahmatulyasāraṇī*

Figure 4.28 shows a table of solar and lunar rising-difference (*udayāntara*) (top row and bottom row, respectively) from a manuscript of the *Brahmatulyasāraṇī*.

Argument: 2 to 90° of doubled tropical solar longitude, for every other degree of argument (corresponding to 1–45° of tropical solar longitude).

Table entries. The table contains the following two rows:

Row 1: Arcseconds and arcthirds of solar rising-difference correction.
Row 2: Arcminutes, arcseconds and arcthirds of lunar rising-difference correction.

Maximum value. Sun: 24″;0. Moon: 5′;42,51.

Method of computation. The table entries are derived by a procedure prescribed in *Karaṇakutūhala* 2.16cd–17, in which the R sin of twice the mean tropical longitude of the sun and the moon is multiplied by 1/5 and 1/21, respectively.

Paratext. The table heading reads *atha ravicandrayor udayāntaraṃ koṣṭhakāḥ* ∥ *dvighnaṃ bhujāṃśopari* "Now, the tabular entries [of] the rising-difference for the sun and the moon. The above [argument is] doubled degrees of arc [of longitude]."

Half-length of daylight tables: *Karaṇakesarī*

Figure 4.29 shows a table of half-length of daylight in *ghaṭīs* (or equivalently fifteen *ghaṭīs* plus or minus ω) for terrestrial latitude approximately 22°;36, from a manuscript of the *Karaṇakesarī*.

Argument: $0^s, 0°$ to $11^s, 29°$ of tropical ecliptic longitude. The argument is split between the vertical axis listing zodiacal signs in the unusual (and unexplained) order 1/Taurus to 11/Pisces followed by 0/Aries, and the horizontal axis listing degrees in each zodiacal sign from 0 to 29.

Table entries. The entries in *ghaṭīs* and *vighaṭīs* increase from 15;50 at Taurus 0° (the beginning of the table) to a maximum of 16;47 at Cancer 0° and then decrease to a minimum of 13;13 at Capricorn 0°, before increasing again until the end of the table. The ratio of longest day to shortest day is thus 33;34 to 26;26, producing a maximum half-equation of daylight of 1;47 *ghaṭīs*, which is equal to the sum of the

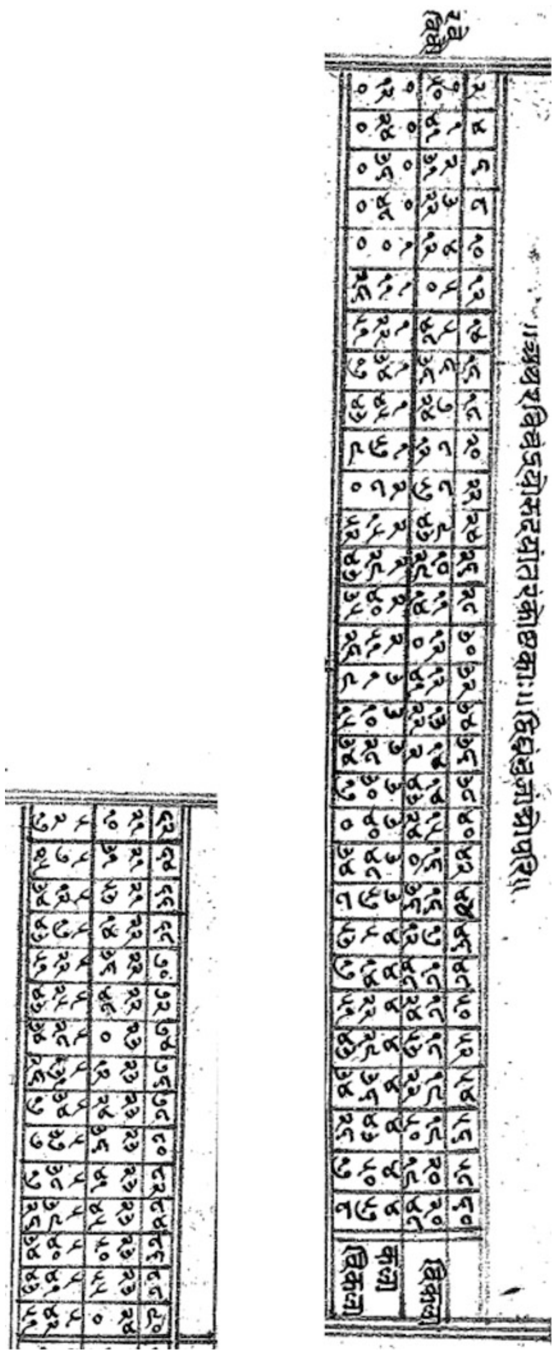

Fig. 4.28 Solar and lunar rising-difference table from the *Brahmatulyasāraṇī* (Smith Indic 45 f. 7v).

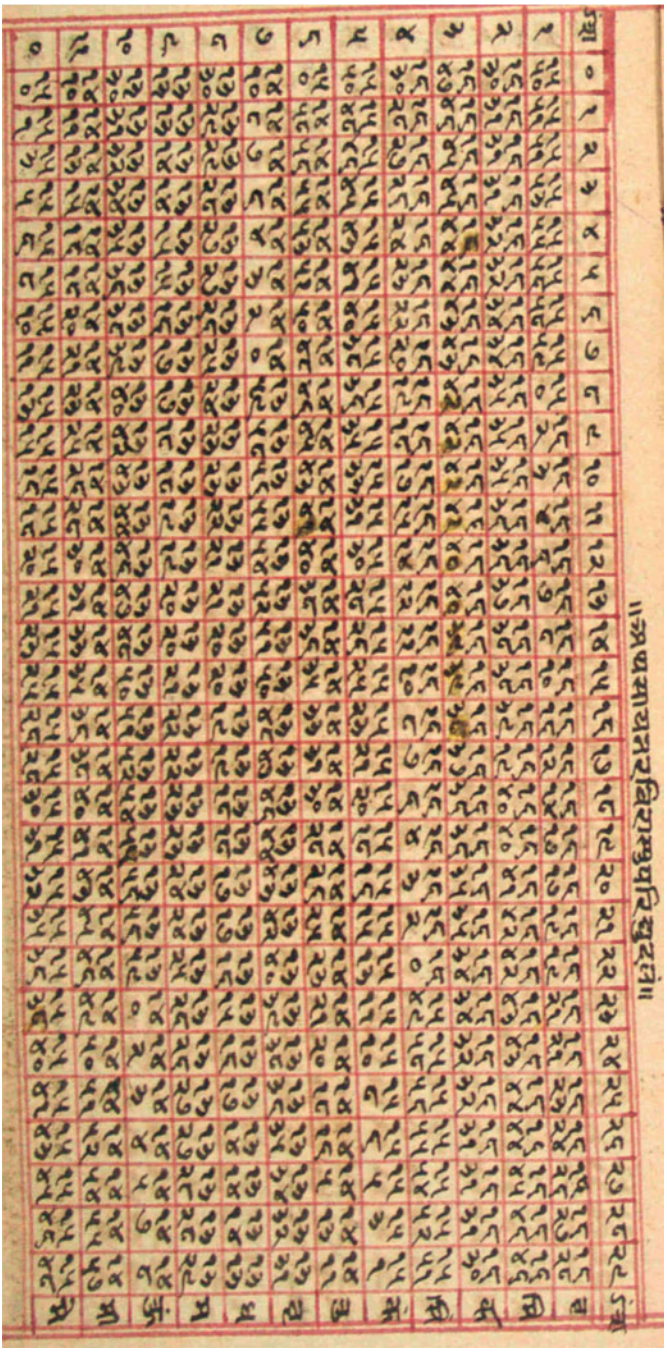

Fig. 4.29 Table of half-length of daylight from the *Karaṇakesarī* (RORI 12792 f. 5r).

three ascensional differences 50, 40, and 17 *vighaṭīs* listed for Ujjain in Table 4.12. The entries between the beginnings of zodiacal signs appear to have been computed by truncating linearly interpolated values (Misra et al. 2016, pp. 24–25).

Paratext. The table heading reads *atha sāyanaravirāśyupari dyudalaṃ* "Now, the half[-length] of day [with its] above [argument] sign [and degree] of tropical solar [longitude]."

Half-length of daylight tables: *Siddhāntasindhu*

Figure 4.30 shows three tables from a manuscript of the *Siddhāntasindhu*, listing values of half-length of daylight in hours rather than *ghaṭīs* at Samarqand, Agra, and Lahore, respectively.

Argument: $0^s, 0°$ to $11^s, 29°$ of tropical ecliptic longitude. The argument is split between the vertical axis (zodiacal signs from 9/Capricorn to 2/Gemini on the left side and from 8/Sagittarius backwards to 3/Cancer on the right) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).

Table entries: Integer and sexagesimal fractional (to two places) hours. The ratios of longest to shortest day for Samarqand, Agra, and Lahore are 7;24,25/4;35,35, 6;50,34/5;9,25, and 7;2,34/4;57,20, respectively.

Paratext. The three table headings read, in order:

- *atha makarāditaḥ samarakaṃde dinārddhahorāḥ* "Now, beginning with Capricorn, the hours of half the day in Samarqand."
- *argālapure makarādito dinārddhahorāḥ* "In Agra the hours of half the day beginning with Capricorn."
- *lābhapure makarādito dinārddhahorāḥ* "In Lahore the hours of half the day beginning with Capricorn."

Time correction function tables: *Jagadbhūṣaṇa*

Figure 4.31 shows a table from a manuscript of the *Jagadbhūṣaṇa* containing the four time correction components and the half-length of daylight, as well as other quantities enumerated in "Table entries" below.

Argument: 1 to 27 *avadhis*. The argument row is repeated in the middle of the second page of the table, with each *avadhi* number accompanied by the integer multiple of 14 days (0, 14, 28, 42, . . . , 350, 364) elapsed at the beginning of the *avadhi*.

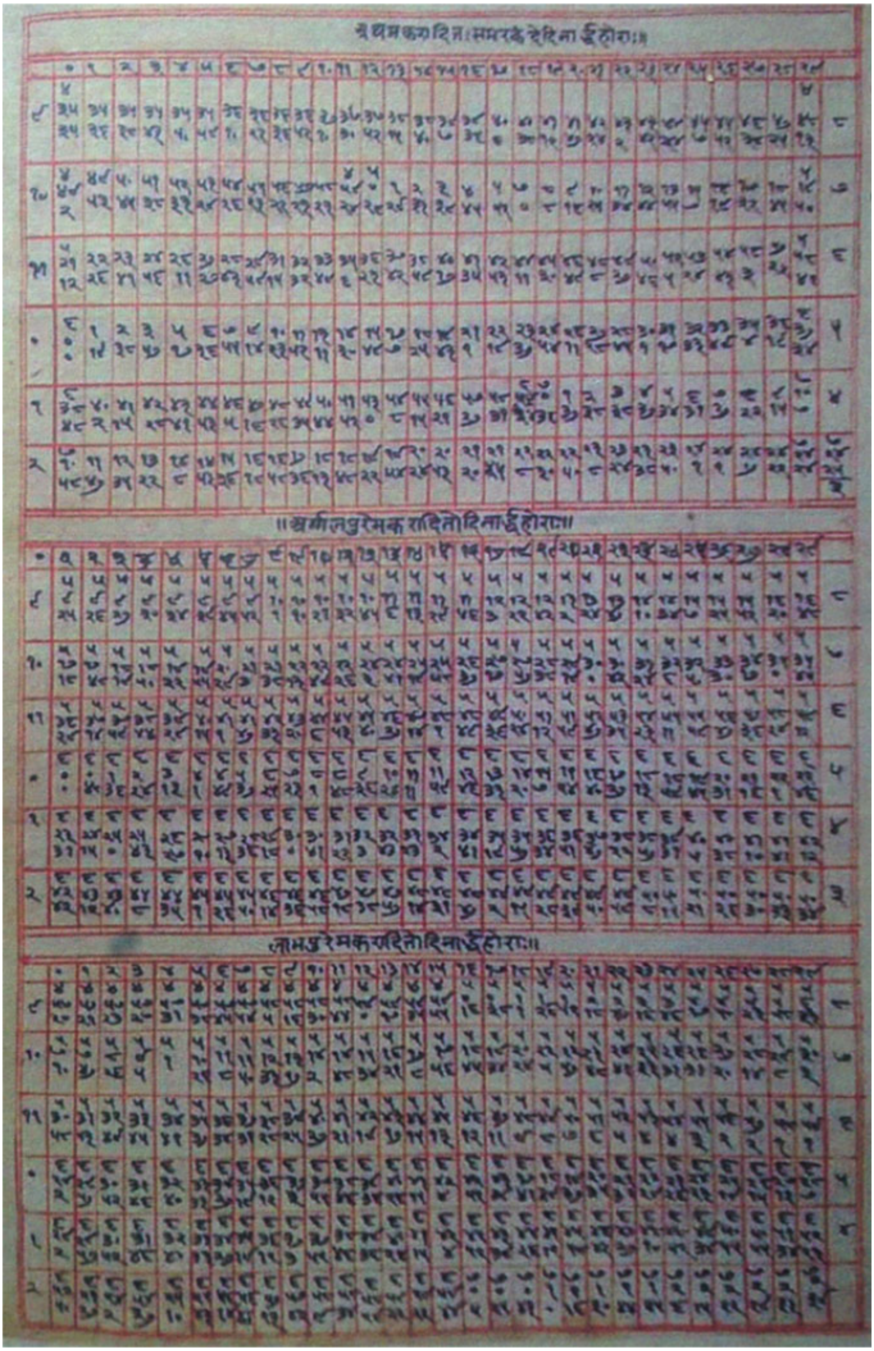

Fig. 4.30 Three tables of half-length of daylight from the *Siddhāntasindhu* (Jaipur Khasmohor 4962 f. 88r).

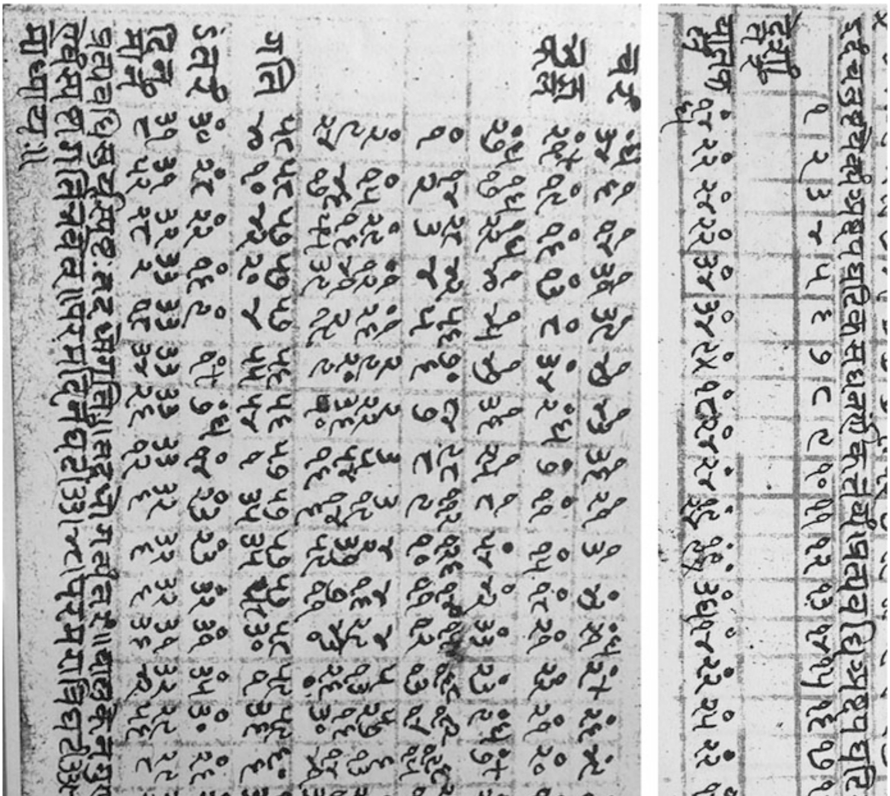

Fig. 4.31 Table of length of daylight and various related solar equations, velocities, and longitudes from the *Jagadbhūṣaṇa* (Smith Indic 146 f. 99r–v).

Table entries. The table includes the following quantities:

deśāntara This row has been left blank, for undetermined reasons. For a given locality, of course, the *deśāntara* value remains constant over time, so there would be no point in tabulating it afresh for each *avadhi* in any case. We may speculate that perhaps the user was intended to fill in the appropriate amount for their own longitude.
yātaphala Also known as *udayāntara* equation.
cara Half-equation of daylight.
bhujaphala Second component of the equation of time correction.
traikya Sum of the previous three rows.
Second argument 0–364 days, and below them 1–27 *avadhis*.
True solar longitudes First entry (for mean solar longitude 0) is 0^s, 2°;9,24.
gati True (?) daily velocity of the sun at that longitude. Maximum tabulated value $v =$ 1°;1,23. Minimum tabulated value $v =$ 0°;56,54.
antara Differences in successive entries in previous row of daily velocities.
dinamāna Length of daylight for the beginning of each *avadhi*. Maximum tabulated length of daylight 34;48 *ghaṭīs*. Minimum tabulated length of daylight 26;28 *ghaṭīs*. These values are produced by adding the appropriate equation of daylight (double the value in the *cara* row) to 30 *ghaṭīs*.

Locality. Given that the longest day is 33;48 *ghaṭīs*, the latitude for which the tables were computed can be reconstructed as $\phi \approx 24$, the terrestrial latitude of Ujjain (Pingree 1968, p. 57).

Paratext. The table heading reads *idaṃ catuṣṭayaikyaṃ abdapaghaṭikāsu dhanarṇaṃ karttavyaṃ praty 'vadhi ghaṭikā bījasaṃskṛtā bhavaṃti* || "This sum of four is to be applied positively or negatively to the *ghaṭīs* of the lord of the year, for each *avadhi*. The *ghaṭīs* become *bīja*-corrected."

Text below the table reads *pratyavadhisuryaspaṣṭaḥ tad adho gatiḥ* || *tad adho gatyaṃtaraṃ* || *cālakena guṇīyatvā śakrair bhājyaṃ 14* || *labdhaṃ gatau dhanarṇaṃ kārya evaṃ spaṣṭā gatir bhavet* || *paramadinaṃ ghaṭī 33* | *48 paramarātrighaṭī 33* | *48 hīnadina 26* | *12* || *iti jagatbhuśaṇe prathamo dhyāyaḥ* || "The true [longitude] of the sun per *avadhi*. Under that, the velocity. Under that, the difference of the velocity. Having multiplied [the argument increment] by the *cālaka*, [the product is] to be divided by 14. The result is to be applied positively or negatively to the velocity. The true velocity is produced. The longest day is 33;48 *ghaṭīs*. The longest night is 33;48 *ghaṭīs*. The shortest day is 26;12 [*ghaṭīs*]. Thus, the first chapter of the *Jagadbhūṣaṇa*."

4.7 Ecliptic declination

A small set of values of the ecliptic or solar declination δ is sometimes listed in verse form in *karaṇa* works,[7] but they are not exhaustively tabulated in most *koṣṭhakas* (unlike Islamic *zījes*, in which tables of declination routinely appear among the standard spherical astronomy functions).

[7] For instance, verses containing values of ecliptic declination appear in the *Khaṇḍakhādyaka* (Misra 1925, p. 103, verse 3.11) and in the *Karaṇakutūhala* (Mishra 1991, p. 43, verses 3.13–14).

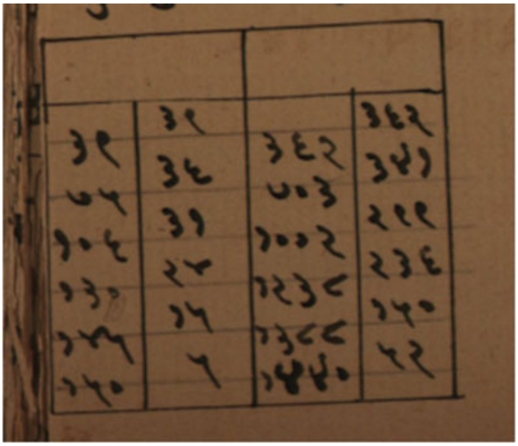

Fig. 4.32 Numerical tables inserted in a late nineteenth-century manuscript of the *Khaṇḍakhādyaka* (Pune 741809 page 26). Left half: *R* sin values and differences with $R = 150$ for every 15° of argument. Right half: Ecliptic declination and differences for every 15° of argument.

Tabular version of declination values in a *karaṇa* manuscript: *Khaṇḍakhādyaka*

Figure 4.32 shows in the two rightmost columns a rough tabular representation of the versified list of values of solar declination in the *Khaṇḍakhādyaka* of Brahmagupta (see footnote 7), along with their successive differences. (The two columns to the left similarly reproduce the *karaṇa*'s simple table of *R* sin values; see Section 2.1.8.) The six entries represent the values of declination in integer arcminutes corresponding successively to 15, 30, 45, 60, 75, and 90 degrees of longitude. The maximum entry at argument 90° is the standard Indian δ_{max} or ε value $1440' = 24°$.

Ecliptic declination tables: *Rājamṛgāṅka*

Figure 4.33 shows two separate tables of ecliptic declination in minutes and seconds from a manuscript of the *Rājamṛgāṅka*.

Argument: Top table: 2 to 90° of tropical longitude (even numbers). Bottom table: 1 to 90° of tropical longitude.

Table entries. Top table: Values of ecliptic declination are computed in minutes and seconds, although the seconds place is almost always zero after the 11th argument. Bottom table: Row 1 contains values of ecliptic declination given to minutes and seconds, and Row 2 the successive differences of the values in Row 1.

Method of computation. Comparison of the table entries suggests that the two tables were produced using different algorithms to compute the declination values, although we have not successfully reproduced either set of computations.

Fig. 4.33 Ecliptic declination tables from the *Rājamṛgāṅka* (Baroda 9476 f. 13v). Top: Condensed table with entries for every second degree of argument. Bottom: Beginning of full table with entries for every degree of argument.

Paratext. The table headings read as follows:

Top: *atha kramakrāṃtikoṣṭhakā* "Now, the tabular values for declination for even [degrees of argument]."

Bottom: *atha śrī pratyaṃśacāpakalā krāṃtau likhyaṃte* "Now [. . .] the minutes of arc in the declination for each degree [of argument] are written."

Ecliptic declination tables: *Grahaṇasāraṇī*

Figure 4.34 shows a table of ecliptic declination in degrees, minutes, and seconds from a manuscript of the anonymous *Grahaṇasāraṇī* mentioned in Section 2.1.5. The values for integer multiples of 10 have been computed trigonometrically, with the intermediate values linearly interpolated.

Argument: 1 to 90° of tropical longitude.

4.8 Tables specific to eclipses

Tables relating to the prediction of eclipses may constitute a separate book in their own right or may be incorporated with other tables in a more general work.[8] The

[8] The former category includes table texts such as the *Gaṇitarāja* of Kevalarāma Pañcānana, the *Grahasāraṇī* (also known as the *Khecaraśīghrasiddhi*) of Gaṅgādhara, and the *Makaranda* of Makaranda. The latter category contains, e.g., the *Karaṇakesarī* of Bhāskara (fl. 1681) (Montelle and Plofker 2013) and the *Parvadvayasādhana* of Mallāri (Pingree 1981, pp. 46, 55). See the summaries of these works in Appendix A.

Fig. 4.34 Ecliptic declination table from the *Grahaṇasāraṇī* (Plofker 58 f. 4r–v).

following sections describe the types of tables typically provided to facilitate the eclipse calculations discussed in Section 2.1.6.

4.8.1 Lunar-nodal elongation

Since the moon's longitudinal separation from its node determines its latitude, which in turn impacts the possibility and characteristics of eclipse occurrence at a syzygy, eclipse table texts generally tabulate lunar-nodal elongation in a form somewhat similar to the mean longitudinal displacement tables described in Section 4.1.

Elongation tables: *Karaṇakesarī*

Figure 4.35 shows an excerpt from the table of elongation between the node and the sun (not the moon) in the *Karaṇakesarī*. (Note that this quantity at a syzygy is identical modulo 180° to lunar-nodal elongation, since at the moment of syzygy the sun and moon are either 0° or 180° apart (Montelle and Plofker 2013, pp. 12–13).)

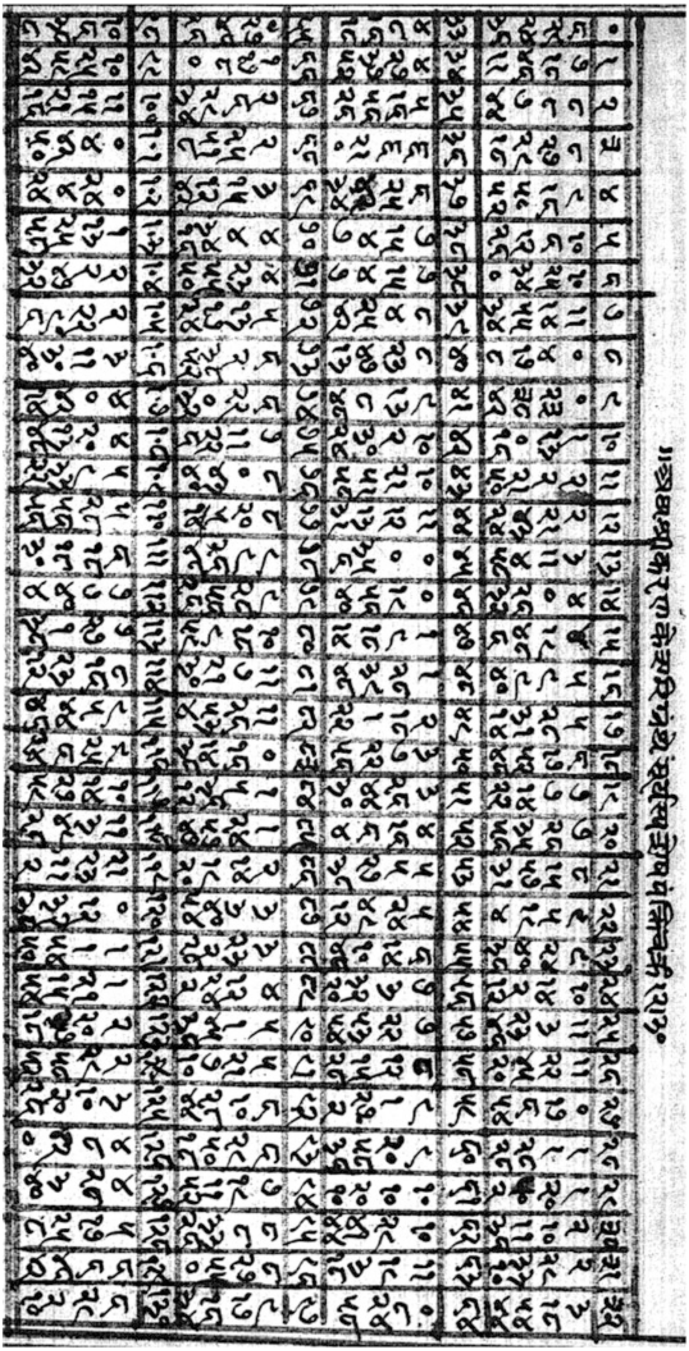

Fig. 4.35 Excerpt for single years from nodal-solar elongation tables of the *Karaṇakesarī* (Smith Indic MB XIV f. 3v).

Table 4.13 Characteristics of the nodal-solar elongation tables of the *Karaṇakesarī* (Smith Indic MB XIV ff. 3r–4r).

Argument range	Time-unit t	Elongation increment $\Delta\bar{\lambda}_t$ (to arcseconds)
1–130	Periods of 130 years	$-3°;16,20$
0–130	Single years	$19°;21,34$
1–27	*Avadhis* in a year	(nonlinear)

The mean elongation increments corresponding to successive multiples of each time-unit are tabulated as shown in Table 4.13, where the mean elongation increment per given time-unit t is denoted $\Delta\bar{\lambda}_t$.

Epoch elongation value. 6^s,29°;24,36 is incorporated into the first value of this table for Śaka 1603 (=1681 CE).

Paratext. The table heading reads *atha śrīkaraṇakeśarigraṃthe sūryasya śeṣapaṃkticakraṃ 12 | 30* "Now, in the book *Karaṇakesarī* the cyclic [elongation value] of the sun for the remainder number [of integer years after division by 130], [modulo] 12 [zodiacal signs and] 30 [degrees in a sign]."

4.8.2 Lunar latitude

Since lunar latitude is occasionally tabulated in other contexts besides eclipse reckoning, we defer the discussion of this quantity and its tables to Section 4.9.

4.8.3 Apparent diameters of sun, moon, and shadow

Apparent diameter tables: *Parvadvayasādhana*

Figure 4.36 shows a table of apparent diameters of the moon and the earth's shadow from a manuscript of the *Parvadvayasādhana*, for use in computing lunar eclipses.

Argument: 54 to 65 *ghaṭīs* in a *tithi*.

Table entries.

Row 1: Sexagesimal integer and fractional (to one place) digits of lunar apparent diameter, from initial maximum value of 11;42 to final minimum value of 9;43.
Row 2: Sexagesimal integer and fractional (to one place) digits of earth's shadow apparent diameter, from initial maximum value of 29;20 to final minimum value of 24;30.
Row 3: Half the sum of the corresponding entries in Row 1 and Row 2.

Paratext. The headers of the argument row and the three entry rows read, respectively, *tithayaḥ* "*tithi*[-lengths]," *caṃdrabiṃbāni* "lunar disks," *bhūbhāpramāṇaṃ* "amount of earth's shadow," and *mānaikyārdhaṃ* "half the sum of the measures."

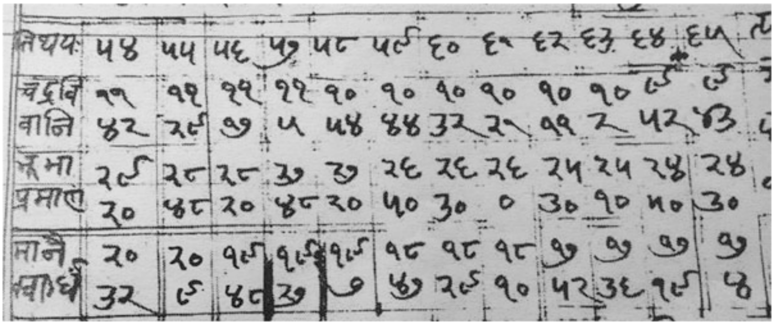

Fig. 4.36 Table of apparent diameters of the moon and the shadow of the earth from the *Parvadvayasādhana* (CSS d. 800 f. 2r).

Fig. 4.37 Tables of apparent diameter of the sun (top) and of the moon and shadow of the earth (bottom) from the *Karaṇakesarī* (RORI 12792 f. 2r–v).

Apparent diameter tables: *Karaṇakesarī*

Figure 4.37 shows tables of the apparent diameters of the sun and of the moon and the earth's shadow from a manuscript of the *Karaṇakesarī.*

Argument.

Sun table: 59;59 to 64;42 arcminutes of daily elongation of the sun from the lunar node.
Moon and shadow table: 52 to 67 *ghaṭīs* in a *tithi*.

Table entries.

Sun table: Sexagesimal integer and fractional (to one place) digits of solar apparent diameter, from initial minimum value of 10;19 to final maximum value of 11;11.

Moon and shadow table.

- **Row 1:** Sexagesimal integer and fractional (to one place) digits of lunar apparent diameter, from initial maximum value of 11;57 to final minimum value of 9;30.
- **Row 2:** Sexagesimal integer and fractional (to one place) digits of earth's shadow apparent diameter, from initial maximum value of 30;45 to final minimum value of 23;11.

Paratext. The headings of the two tables read as follows:

Sun table. *atha sapātacaṃdragatyupari ravibiṃbāṃgulādi* "Now, digits etc. of the solar disk [with] above [argument] velocity of the moon plus the node."

Moon and shadow table. *atha tithīr mānaghaṭyopari caṃdrabiṃba tathā bhū-bhāṃgulādi* "Now, the lunar disk, likewise the digits etc. of the earth's shadow, [with] above [argument] the measure in *ghaṭīs* of the *tithi*."

4.8.4 Duration of eclipse stages

The half-duration (*sthityardha*/*sthiti*) and half-duration of totality (*marda*), being dependent on the magnitude of the eclipse, are generally tabulated as a function of it.

Eclipse duration tables: *Karaṇakesarī*

Figure 4.38 shows tables of duration of eclipse stages from a manuscript of the *Karaṇakesarī*.

Argument.

Lunar eclipse half-totality table: 1 to 9 digits of "sky-obscuration," or excess of earth shadow diameter over lunar disk diameter.

Fig. 4.38 Eclipse duration tables from the *Karaṇakesarī* (RORI 12792). Top: Half-duration of totality of lunar eclipse (f. 2v). Middle: Half-duration of lunar eclipse (f. 2v). Bottom: Half-duration of solar eclipse (f. 7v).

Lunar eclipse half-duration table: 1 to 21 digits of lunar obscuration.
Solar eclipse half-duration table: 1 to 12 digits of solar obscuration.

Table entries.

Lunar eclipse half-totality table. Sexagesimal integer and fractional (to two places, with final digit of each entry zero) *ghaṭīs* of the 60-complement of half-duration of totality.
Lunar eclipse half-duration table. Sexagesimal integer and fractional (to one place) *ghaṭīs* of the 60-complement of half-duration.
Solar eclipse half-duration table. Sexagesimal integer and fractional (to one place) *ghaṭīs* of half-duration.

Method of computation. See Misra et al. (2016, pp. 17–18 and 30–31) respectively for the algorithms used to find the lunar and solar eclipse durations.

Paratext. The three table headings read as follows:

Lunar eclipse half-totality table. *khachinnāṃgulopari mardaghaṭīkā cakraṃ 60* "Complement [up to] 60 of the *ghaṭīs* of half-duration of totality, [with] above [argument] digits of sky-obscuration."
Lunar eclipse half-duration table. *atha candrachinnāṃgulopari caṃdrasya madhyasthitighaṭikādi cakraṃ 60* "Now, complement [up to] 60 of the *ghaṭīs* of mean half-duration [of the eclipse] of the moon, [with] above [argument] digits of lunar obscuration."
Solar eclipse half-duration table. *atha raviḥchannāṃgulāt madhyasthitighaṭikādi ravimadhyasthiti sparśakāle gaṇītagatakāle darśe hīnaṃ mokṣakāle yutaṃ | paścāl laṃbanaṃ deyaṃ* "Now, the *ghaṭīs* etc. of the mean half-duration with respect to the digits of solar obscuration. The solar [eclipse] mean half-duration, in [computing] the time of contact, is subtracted from the computed elapsed time [of] mid-eclipse; in [computing] the time of release, added. Then longitudinal parallax is to be applied."

Eclipse duration tables: *Parvadvayasādhana*

Figure 4.39 shows tables of duration of eclipse stages from a manuscript of the *Parvadvayasādhana*.

Argument.

Lunar eclipse half-totality table: 1 to 9 digits of "sky-obscuration."
Lunar eclipse half-duration table: 1 to 21 digits of lunar disk obscuration. (The scribe appears to have erroneously added a column for argument value 22, which is left blank.)
Solar eclipse half-duration table: 1 to 12 digits of solar disk obscuration.

Table entries.

Lunar eclipse half-totality table. Sexagesimal integer and fractional (to one place) *ghaṭīs* of half-duration of totality, from initial minimum value of 1;0 to final maximum value of 2;6 beginning at argument value 8 digits.
Lunar eclipse half-duration table. Sexagesimal integer and fractional (to one place) *ghaṭīs* of half-duration, from initial minimum value of 1;27 to final maximum value of 4;40 beginning at argument value 17 digits.

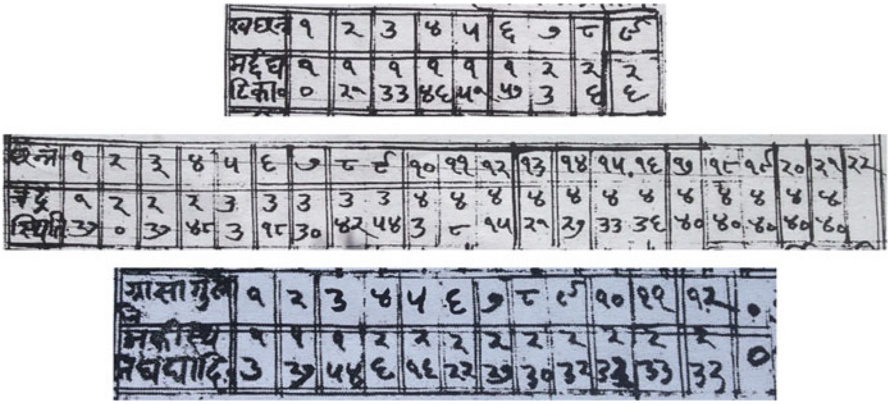

Fig. 4.39 Eclipse duration tables from the *Parvadvayasādhana* (CSS d. 800). Top: Half-duration of totality of lunar eclipse (f. 3r). Middle: Half-duration of lunar eclipse (f. 2v). Bottom: Half-duration of solar eclipse (f. 3r).

Solar eclipse half-duration table. Sexagesimal integer and fractional (to one place) *ghaṭīs* of half-duration, from initial minimum value of 1;3 to final maximum value of 2;33 beginning at argument value 11 digits.

Paratext. The row headers of the three tables read as follows:

Lunar eclipse half-totality table. Argument row: *khachanna* "sky-obscuration." Entry row: *marddaghaṭikā* ○ "*ghaṭīs* of half-duration of totality."
Lunar eclipse half-duration table. Argument row: *khachanna* "sky-obscuration." Entry row: *caṃdrasthitiḥ* "half-duration of lunar [eclipse]."
Solar eclipse half-duration table. Argument row: *grāsāṃgula* "digits of [solar disk] obscuration." Entry row: *arkasthitighaṭyādiḥ* "*ghaṭīs* of half-duration of solar [eclipse]."

4.8.5 Eclipse deflection (*valana*)

The following examples illustrate some of the ways the quantity known as *valana* or deflection in direction is tabulated for eclipse computation.

Deflection tables: *Grahaṇasāraṇī*

Figure 4.40 shows a table for deflection from a manuscript of the *Grahaṇasāraṇī*. Although the manuscript identifies the quantity only as *valana* "deflection," the table entries reveal that it is specifically the component of deflection due to terrestrial latitude ϕ (*akṣavalana*).

Argument: 0 to 90° of zenith distance ζ.

Fig. 4.40 Deflection table from the *Grahaṇasāraṇī* (Plofker 58 f. 5r–v).

Table entries: Sexagesimal integer and fractional (to one place) digits of *akṣavalana*, from initial minimum value of 0;0 to final maximum value of 13;0.

Method of computation. The pattern in the fractional digits suggests that the entries were computed directly for argument values 30°, 60°, and 90° according to the following algorithm, with linear interpolation used to get intermediate values:

$$ak\d{s}avalana = \arcsin(\sin\zeta \cdot \sin\phi)$$

with $\phi = 13°$.

Paratext. The table heading reads *valanam aṃgulādyaṃ* "The deflection beginning with digits."

Deflection tables: *Karaṇakesarī*

Figures 4.41, 4.42, and 4.43 show respectively tables for the component of deflection due to terrestrial latitude (*akṣavalana*), the component due to ecliptic obliquity (*ayanavalana*), and the sum of the two components, from a manuscript of the *Karaṇakesarī*.

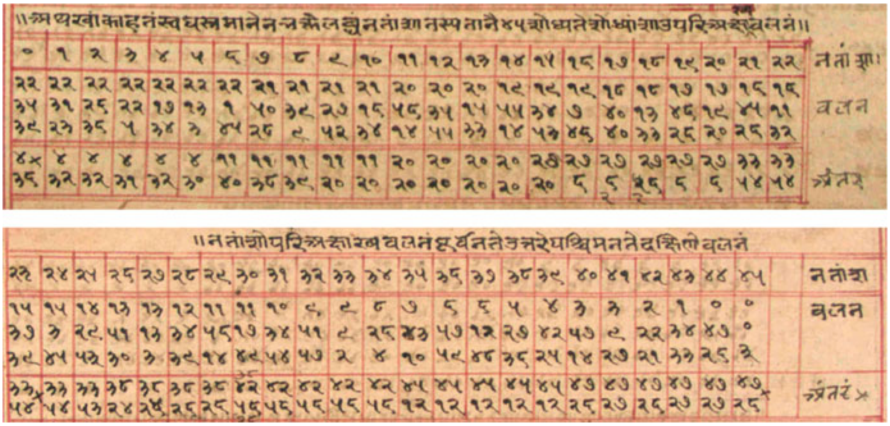

Fig. 4.41 Table of deflection due to terrestrial latitude (*akṣavalana*) from the *Karaṇakesarī* (RORI 12792 f. 3r–v).

akṣavalana

Argument: 0 to 45° of the 45-complement of half the zenith distance, $45° - \zeta/2$.

Table entries.

Row 1: Degrees, arcminutes and arcseconds of *akṣavalana* from initial maximum value of 22°;35,39 at argument value 0 (corresponding to $\zeta = 90°$) to final minimum value of 0°;0,3 at argument value 45 (corresponding to $\zeta = 0°$).
Row 2: Differences between successive entries in Row 1.

Method of computation. The algorithm used appears to be based on the same relation as in the above *Grahaṇasāraṇī* example, but with $\phi \approx 22°;35,39$. See Montelle and Plofker (2013, pp. 23–26) for a reconstruction of the computation.

Paratext. The table heading on the first page reads *atha khāṃkāhataṃ svaghasramānena bhakte labdhaṃ natāṃśā tasya tānai 45 śodhyate śodhyāṃśā upari akṣākhyavalanaṃ* "Now, when [the hour-angle] is divided by the amount of its own day measured [and] multiplied by 90, [there is] obtained the degrees of [half] the zenith distance. Its [amount] is to be subtracted from 45; the degrees from the subtraction are the above [argument of] the *valana* called latitudinal."

Its continuation on the second page reads *nataṃśopari akṣākhyavalanaṃ pūrvanate uttare paścimanate dakṣiṇe valanaṃ* "The *akṣavalana* [with its] above [argument] degrees of [half] the zenith distance. The *valana* [is to be applied] in the north when the zenith distance [is] eastern, in the south when the zenith distance [is] western."

ayanavalana

Argument: 0 to 90° of the complement of tropical solar longitude.

Fig. 4.42 Table of deflection due to ecliptic obliquity (*ayanavalana*) from the *Karaṇakesarī* (RORI 12792 f. 4r).

Fig. 4.43 Tables of combined deflection for solar and lunar eclipses from the *Karaṇakesarī* (RORI 12792 f. 4v).

Table entries.

Row 1: Degrees, arcminutes and arcseconds of *ayanavalana* from initial minimum value of 0°;0,3 at argument value 0 to final maximum value of 24°;0,1 at argument value 90.
Row 2: Differences between successive entries in Row 1.

Method of computation. Every tenth table entry appears to have been calculated using the relation

$$ayanavalana = \arcsin(\sin(90° - \lambda) \cdot \sin 24°)$$

(where λ is tropical solar longitude), while the intervening values are linearly interpolated.

Paratext. The table heading reads *atha sparśakāle tathā mokṣakāle sāyanagrahasya koṭyāṃśopari aṃśādyaṃ āyanajaṃ valanaṃ sāyanagrahakarkādau dakṣiṇe makarādau uttare valano deyaṃ* "Now, [with] above [argument] degrees of the complement of the precession-added [longitude] of the planet at the time of contact, likewise at the time of release, the *ayanavalana* in degrees etc. The *valana* is to be applied to the south when the precession-corrected planet [?] is in [the six signs] beginning with Cancer, to the north when in [the six signs] beginning with Capricorn."

Sum of *valana* components

Argument: 0 to 47° of combined deflection (in both tables).

Table entries.

Solar *valana* table. Sexagesimal integer and fractional (to one place) digits of converted total solar valana, from initial minimum value of 0;7 to final maximum value of 8;2.
Lunar *valana* table. Sexagesimal integer and fractional (to one place) digits of converted total solar valana, from initial minimum value of 0;4 to final maximum value of 13;51.

Method of computation. The two tables' entries represent the algebraic sum of the *akṣavalana* and *ayanavalana* components scaled for solar and lunar eclipse diagrams, respectively. The algorithm used to determine the relative scales is unknown.

Paratext. The headings of the two tables, which include detailed instructions for representing the deflection direction in eclipse diagrams, read as follows:

Solar *valana* table. *atha sūryasya valanaṃ spaṣṭaṃ akṣākhyaṃ tathā āyanajaṃ valanayor yogāṃtarāṃśopari aṃgulādyaṃ valanaṃ spaṣṭa sūryasya sūryasya grāsavalanaṃ paścime deyaṃ mokṣavalanaṃ pūrve deyaṃ sūryasya sparśavalanaṃ viparītaṃ deyaṃ* [inserted from right margin: *uttare jātaṃ*] *dakṣiṇe deyaṃ dakṣiṇe jātaṃ uttare deyam* "Now, the accurate *valana* of the sun. The accurate *valana* of the sun in digits etc., [with] above [argument] degrees of the sum or difference of the two *valanas*, [the one] called latitudinal [and] likewise [the one] produced by the *ayana*. The sun's contact *valana* is to be applied to the west; the release *valana* is to be applied to the east. The sun's contact *valana* is to be applied in reverse: occurring in the north, applied to the south; occurring in the south, applied to the north."

Lunar *valana* table. *atha caṃdrasya valanaṃ spaṣṭaṃ akṣākhyavalanaṃ tathā āyanajaṃ valanayor yogāṃtarāśopari aṃgulādyaṃ valanaṃ spaṣṭaṃ caṃdragrahaṇe sparṣavalanaṃ pūrve deyaṃ mokṣavalanaṃ paścime deyaṃ caṃdrasya mokṣavalanaṃ viparītaṃ deyaṃ uttare jātaṃ dakṣiṇe deyaṃ dakṣiṇe jātaṃ uttare deyam* "Now, the accurate *valana* of the moon. The accurate *valana* in a lunar eclipse in digits etc., [with] above [argument] degrees of the sum or difference of the two *valanas*, [the one] called latitudinal [and] likewise [the one] produced by the *ayana*. The contact *valana* is to be applied to the east; the release *valana* is to be applied to the west. The moon's release *valana* is to be applied in reverse: occurring in the north, applied to the south; occurring in the south, applied to the north."

4.8.6 Lunar parallax

As discussed in Section 2.1.6, the longitudinal (*lambana*) and latitudinal (*nati*, *avanati*) components of lunar parallax are tabulated as separate quantities.

Parallax tables: *Parvadvayasādhana*

Figure 4.44 shows tables of longitudinal and latitudinal parallax from a manuscript of the *Parvadvayasādhana*.

Argument.

Longitudinal parallax table. No argument row is given, but the implied argument is 0–3 30° increments of tropical longitudinal elongation between the sun and the ascendant.]

Latitudinal parallax table. 1–12 zodiacal signs of tropical longitude of the ascendant.

Table entries.

Longitudinal parallax table. Sexagesimal integer and fractional (to one place) *ghaṭīs* of *lambana*, from initial maximum value of 3;30 to final minimum value of 0;45.

Latitudinal parallax table.

Row 1: Sexagesimal integer and fractional (to one place) digits of *nati*, decreasing from initial maximum value of 11;0 (south) to maximum value of −1;0 (north) at argument value 6/Virgo before returning to final maximum value of 11;0 (south).

Fig. 4.44 Tables of longitudinal (left) and latitudinal (right) parallax from the *Parvadvayasādhana* (UPenn 390 1888 f. 2v).

Row 2: Direction indicator for corresponding entries in Row 1.

Method of computation. We think the calculation of these values is based on techniques prescribed in the *Grahalāghava* (see Section 2.1.6; this also appears likely from the use of *natihārā* for *nati* in the paratext mentioned below), but we have not managed to reconstruct its details.

Paratext. The entry row headers in the two tables read as follows:

Longitudinal parallax table. *laṃbanāni* "[Values of] longitudinal parallax."
Latitudinal parallax table. *natihārā* "Latitudinal parallax."

Parallax tables: *Karaṇakesarī*

Figures 4.45 and 4.46 show respectively tables of longitudinal and latitudinal parallax from a manuscript of the *Karaṇakesarī*. See Montelle and Plofker (2013, pp. 39–45) and Misra et al. (2016, pp. 28–30) for discussions of their construction.

Longitudinal parallax ('mean')

Argument: 0 to 91(?) degrees of tropical longitudinal elongation between the sun and the ascendant.

Table entries: Sexagesimal integer and fractional (to one place) *ghaṭīs* of "mean" *lambana*, increasing from initial value of 3;40 to maximum value of 4;0 at argument value 24° before decreasing to final minimum value of 0;1.

Paratext. The table heading reads *atha darśāṃtalagnārkayor vivarabāhubhāga-pramite koṣṭake madhyamalaṃbanaṃ ghaṭikādi* "Now, in [each] table entry commensurate with the degree of the difference-arc [between the longitudes] of the sun and the ascendant at the end of the *tithi* [i.e., at mid-eclipse], the mid[-eclipse] longitudinal parallax in *ghaṭīs*."

Longitudinal parallax correction

Argument: $0^s, 0°$ to $11^s, 29°$ of tropical longitude of ascendant. The argument is split between the vertical axis (zodiacal signs from 0/Aries to 11/Pisces) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).

Table entries: Sexagesimal integer and fractional (to one place) correction factors, increasing from initial value of 27;22 to maximum value of 39;59 at argument value 5/Virgo 9° before decreasing to 39;54 at argument value 5/Virgo 29°, increasing to the same maximum 39;59 at argument value 6/Libra 17°, and returning to final value of 27;21.

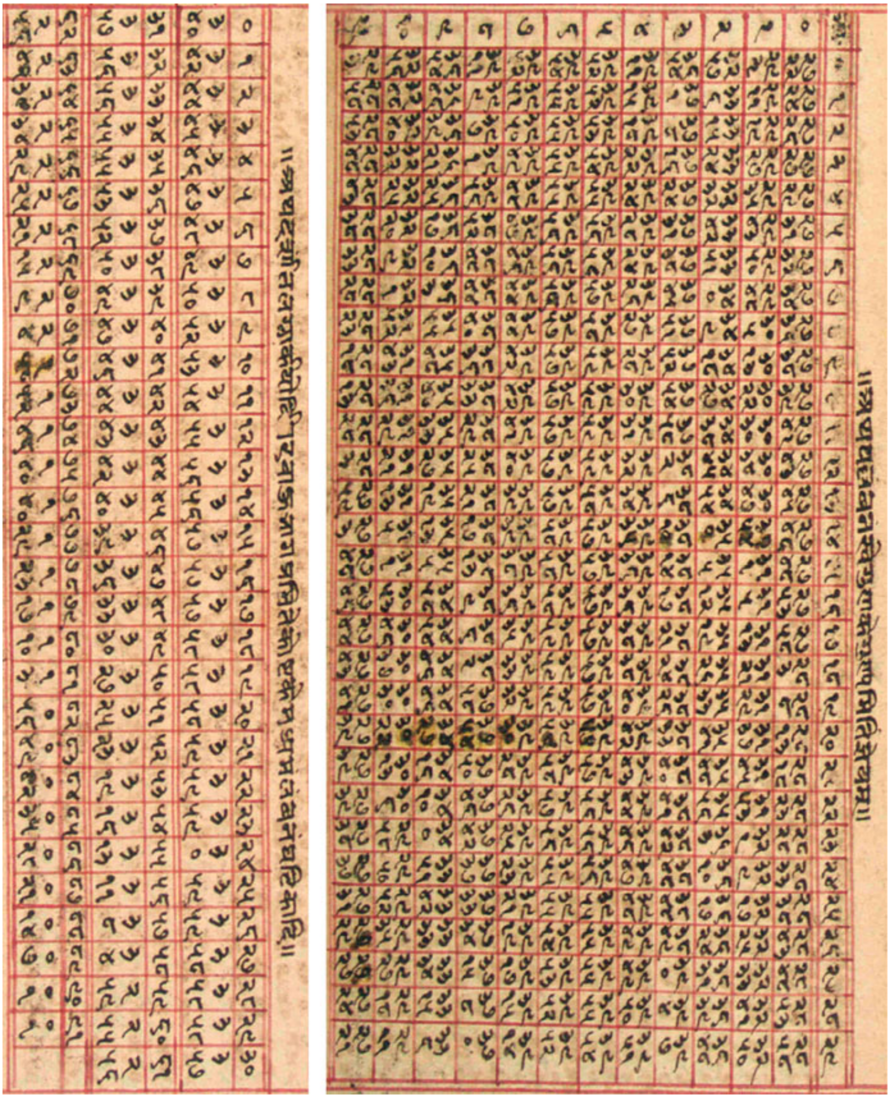

Fig. 4.45 Longitudinal parallax tables from the *Karaṇakesarī* (RORI 12792): "mean" *lambana* (left, f. 7v) and *lambana* correction factors (right, f. 8r).

Paratext. The table heading reads *atha yal laṃbanaṃ sveguṇake guṇyam iti jñeyam* "Now, each *lambana* [value] is to be multiplied [by] its own multiplier: thus it is understood."

Fig. 4.46 Latitudinal parallax table from the *Karaṇakesarī* (RORI 12792 f. 8v).

Latitudinal parallax

Argument: $0^s, 0°$ to $11^s, 29°$ of tropical longitude of ascendant. The argument is split between the vertical axis (zodiacal signs from 0/Aries to 11/Pisces) and the horizontal axis (degrees in each zodiacal sign from 0 to 29).

Table entries. Sexagesimal integer and fractional (to one place) *ghaṭīs* of latitudinal parallax, decreasing from initial value of 11;46 (south) to minimum value of 0;1 at argument value 5/Virgo 12° before increasing to maximum north value of 0;24 at argument value 5/Virgo 29°, decreasing to minimum 0;1 at argument value 6/Libra 18°, and returning to final maximum value of 11;46.

Paratext. The table heading reads *atha sāyanalagnarāśyaṃśopari nati aṃgulādi kaṃnyāṃnām aṃśa 11 tha kīte tulanā 'ṃśa 19 lageṃ uttare 'ṃnyathā sarvadakṣiṇe* "Now, the digits etc. of latitudinal parallax [with] above [argument] the sign and degree of the precession-corrected ascendant. [Between] degree 11 of [the entries for] Virgo/6 [. . .] degree 19 [of the entries for] Libra/7 [. . .] in the north; otherwise, entirely in the south."

4.9 Lunar latitude

As noted above in Section 4.8, tables of lunar latitude are crucial for eclipse reckoning. In this context their table argument is usually limited to the first 16 or so degrees of lunar-nodal elongation, since eclipses are theoretically possible only when the moon is close to a node. But some works also tabulate the full range of lunar latitude values separately from eclipse procedures, for purposes currently unclear to us. The following examples include both eclipse-specific and general tables of lunar latitude.

Lunar latitude tables: *Karaṇakesarī*

Figure 4.47 shows a table of lunar latitude for eclipse calculations from the *Karaṇakesarī*.

Fig. 4.47 Lunar latitude table from the *Karaṇakesarī* (RORI 12792 f. 2r).

Argument: 0 to 16° of nodal elongation. The table does not specify whether it is the sun or moon whose nodal elongation is considered here, but as discussed in Section 4.8.1, the absolute value of the length of the arc between either luminary and the nearest lunar node will be the same.

Table entries. The table contains the following two rows:

Row 1: Sexagesimal integer and fractional (to one place) digits of lunar latitude.
Row 2: Differences between successive entries in Row 1.

Maximum latitude: 24;45 digits at argument value 16°.

Method of computation. The latitude entries for argument values 8 and 16 appear to have been produced by the algorithm described in Section 2.1.6, implying the standard global maximum of 90 digits of latitude for elongation value 90° (Misra et al. 2016, p. 15). The remaining entries are produced by linear interpolation.

Paratext. The table heading reads *atha bhujāṃśopari śarāṃgulādi | śarāṃtara āveṃte bhujakalāvikalāgomūtrayādi guṇya* "Now, the digits of latitude [with its] above [argument] degrees of arc [of elongation]. The latitude difference [...] [and] the minutes and seconds of the arc [of elongation are] to be multiplied by the *gomūtrika* [multiplication technique]:" i.e., the heading is explaining how to interpolate linearly within the table.

Lunar latitude tables: *Makaranda*

Figure 4.48 shows a lunar latitude table from the eclipse section of the *Makaranda*. Unlike the previous example, this table shows a complete range of latitude values.

Argument: 0 to 90° of lunar-nodal elongation.

Table entries. The table's one row contains values in decimal integer and sexagesimal fractional (to one place) arcminutes of lunar latitude. They all appear to have been computed directly by an unidentified algorithm, rather than as a mix of algorithm-derived values and linearly interpolated ones.

Maximum latitude: $270';0$ at argument value 90°.

Paratext. The table heading reads *pratyāṃśaṃ vikṣepakalā caṃdrasya sapātacaṃdrasya bhujāṃśamitikoṣṭhasthaḥ śaraḥ sānupāto grāhyaḥ ayaṃ śaraḥ sūkṣma kalādi* "The minutes of latitude of the moon for each degree: the latitude, situated in the [table] cell commensurate with the degree of arc [of longitude] of the moon plus [its] node, is obtained by proportion. This latitude is accurate, beginning with arcminutes." An abbreviated heading on the second page reads *pratyāṃśaṃ śaraḥ* "Latitude for each degree."

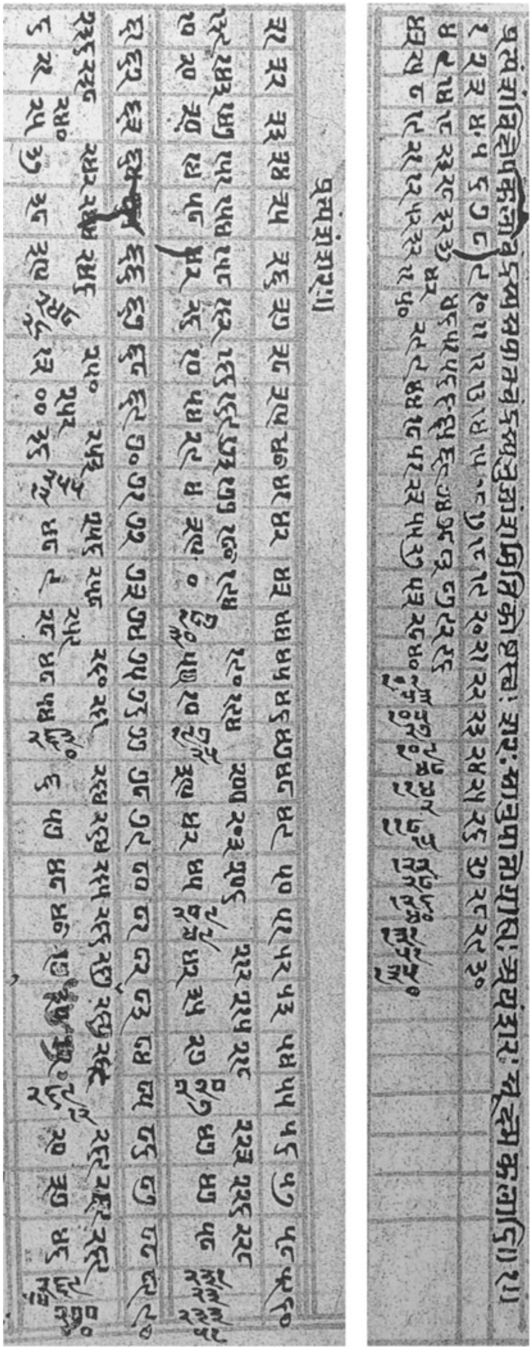

Fig. 4.48 Lunar latitude table from the *Makaranda* (BORI 546 f. 12r–v): argument values 1–30 (right) and 31–90 (left). (Note the scribe's rotation of entry numbers that proved too wide for his table columns.)

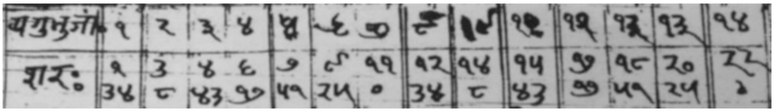

Fig. 4.49 Lunar latitude table from the *Parvadvayasādhana* (CSS d. 800 f. 2r).

Lunar latitude tables: *Parvadvayasādhana*

Figure 4.49 shows a lunar latitude table for eclipse calculations from the *Parvadvayasādhana*.

Argument: 1 to 14° of lunar-nodal elongation.

Table entries. The table's one row contains sexagesimal integer and fractional (to one place) digits of lunar latitude. They appear to have been computed via linear interpolation as fourteen evenly spaced values.

Maximum latitude: 22;0 digits at argument value 14°.

Paratext. The argument row header reads *vyagubhujāṃ*∘ "degrees of arc [of elongation of moon] from node," and the entry row header reads *śaraḥ* "latitude."

Lunar latitude tables: *Rājamṛgāṅka*

Figure 4.50 shows a full table of lunar latitude from a manuscript of the *Rājamṛgāṅka*, which does not tabulate any other quantities specifically relevant to eclipse computations.

Argument: 1 to 90° of lunar-nodal elongation.

Table entries.

Row 1: Arcminutes and arcseconds of lunar latitude.
Row 2: Differences between successive entries in Row 1.

Maximum latitude: $270';0$.

Method of computation. All the lunar latitude values appear to have been computed directly by an unidentified algorithm, rather than as a mix of algorithm-derived values and linearly interpolated ones.

Paratext. The table heading on the second page reads *caṃdravikṣepakalāḥ* "Minutes of lunar latitude." Squeezed into an empty column just before the start of the table on the first page is the abbreviated form *caṃdravikṣe*.

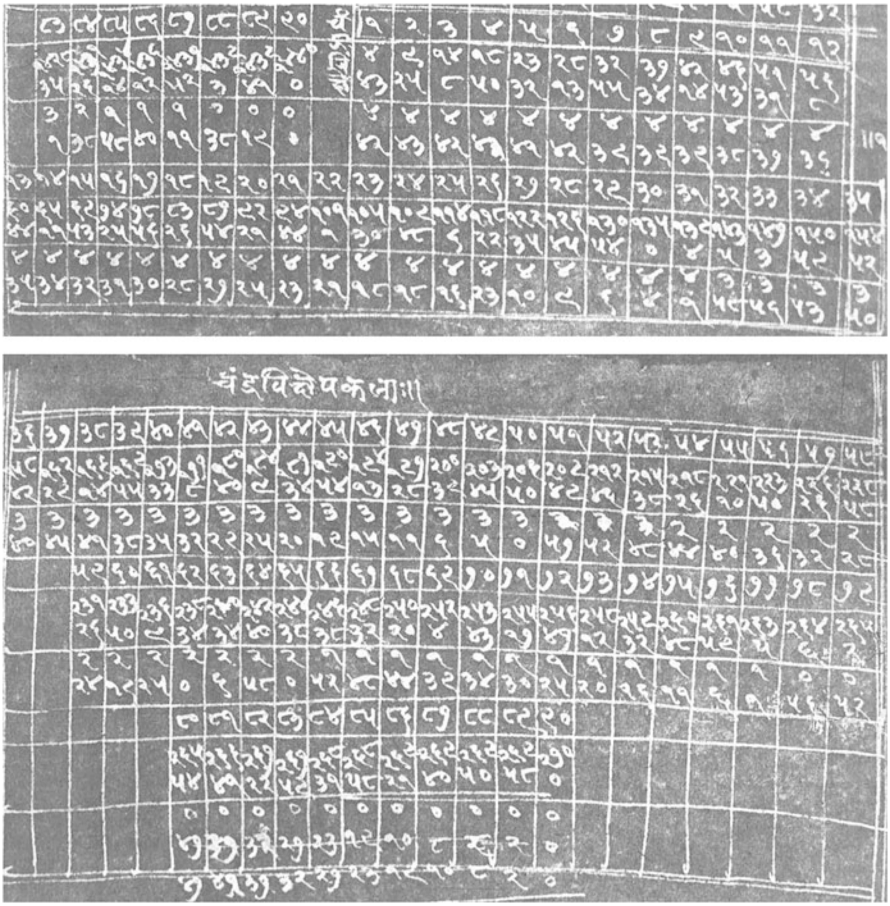

Fig. 4.50 Lunar latitude table from the *Rājamṛgāṅka* (Baroda 9476 ff. 14v–15r).

4.10 Calendric tables

This category of tables is primarily designed to support construction of the classic Indian calendar or *pañcāṅga* discussed in Section 1.4.3.

Calendar tables for *tithi/nakṣatra/yoga*: *Makaranda*

In this work, the weekday and time corresponding to the beginnings of the initial *tithis*, *nakṣatras* and *yogas* of some time interval are tabulated separately. Correction terms are also tabulated for converting from the beginning of a mean time-unit to that of a true one. (See Section 5.5 for a note on the various tabular epochs found in manuscripts of the *Makaranda*, and Section 5.5.1 for a synopsis of Makaranda's idiosyncratic terminology.)

***Tithis*.** Figure 4.51 shows an excerpt from tables of weekday and time corresponding to the start of the initial *tithis* of various time-units, from a manuscript of the *Makaranda*.

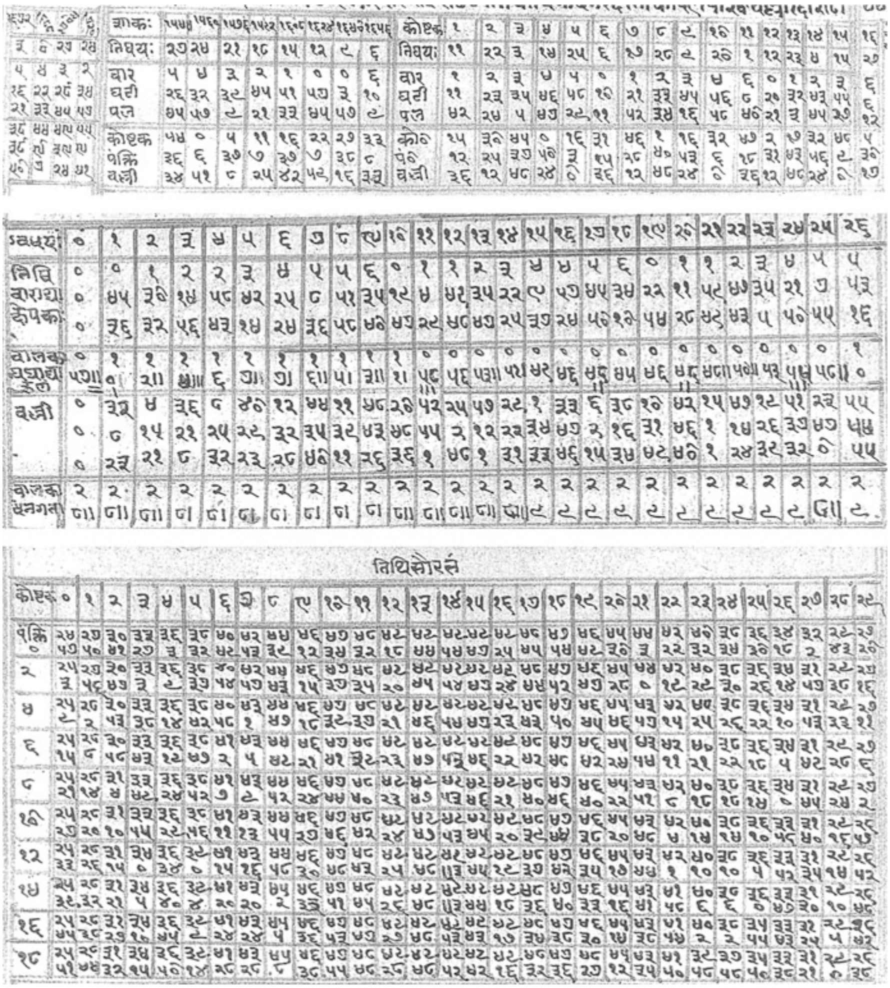

Fig. 4.51 Tables of weekday/time at start of *tithi* from the *Makaranda* (BORI 446). Top (f. 2v): Table for initial *tithis* of 16-year periods (left), and 16 single years (right). Middle (f. 3r): Table for initial *tithis* of *pakṣas* (called *'vadayaḥ* = *avadhis* in the row header). Bottom (f. 3v): Beginning of *saurabha* table for determining day/time adjustment for true *tithi*.

Argument.

Table of 16-year periods: Year numbers (out of order) from Śaka 1544 to 1656 and to the left of that, 1672 to 1720 (1622–1798 CE).

Table of single years: 1 to 16 solar years beginning approximately with Meṣasaṅkrānti (actually with the start of the mean *tithi* in which Meṣasaṅkrānti falls).

Table of *pakṣas*: 0 to 26 *pakṣas* (apparently confused with *avadhis* by the scribe in Figure 4.51).

Table entries.

Table of 16-year periods:

Row 1: The number of the initial *tithi* of each 16-year period.

Row 2: The integer and sexagesimal fractional (to two places) weekday corresponding to the start of the mean *tithi* in Row 1 (offset by a vertical shift of −0;24,57 days; see "Method of computation" below).

Row 3: The *vallī* or quantity determining the amount of the offset between the initial instants of the mean and the true initial *tithi*, in integer and sexagesimal fractional (to two places) 6° arc-units. For periods of integer years, the *vallī* depends only on the amount of lunar anomaly since the net effect of solar anomaly over an integer number of years is zero.

Table of single years:

Row 1: The number of the initial *tithi* of each solar year.

Row 2: The integer and sexagesimal fractional (to two places) weekday increment modulo 7 corresponding to the offset start of the mean *tithi* in Row 1.

Row 3: The *vallī* (or lunar anomaly) in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of that initial *tithi*.

Table of *pakṣas*:

Row 1: The integer and sexagesimal fractional (to two places) weekday increment modulo 7 corresponding to the offset start of the "true-mean" *tithi* (reflecting the motion of true sun and mean moon) initiating the current *pakṣa*.

Row 2: The negative interpolation factor for the deficit up to integer days of the integer and fractional weekday increment for each *tithi* in the current *pakṣa*. That is, one-fifteenth of the 60-complement of the difference between successive entries in Row 1.

Row 3: The *vallī* in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of each *pakṣa*. These non-uniform *vallī* increments appear to take into account changing solar anomaly during the course of the year, as well as the approximately monthly cycle of lunar anomaly.

Row 4: The positive interpolation factor for the *vallī* for each *tithi* in the current *pakṣa*: that is, one-fifteenth of the difference between successive entries in Row 3.

Table of weekday/time corrections for true *tithi* (*tithi-saurabha*).

Argument: 0;0 to 59;59 6° arc-units of *vallī*. The argument is split between the horizontal axis (6° arc-units from 0 to 59) and the vertical axis (sixtieths of 6° arc-units from 0 to 59). (The bottom table in Figure 4.51 tabulates the vertical-axis argument only for even values.)

Table entries. The entries in *ghaṭīs* and *vighaṭīs* represent the offset between the start of the desired true *tithi* and that of the corresponding mean *tithi*. They increase from the mean value 24;57 at argument 0;0 (the beginning of the table) to the maximum value 49;54 starting at 13;54, decrease through the mean again at 30;0 to the minimum value 0;0 at 45;48 and then increase again to the mean at the end of the table.

Paratext. The table heading reads *tithisaurabhaṃ* "The 'fragrance' of *tithis*."

Method of computation. These luni-solar calendar synchronization tables are designed to account for three phenomena:

1. the fractional time-units of difference between integer mean *tithis* and integer solar years;
2. the (small) variation in the synchronization of true and mean *tithis* over the course of a year due to the annual cycle of solar anomaly;
3. the (much greater) variation in the synchronization of true and mean *tithis* over the course of an anomalistic month due to the cycle of lunar anomaly.

The difference in time between twelve complete synodic months (or 360 mean *tithis*) and a solar year, called the annual epact (Sanskrit *śuddhi*), is approximately 11;3,52,30 mean *tithis*. So the *tithi*-number in the tables of weekday/time values increases by 11 for every single year and by 27 ($\approx$ 11;3,52,30 $\times$ 16, modulo integer 30-*tithi* months) for every 16-year period. Recall that the entries in the weekday/time tables for single years and for 16-year periods each correspond to the completion of some integer number of mean *tithis*, which will not perfectly sync up with an integer number of solar years. Thus the tables' constant differences (modulo 7 weekdays) are not identical to integer multiples (modulo 7 weekdays) of the mean solar year length: rather, 371 *tithis* is approximately 365;11,41,40 days. This single-year constant difference 1;11,41,40 times 16, plus the 0;59,3 days equivalent to an extra mean *tithi*, gives (modulo 7 weekdays) the 6;6,12 value for the constant difference in the 16-year table. (These successive-difference values are not shown in the table layout as there is no need to interpolate within these integer-year tables.)

The *pakṣa* table, on the other hand, does require interpolation between its tabulated entries in order to determine the weekday/time of a given *tithi* within a given *pakṣa*. Somewhat unusually, the quantity used for interpolation (*phala*, *cālaka* or *cālana*) is negative: namely, the 60-complement or 60 minus the difference of two successive weekday/time entries in the *pakṣa* table, instead of the difference itself. One-fifteenth of that 60-complement is tabulated as the negative interpolation factor.

The weekday/time entries in the *pakṣa* table are nonlinear, evidently following the variation in solar anomaly during the year. When the sun is near its apogee, between two and three months after Meṣasaṅkrānti, its true velocity is minimum. Therefore the amount of time required to complete 15 "true-mean" *tithis*, where the mean moon gains 12° of elongation from the true sun, is also minimum. Consequently, the 60-complement of that amount of time is maximum, as is the corresponding absolute value of the interpolation factor. The subsequent increase in weekday/time entries reflects the greater apparent solar velocity later in the year and produces smaller absolute values for the interpolation factor.

The tabulated *vallī* values in all three tables represent amounts of lunar and solar anomaly corresponding to their respective amounts of solar time. As noted above, net solar anomaly contribution to *vallī* in a period of integer solar years is zero, but within a single year it is nonzero. The constant differences in *vallī* entries in the first two tables represent the excess of lunar anomaly over integer cycles in integer years: about 15°;12,36,6 arc-units for a single year, and 5°;30,17,6 arc-units for 16 years. The *vallī* entries in the *pakṣa* table appear to represent amounts of combined lunar and solar anomaly in the course of a year. The contribution from lunar anomaly, as noted above, is periodic with period one anomalistic month or approximately two *pakṣas*. The contribution from solar anomaly is positive during the sun's course from apogee to perigee, when the sun's velocity is increasing and lengthening the time required to complete a true *tithi*; it is negative when the sun's progress slows down between perigee and apogee and the requisite 12° of luni-solar elongation is attained more quickly. The interpolation factors for these *vallī* entries are positive.

The true-*tithi* correction terms in the *tithi-saurabha* table serve to convert the weekday/time entries from mean or "true-mean" *tithi*-beginnings to true ones. They represent the time adjustment in fractional days corresponding to a given amount of *vallī*. Note that these periodic corrections are vertically shifted in the positive direction to make all entries nonnegative; the first weekday/time entry in the table of 16-year periods is equally offset in the negative direction to account for this shift. In other words, when the *vallī* is 0 or 30 6° arc-units and the current true *tithi* is synchronized with the corresponding mean *tithi*, the mean correction 0;24,57 days is added to the tabulated weekday/time value to cancel out the initial offset.

Notes on table use. The weekday and time for the beginning of any mean *tithi* in a desired year are added up (modulo 7 weekdays) from the incremental entries in the tables for 16-year periods, individual years, and *pakṣas* of the current year (see the worked examples in Rupa et al. (2014)). Recall that the interpolation procedure in the *pakṣa* table is "backwards": that is, a user seeking the weekday/time for the beginning of *tithi* number n in a given *pakṣa* must add n to the weekday/time tabulated for the start of the first mean *tithi* in that *pakṣa*, and then subtract from that sum n times the corresponding interpolation factor, to account for the excess of day-length over *tithi*-length.

The correction procedure to bring the weekday/time up to the start of the true *tithi* instead of the offset mean or mean-true *tithi* begins with adding up the *vallī* increments produced for the given 16-year period, individual year, and location within the current year and *pakṣa*. Then this sum is entered into the true-*tithi* correction or *saurabha* table, whose corresponding entry is added to the previously determined weekday/time for the desired *tithi*.

***Nakṣatras*.** Figure 4.52 shows related *Makaranda* tables from a different manuscript, indicating the weekday and time corresponding to the start of initial *nakṣatras*.

Argument.

Table of 24-year periods: Year numbers from Śaka 1448 to 1880 (1526–1958 CE). (As noted in Section 5.5, the year-number ranges in various manuscripts/recensions of the work cover different periods.)

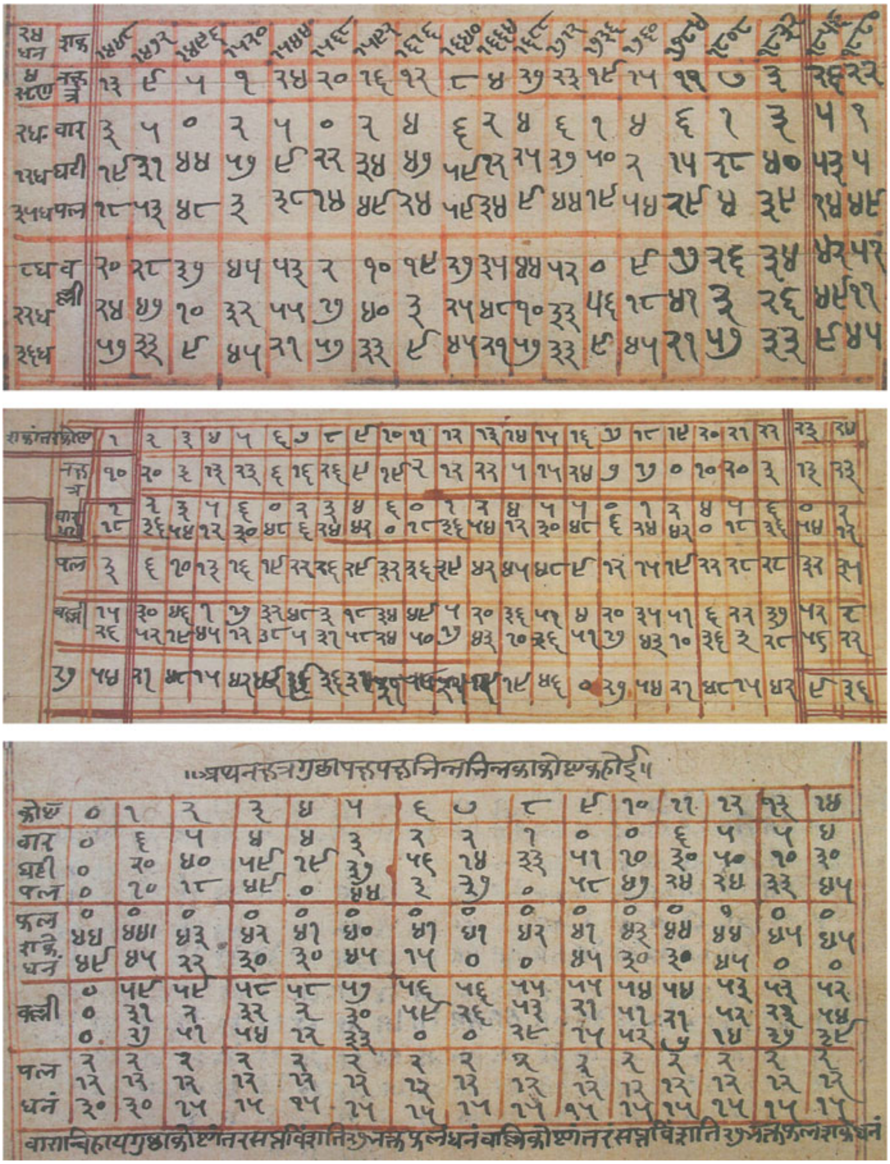

Fig. 4.52 Tables of weekday/time at start of *nakṣatra* from the *Makaranda* (RORI 5498). Top (f. 6r): Table for initial *nakṣatras* of 24-year periods. Middle (f. 6v): Table for initial *nakṣatras* of single years. Bottom (f. 7r): Table for initial *nakṣatras* of sidereal months.

Table of single years: 1 to 24 solar years beginning with Meṣasaṅkrānti.
Table of sidereal months: 0 to 14 sidereal months or *nakṣatra* cycles.

Table entries.

Table of 24-year periods:
Row 1: The number of the initial *nakṣatra* of each 24-year period.
Row 2: The integer and sexagesimal fractional (to two places) weekday corresponding to the start of the mean *nakṣatra* in Row 1.
Row 3: The *vallī*, or lunar anomaly in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of that initial *nakṣatra*.

Table of single years:
Row 1: The number of the initial *nakṣatra* of each solar year.
Row 2: The integer and sexagesimal fractional (to two places) weekday increment modulo 7 corresponding to the start of the mean *nakṣatra* in Row 1.
Row 3: The *vallī*, or lunar anomaly in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of that initial *nakṣatra*. (The first five columns shown in the manuscript image have the last sexagesimal place of the *vallī* mistakenly shifted one column to the left.)

Table of sidereal months:
Row 1: The integer and sexagesimal fractional (to two places) weekday increment modulo 7 corresponding to what appears to be the offset start of the *nakṣatra* beginning the current sidereal month. Their successive differences represent the sidereal month length of three integer weeks plus about 6;20 days. But this quantity seems to fluctuate slightly over the course of the year with its minimum about month 7, suggesting that it takes into account somehow an annual change in the solar day as well as the lunar motion increments.
Row 2: The positive *phala* or interpolation factor for the deficit up to integer days of the integer and fractional weekday for each *nakṣatra* in the current sidereal month. That is, 1/27 of the 60-complement of the difference between successive entries in Row 1.
Row 3: The *vallī* in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of each sidereal month. The differences of successive *vallī* entries, like those of the weekday entries, are slightly non-uniform, reaching their minimum about halfway through the year. This suggests again that annual changes in solar anomaly are being accounted for, as well as the approximately monthly cycle of lunar anomaly.
Row 4: The positive *phala* or interpolation factor for the *vallī* for each *nakṣatra* in the current sidereal month: that is, 1/27 of the difference between successive entries in Row 3.

Paratext. The table heading for the table of sidereal months reads as follows:

atha nakṣatragucchā pakṣapakṣabhinnabhinnakā koṣṭhaka hoī

"Now the 'shrub' of *nakṣatras*: table entries [for] each two *pakṣas* [?]."

The note at the bottom of the table reads:

vārān vihāya gucchākoṣṭhāṃtara saptaviṃśati 27 bhakta phalaṃ dhanaṃ vallikoṣṭhāṃtaraṃ saptaviṃśati 27 bhakta phalaśake dhanaṃ

"Disregarding the weekdays, the *guccha*-tabular difference is to be divided by 27. The result [is to be applied] positively. The *vallī*-tabular difference is to be divided by 27 [. . . ?] positively."

Fig. 4.53 Beginning of *saurabha* table for determining day/time adjustment for true *nakṣatra* from the *Makaranda* (RORI 5498 f. 7v).

Table of weekday/time corrections for true *nakṣatra* (*nakṣatra-saurabha*).

Argument: 0;0 to 59°;59 6 arc-units of lunar anomaly. The argument is split between the horizontal axis (6° arc-units from 0 to 59) and the vertical axis (sixtieths of 6° arc-units from 0 to 59). (The table in Figure 4.53 tabulates the vertical-axis argument only for every sixth value.)

Table entries. The entries in *ghaṭīs* and *vighaṭīs* represent the offset between the start of the desired true *nakṣatra* and that of the corresponding mean *nakṣatra*. They increase from the mean value 23;4 at argument 0;0 (which appears to be a vertical shift at the beginning of the table to make all the entries nonnegative, like the 0;24,57 offset in the *tithi-saurabha* table) to the maximum value 46;8 at 13;54, decrease through the mean again at 30;0 to the minimum value 0;0 at 45;48 and then increase again to the mean at the end of the table.

Paratext. The table heading reads *nakṣatrasaurabhaṃ* "The 'fragrance' of *nakṣatras*."

***Yogas*.** Figure 4.54 shows a selection of *yoga* tables from a third manuscript of the *Makaranda*.

Argument.

Table of 24-year periods: Year numbers from Śaka 1400 to 1664 (1478–1742 CE).
Table of single years: 1 to 24 solar years beginning with Meṣasaṅkrānti.
Table of *yoga* cycles: 0 to 14 *yoga* cycles.

Table entries.

Table of 24-year periods:
Row 1: The number of the initial *yoga* of each 24-year period. The table shown in Figure 4.54 has the row header mistakenly labelled *nakṣatra* instead of *yoga*. (Note,

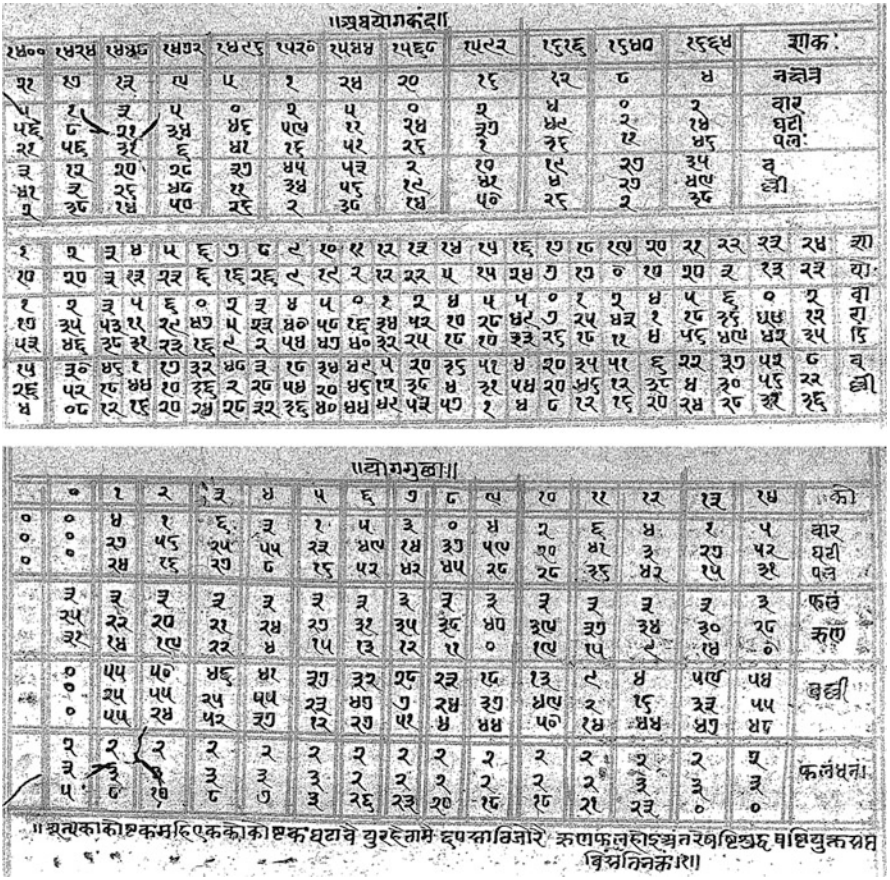

Fig. 4.54 Tables of weekday/time at start of *yoga* from the *Makaranda* (BORI 546). Top (f. 8v): Tables for initial *yogas* of 24-year periods (upper) and 24 single years (lower). Bottom (f. 9r): Table for initial *yogas* of *yoga* cycles.

however, from comparison with Figure 4.52 that the Row 1 entries in the tables of 24-year periods and single years are exactly the same for *yogas* and *nakṣatras*, owing to the fact that a solar year contains almost exactly the same fractional excess over integer cycles for both: namely, about 9.9 time-units. So both *nakṣatra* and *yoga* numbers for the start of the year increase by 10 annually.)

Row 2: The integer and sexagesimal fractional (to two places) weekday corresponding to the start of the mean *yoga* in Row 1.

Row 3: The *vallī*, or lunar anomaly in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of that initial *yoga*.

Table of single years:

Row 1: The number of the initial *yoga* of each solar year.

Row 2: The integer and sexagesimal fractional (to two places) weekday increment modulo 7 corresponding to the start of the mean *yoga* in Row 1.

Row 3: The *vallī*, or lunar anomaly in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of that initial *yoga*.

Table of *yoga* cycles:

Row 1: The integer and sexagesimal fractional (to two places) weekday increment modulo 7 corresponding to the offset start of the "true-mean" *yoga* (reflecting the motion of true sun and mean moon) beginning the current cycle. (The increment is largest when the sun is moving slowly near its apogee and the completion of a *yoga* takes more time.)

Row 2: The negative interpolation factor for the deficit up to integer days of the integer and fractional weekday for each *yoga* in the current cycle, i.e., the 60-complement of 1/27 of the difference between successive entries in Row 1.

Row 3: The *vallī* in integer and sexagesimal fractional (to two places) 6° arc-units for the beginning of each *yoga* cycle.

Row 4: The positive *phala* or interpolation factor for the *vallī* for each *yoga* in the current cycle, that is, 1/27 of the difference between successive entries in Row 3.

Paratext. The table headings read as follows:

Top: *atha yogakaṃda* "Now, the 'bulb' of *yogas*."
Bottom: *yogagucchā* "The 'shrub' of *yogas*."

We have not translated the note in a Sanskrit-related vernacular underneath the bottom table.

Table of weekday/time corrections for true *yoga* (*yoga-saurabha*)

Argument: 0;0 to 59°;59,6 arc-units of lunar anomaly. The argument is split between the horizontal axis (6° arc-units from 0 to 59) and the vertical axis (sixtieths of 6° arc-units from 0 to 59). (The table in Figure 4.55 tabulates the vertical-axis argument only for every sixth value.)

Table entries. The entries in *ghaṭīs* and *vighaṭīs* represent the offset between the start of the desired true *yoga* and that of the corresponding mean *yoga*. They

Fig. 4.55 Beginning of *saurabha* table for determining day/time adjustment for true *yoga* from the *Makaranda* (BORI 546 f. 9v).

increase from the mean value 21;27 at argument 0;0 (which again appears to be a vertical shift at the beginning of the table to make all the entries nonnegative) to the maximum value 42;54 at 13;54, decrease through the mean again at 30;0 to the minimum value 0;0 at 45;48 and then increase again to the mean at the end of the table.

Paratext. The table heading reads as follows:

> *yogasaurabhaṃ caṃdramaṃdaphalaṃ ravicaṃdragatiyogena bhaktaṃ yogasaurabhaṃ syāt*
>
> "The 'fragrance' of *yogas*: The moon's *manda*-equation divided by the sum of the solar and lunar velocities should be the *yoga-saurabha*."

Calendar tables for *tithi*/*nakṣatra*/*yoga*: *Tithicintāmaṇi* of Gaṇeśa

Figures 4.56, 4.57, and 4.58 show excerpts from tables from the *Tithicintāmaṇi* of Gaṇeśa, listing values of weekday and time corresponding to the ends of successive *tithis*, *nakṣatras* and *yogas* respectively. See Ikeyama and Plofker (2001) for details.

Table of *tithis*

Argument: 0 to 400 successive *tithis* beginning at Meṣasaṅkrānti.

Table entries.

Row 1: Integer and sexagesimal fractional (to two places) weekdays (modulo 7) of the time interval from the zeroth to the current *tithi*.

Fig. 4.56 Excerpt from weekday table for *tithis* from the *Tithicintāmaṇi* (UPenn 390 1891 f. 1v).

Fig. 4.57 Excerpt from weekday table for *nakṣatras* from the *Tithicintāmaṇi* (UPenn 390 1891 f. 6r).

Fig. 4.58 Excerpt from weekday table for *yogas* from the *Tithicintāmaṇi* (UPenn 390 1891 f. 10v).

Row 2 (*parākhya*): Sexagesimal integer and fractional (to one place) *ghaṭīs* of correction to the corresponding Row 1 entries to account for the effect of lunar anomaly.

Row 3 (*hāra* "divisor"): Divisors, in integer and *daṇḍa*-quarter-units, for further correcting the corresponding Row 2 entries.

Paratext. The table heading reads *tithipatraṃ* "*tithi*-table."

Table of *nakṣatras*

Argument: 0 to 390 successive *nakṣatras* from the beginning of the anomalistic month.

Table entries.

Row 1: Integer and sexagesimal fractional (to two places) weekdays (modulo 7) of the time interval from the zeroth to the current *nakṣatra* of the anomalistic month.
Row 2 (*hāra* "divisor"): Divisors, in integer and *daṇḍa*-quarter-units, for correcting the corresponding Row 1 entries.

Paratext. The table heading reads *nakṣatrapatrāṇi* "*nakṣatra*-tables."

Table of *yogas*

Argument: 0 to 390 (Row 1) or 0 to 419 (Rows 2–3) successive *yogas*.

Table entries.

Row 1: Integer and sexagesimal fractional (to two places) weekdays (modulo 7) of the time interval from the zeroth to the current *yoga*.
Row 2 (*parākhya*): Sexagesimal integer and fractional (to one place) *ghaṭīs* of correction to the corresponding Row 1 entries to account for the effect of lunar anomaly.
Row 3 (*hāra* "divisor"): Divisors, in integer and *daṇḍa*-quarter-units, for further correcting the corresponding Row 2 entries.

Paratext. The table heading reads *yogapatraṃ* "*yoga*-table."

Method of computation. What we know so far about the production of these table entries is described in detail in Ikeyama and Plofker (2001). In brief, they appear to have been created by computing mean solar and lunar longitudes corresponding to successive mean time-units measured in days, and then applying the appropriate true-motion corrections to adjust the length of the tabulated intervals based on true solar and mean lunar positions. Further adjustments to account for the lunar anomaly are tabulated in subsequent rows, and apparently can be applied separately at the user's discretion.

Calendar tables for *saṃkrānti*: *Makaranda*

Figure 4.59 shows tables of weekday and time for *saṃkrānti*, the sun's entry into various divisions of its cycle, from a manuscript of the *Makaranda*.

Argument.

Table of 24-year periods: Year numbers from Śaka 1496 to 1664 (1574–1742 CE).
Table of single years: 1 to 24 solar years.
Table of zodiacal signs: Aries–Pisces (1 to 12 zodiacal signs).
Table of *nakṣatras*: Aśvinī–Revatī (1 to 27 *nakṣatras*).

Table entries. The integer and sexagesimal fractional (to two places) weekday corresponding to the *saṃkrānti* in question.

Fig. 4.59 Tables of weekday/time at *saṃkrānti* from the *Makaranda* (BORI 546 f. 10v).

Fig. 4.60 Tables of accumulated civil days from the *Makaranda* (BORI 446 f. 14r). Top: Sexagesimal *ahargaṇa* for 57-year periods (left), and *ahargaṇa* increments for 57 single years (right and middle row). Bottom: Sexagesimal *ahargaṇa* increments for 24 *pakṣas*.

Paratext. The table headings read as follows:

24-year periods: *makaraṃde saṃkrāṃtiśakaḥ* "Year [at] *saṃkrānti* in the *Makaranda*."
Single years: *makaraṃde saṃkrāṃtiśaka aṃtarakoṣṭakā* "Table entries [at] intervals [of a] year [at] *saṃkrānti* in the *Makaranda*."
***Nakṣatras*:** *mahānakṣatrāṇi koṣṭaṃ* "Table [of] the great [?] *nakṣatras*."

We have not translated the two accompanying Sanskrit/vernacular notes.

Calendar tables for accumulated civil days: *Makaranda*

Figure 4.60 contains tables of the accumulated civil days or *ahargaṇa* since the beginning of the Kaliyuga (see Section 1.1.4) from a manuscript of the *Makaranda*.

Argument.

Table of 24-year periods: Year numbers from Śaka 1400 to 1799 (1478–1877 CE).
Table of single years: 1 to 57 luni-solar calendar years.
Table of *pakṣas*: Caitra *śuklapakṣa* to Caitra *kṛṣṇapakṣa*, followed by *adhimāsa kṛṣṇapakṣa* and *adhimāsa śuklapakṣa* (1 to 26 *pakṣas*).

Table entries.

Table of 57-year periods:

Row 1: The *vallī*, here meaning the integer sexagesimal (to four places) civil days of the Kaliyuga accumulated at the end of the numbered year. The constant difference between entries is $20819 = 0, 5, 46, 59$ civil days (almost exactly 57 solar years or 705 synodic months).

Row 2: The *vāra* or weekday number of the first day of the following year. The constant difference between entries is $1 \mod 7$ since $20819 = 1 \mod 7$. The first entry 3 represents Wednesday because the Kaliyuga began on a Friday and the *ahargaṇa* value in Row 1 is $1672494 = 5 \mod 7$, indicating that the initial *ahargaṇa* ended on Tuesday or weekday 2.

Table of single years:

Row 1: The *vallī* or integer sexagesimal (to four places) civil days of the current 57-year period accumulated at the end of the numbered calendar year. The difference between entries is either 353, 354, 355, 383, 384, or 385 civil days, depending on whether the year in question contains an intercalary month and/or an omitted or additional *tithi*.

Row 2: The increment (mod 7) to the *vāra* or weekday number accumulated at the end of the numbered calendar year. The difference between entries is the difference (mod 7) between corresponding Row 1 entries.

Table of *pakṣas*:

Row 1: The *vallī* or integer sexagesimal (to four places) civil days of the current calendar year accumulated at the end of the current *pakṣa*. The difference between entries is either 14 or 15 civil days, depending on whether the *pakṣa* in question does or does not contain an omitted *tithi*. Oddly, in the manuscript exemplar shown in Figure 4.60 the *kṛṣṇapakṣa* and *śuklapakṣa* of the potential intercalary month or *adhimāsa* are appended in that order after the 24th column Caitra *kṛṣṇapakṣa*, and their *ahargaṇa* values increase correspondingly. This would imply an astronomically impossible situation with two waning fortnights in a row followed by two waxing fortnights.

Row 2: The increment (mod 7) to the weekday number (here somewhat puzzlingly called the *cālana*) accumulated at the end of the current *pakṣa*. The difference between entries is the difference (mod 7) between corresponding Row 1 entries.

Paratext. The table heading reads *grahadinavallī bhṛguvārādicaitrāmāvāsyāsannā pakṣavallyāṃ grahadinavallyāṃ yogaḥ pakṣacālanāni vallyāṃ datvā 'gretanā vallī jāyaṃte evaṃ 'gre pi emā sayukta 'hargaṇā hi te aṃtaranāma* "The accumulated-days *vallī* [is] set at new moon of Caitra beginning with weekday Friday. The sum [of] the *pakṣa vallī* [and] the accumulated-days *vallī* [is made]. Putting the *pakṣa cālanas* [together with] the *vallī*, the subsequent [time-unit] *vallī* [is] produced. Thus at the beginning too, these added accumulated days [are] called the difference [?]."

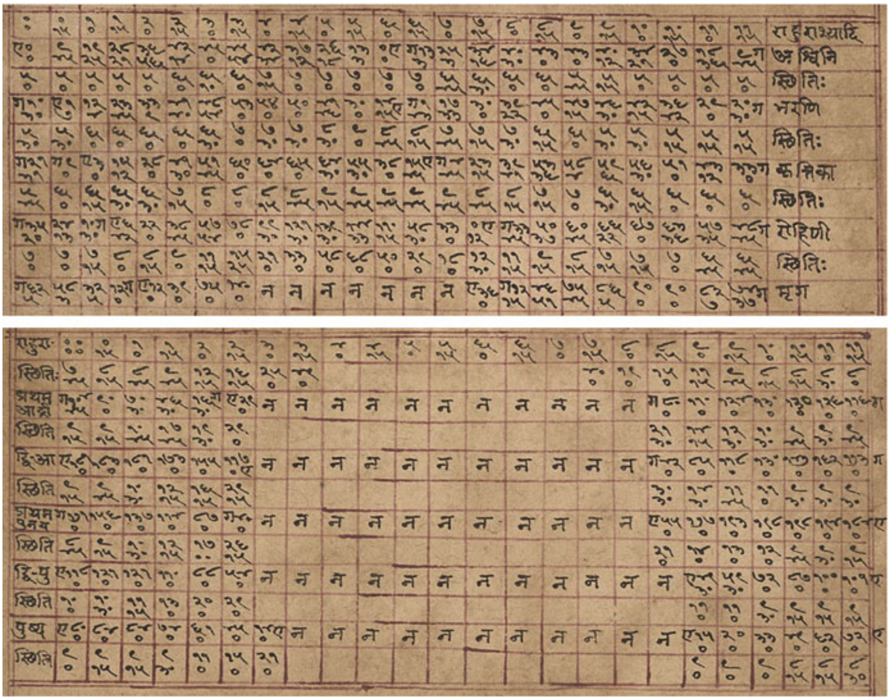

Fig. 4.61 Excerpt from a *mahāpāta* table from the *Pātasāraṇī* of Gaṇeśa (UPenn 390 697 f. 6v–7r).

4.11 The *mahāpātas*

Most Sanskrit *siddhāntas* and *karaṇas* address this topic of the so-called parallel aspects of the sun and moon, with equal declinations and symmetrical longitudes (see Section 2.1.7). But the only astronomical table text devoted to them that we know of is the *Pātasāraṇī* by Gaṇeśa (Section 5.6). This work has not yet been published, and the extent of its overlap with related procedures in the same author's *Grahalāghava* (Jośī 1994, p. 307) is yet to be determined. The following discussion is a tentative preliminary analysis of Gaṇeśa's *mahāpāta* table.

4.11.1 *Pātasāraṇī* of Gaṇeśa

Figure 4.61 shows a table from a manuscript of the *Pātasāraṇī*, enabling the user to determine the time interval between the so-called mean *mahāpāta*, or the moment when the ecliptic declinations of the moon and the sun have the same magnitude, and its "true" form, or the moment of equality of their true declinations.

Argument: The table employs a double argument: the 24 columns designate 0 to $11^s 15°$ zodiacal half-signs of nodal longitude, and the 30 rows *nakṣatras* 1–5, *nakṣatra* 6 (first half), *nakṣatra* 6 (second half), *nakṣatra* 7 (first half), *nakṣatra* 7 (second half), *nakṣatras* 8–19, *nakṣatra* 20 (first half), *nakṣatra* 20 (second half), and *nakṣatras* 21–27.

Table entries. Each *nakṣatra* or half-*nakṣatra* argument row in the table contains the following two rows:

Row 1: Integer and sexagesimal fractional (to one place) *ghaṭīs* of the time interval between the moments of "mean" *mahāpāta* (computed by the user for the given *nakṣatra* and nodal longitude) and accompanying "true" *mahāpāta*. Entries for the "true" *mahāpāta* occurring before or after its "mean" counterpart are marked by *ga* for *gata* "past" or *e* for *eṣya* "future," respectively. For combinations of nodal longitude and *nakṣatra* for which a "true" *mahāpāta* is impossible, the scribe has noted *na* "not" in the empty table cell.

Row 2: Integer and sexagesimal fractional (to one place) *ghaṭīs* of the *sthiti* "duration" of the corresponding *mahāpāta*, apparently referring to the time during which the luminaries' disks have a nonzero overlap in declination.

Paratext. The column-argument row heading reads *rāhurāśyādi* " [Longitude of] Rāhu beginning with zodiacal signs."

4.12 Trigonometric functions

As discussed in Section 2.1.8, purely mathematical trigonometric functions generally are not tabulated separately in Sanskrit astronomical table texts: instead, they are incorporated into the results of more specific formulas for computing astronomical quantities such as declination or parallax. Some of the few known exceptions to this trend are described in this section.

Table of *R* sines: *Rājamṛgāṅka*

Figure 4.62 shows two nearly identical forms of a table of *R* sines, presented together in a manuscript of the *Rājamṛgāṅka*.

Argument: 1 to 90° of arc, in both versions.

Table entries.

Row 1 (in both versions): Decimal integer and sexagesimal fractional (to one place) $R\sin$ values up to $R\sin 90° = R = 1000;0$.

Row 2 (only in first version): Differences between successive entries in Row 1.

Paratext. The table headings read as follows:

First: *saṃkalitajīvākoṣṭhakāḥ* "Tabulated [values of] cumulative $R\sin$"
Second: *atha kramajīvākoṣṭhakāḥ* "Now, tabulated [values of] $R\sin$."

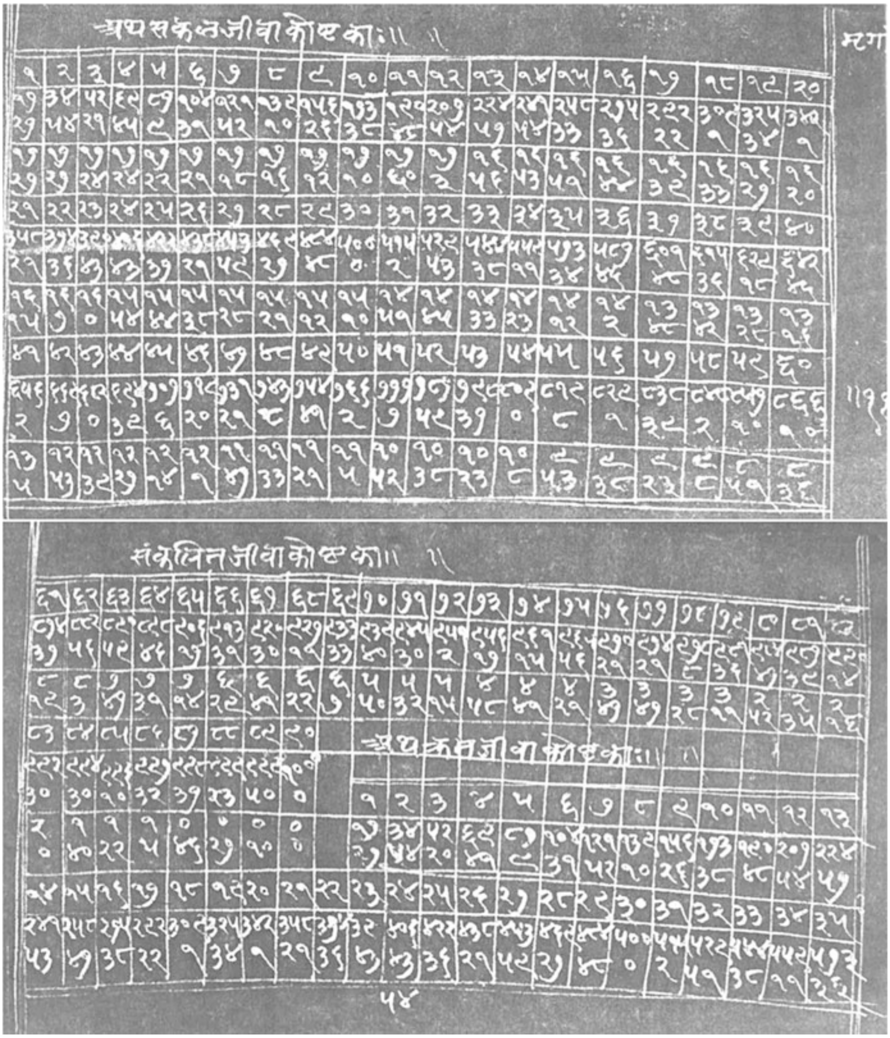

Fig. 4.62 Two versions of the *R* sin table from the *Rājamṛgāṅka* (Baroda 9476 ff. 11v–12r); only part of the second version is shown in this image.

Table of *R* versines: *Rājamṛgāṅka*

Figure 4.63 shows the same *Rājamṛgāṅka* manuscript's table of *R* versines.

Argument: 1 to 90° of arc.

Table entries: Decimal integer and sexagesimal fractional (to one place) *R* vers values up to *R* vers 90° $= R = 1000;0$.

Fig. 4.63 Table of *R* versines from the *Rājamṛgāṅka* (Baroda 9476 ff. 12v–13r). The entry for 90° is erroneously written as 1000;1 instead of 1000;0.

Paratext. The table heading reads *atha utkramajīvākoṣṭhakāḥ* “Now, tabulated [values of] versine.”

Table of *R* sines: *Sarvasiddhāntarāja*

Figure 4.64 shows a table of *R* sines from a manuscript of the *Sarvasiddhāntarāja* of Nityānanda.

Argument: 1 to 90° of arc.

Table entries: Decimal integer and sexagesimal fractional (to four places) *R* sin values up to $R \sin 90° = R = 60;0,0,0,0$.

Table of *R* sines (and *R* cosines): *Siddhāntasindhu*

Figure 4.65 shows a single page of a table of *R* sines, which also functions as a table of *R* cosines, from a manuscript of the *Siddhāntasindhu* of Nityānanda.

Argument: 0°0′ to 89°59′ and 180°0′ to 269°59′ of arc. The argument is split between the horizontal axis (0 to 90° or 180 to 270°) and the vertical axis (0 to 59 minutes in each degree).

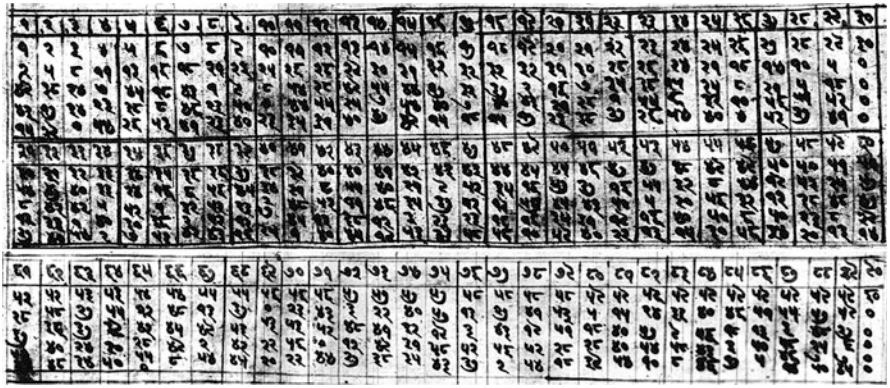

Fig. 4.64 Table of *R* sines from the *Sarvasiddhāntarāja* (Nepal B534 f. 15r–v). The last value includes a spurious fifth sexagesimal place.

Table entries.

Black columns: Sexagesimal integer and fractional (to four places) *R* sin values up to *R* sin $89°59' = 59;59,59,27,6$.

Red columns: Differences between successive entries in adjacent black columns.

Paratext. The table heading reads *atha pratikalājyākoṣṭakāḥ* "Now, tabulated [values of] *R* sin for each minute."

Method of computation. The trigonometric radius *R* in both of Nityānanda's works is 60 digits, following his Islamic sources.

***Sarvasiddhāntarāja*:** The *R* sin table values are derived from procedures found in *Sarvasiddhāntarāja* 3.19–85.[9] Arcs that are integer multiples of three degrees have been computed via traditional geometric techniques. The value of the *R* sin of 1°, however, being impossible to determine geometrically, has been computed using a recursive algorithm based on a cubic equation. This procedure appears to be based on a method originally put forth by al-Kāshī.[10]

***Siddhāntasindhu*:** The trigonometric radius *R* is taken to be 60 digits. Entries are given to a precision of five sexagesimal places. (This table can also function as a table of *R* cosines by using the appropriate symmetries as indicated in the argument.)[11] The *R* sines have been computed down to single degrees of argument using the techniques discussed above. Values down to single minutes have been computed using several ingenious approximation techniques which are

[9] These verses have been critically edited, translated, and analyzed in Dhammaloka et al. (2016) and Montelle and Ramasubramanian (2018).

[10] See Van Brummelen (2009, pp. 147–149), Rosenfeld and Hogendijk (2003), and Aaboe (1954). The value employed for the *R* sin of one degree is 1;2,49,43,15.

[11] For further information about these tables see van Dalen (2002a) and their transmission.

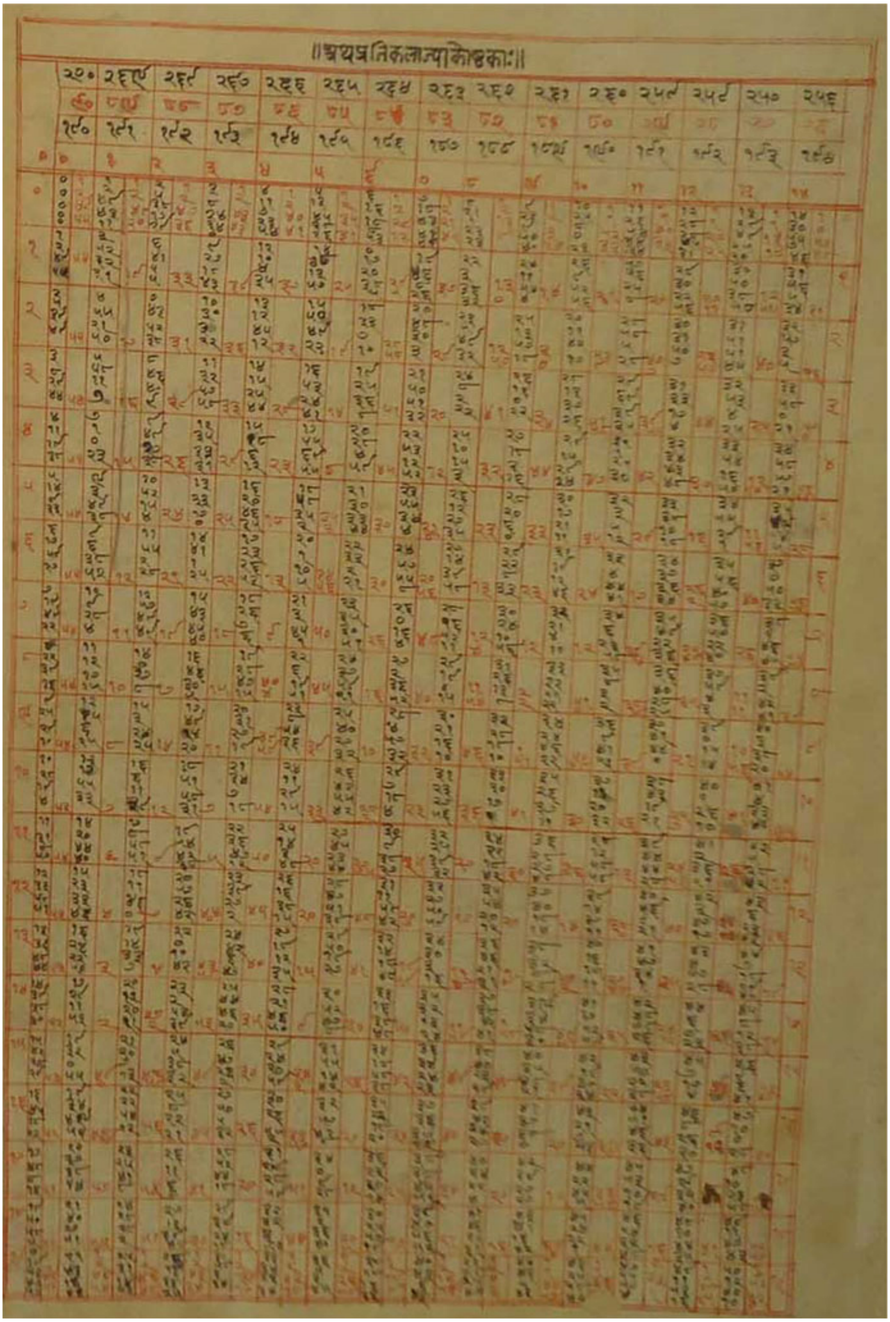

Fig. 4.65 Excerpt from the $R\sin/R\cos$ table from the *Siddhāntasindhu* of Nityānanda (Jaipur Khasmohor 4962 f. 29r), entries for arcminutes 0–14 of either of the following: degrees 0–14 or 180–194 for $R\sin$ (read left to right), or degrees 76–90 or 256–270 for $R\cos$ (read right to left).

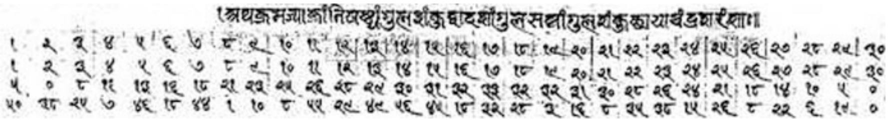

Fig. 4.66 Excerpt from the first part of a table of *R* sines and other quantities from the *Amṛtalahāri* (Tokyo MF 13 f. 40v).

outlined in the *Sarvasiddhāntarāja* (3.83–85) (Montelle and Ramasubramanian 2018). This table appears to have been copied from the *Zīj-i Shāh Jahānī*'s sine table, but has made some modifications to the layout of the argument. There are four rows of arguments. The first two tabulate the *R* sines for quadrants one (0–89) and three (180–269), using red and black to indicate the tabulated values are positive and negative, respectively. The second two tabulate *R* cosines in the reverse order for quadrants one (90–1) and three (270–181), also using black and red ink. Effectively the table does not explicitly contain arguments for the second and fourth quadrants (as the *Zīj-i Shāh Jahānī* has done), but these can be supplied by symmetry. The *R* sin of one degree is 1;2,49,43,11.

Table of *R* sines: *Amṛtalahāri*

Figure 4.66 shows a table of *R* sines from a manuscript of the *Amṛtalahāri*, which forms part of a more elaborate table including several other types of data entries (not shown) also sharing the same argument.

Argument: 1 to 90° of arc.

Table entries. Decimal integer and sexagesimal fractional (to two places) *R* sin values up to $R \sin 90° = R = 60;0,0$.

Paratext. The table heading reads *atha kramajyākrantiṣaṣṭyaṃgulaśaṃku-dvādaśāṃgula saptāṃgulaśaṃkuchāyācaṃdraśaraṃśāḥ* "Now, *R* sines, declination, shadows of a 60-digit gnomon, a 12-digit gnomon [and] a 7-digit gnomon, [and] the degrees of the latitude of the moon."

4.13 Astrological tables

Almost all Sanskrit astronomical tables are to some extent astrological, as they involve the computation of perceived positions of celestial bodies which always have astrological significance. But some tables may be considered solely or explicitly astrological, e.g., if they record the prognosticative characteristics assigned to celestial phenomena or are created specifically for casting horoscopes. The

Fig. 4.67 Astrological data from a nativity horoscope (UC MB 680). Left: Horoscope diagram or birth-chart (*janmapattra*) depicting locations of the nine planets and the ascendant in zodiacal signs 1–12 at the moment of nativity. Right: Table of true longitudes and velocities of planets with their synodic phases at that moment.

following examples of such tables illustrate their relation to astrological practice as briefly outlined in Section 1.3.3.

Figure 4.67 shows on the left a nativity horoscope diagram (birth-chart or *janmapattra*) for an unidentified individual[12] indicating the approximate locations of the nine planets, and on the right an accompanying table listing their precise locations, both computed for the moment of birth. The table specifies the planets' true longitudes in zodiacal signs (0/Aries–11/Pisces), degrees, minutes, and seconds, as well as their instantaneous velocities in minutes and seconds and their synodic phases (direct or retrograde motion, rising or setting). The table heading reads *atha śrībhāskarādayo grahā spaṣṭā* "Now, the true [longitudes of] the planets of the sun and so on."

The same horoscope manuscript includes a table of astrological houses or *bhāvas* beginning from the ascendant at the nativity moment, reproduced in Figure 4.68. The table entries contain for each of the 12 *bhāvas* the longitude of its cusp or central point, followed by that of its *saṃdhi* or junction with the following house, in zodiacal signs (0/Aries–11/Pisces), degrees, minutes, and seconds.[13] The table heading reads *atha śrī aṃgādayo dvādaśabhāvā* "Now, the twelve houses beginning at the ascendant."

A diagram of the influences of the *dreṣkāṇas*, or thirds of the zodiacal signs, at that same nativity moment is shown in Figure 4.69, entitled *atha bhrātraja-*

[12]The horoscope is cast for Saṃvat 1903/Śaka 1768, Winter, Kārttika-*māsa*, *kṛṣṇapakṣa*, 12th *tithi*, 48th *ghaṭī*, 49th *pala*, Sunday (15th November 1846).

[13]Note that this table illustrates the astrological convention of unequal house division: In quadrants 1 and 3, i.e., houses 1–3 and 7–9, the total length of each house is 2 × 12°;54,20 or one-third of the ecliptic arc between the ascendant and the meridian. The length of each of the remaining houses in the other two quadrants is 2 × 17°;5;40.

Fig. 4.68 Astrological houses or *bhāvas* from a nativity horoscope (UC MB 680). The longitude at the cusp of each numbered house is immediately followed by that of the *saṃdhi* or junction between that and the next house.

sukhavilokanārthe dreṣkānaṃ “Now, the *dreṣkāṇa* for the sake of investigating the happiness derived from brothers.” In the diagram, the abbreviation for each planet as well as the ascendant is marked in the sign allotted to the particular *dreṣkāṇa* within which that planet’s (or ascendant’s) longitude falls. The underlying scheme is laid out in the table of *dreṣkāṇas*, grouped with tables of other *vargas* or subdivisions of the zodiac, shown in the top left of Figure 4.70 derived from a different manuscript.

In each of the *varga* tables in Figure 4.70 the argument is split between the horizontal axis (zodiacal signs 0–11/Aries–Pisces) and the vertical axis (integer and fractional degrees of the subdivision of the sign). The table entries are the sign number and corresponding planetary lord (as enumerated in Table 1.1) assigned to each successive subdivision of each sign.

Figure 4.70 also shows a table of planetary relationships, with the argument split between the horizontal axis (abbreviated names of the planets in weekday order) and the vertical axis (the relations of friendship, neutrality, and enmity). Each entry contains the abbreviated name of the planet that stands in the given relation to the given planet.[14]

[14] These table entries reflect the traditional associations attested in the *Bṛhajjātaka* (2.15–17) except for the inclusion of Rāhu as the enemy of sun and moon (Jhā 1934, pp. 33–37; Aiyar 1905, pp. 20–21).

Fig. 4.69 Diagram of the planets' *dreṣkāṇas* from a nativity horoscope (UC MB 680).

The astrological importance of the *nakṣatras* is illustrated by the tables shown in Figure 4.71. The upper folio, from a fragment of an unidentified astrological manuscript, contains two purely cartographic tables of the boundaries of the 28 *nakṣatras* and their quarters. The first of these, entitled *nakṣatracārakoṣṭhakāḥ* "table of the *nakṣatra*-sequence," tabulates the beginnings of the 27 *nakṣatras* in zodiacal signs (0/Aries–11/Pisces), degrees, and minutes, with a constant length of 13°;20. Interspersed among them are the boundaries of the zodiacal signs from the end of Aries to the end of Aquarius: where a *nakṣatra*-boundary and sign-boundary coincide at longitudes 120° (beginning of Māgha/end of Cancer) and 240° (beginning of Mūla/end of Scorpio), the sign abbreviation is written above the *nakṣatra* one. The following table, entitled *atha pādacārakoṣṭhakāḥ* "Now, the table of the sequence of the quarters," tabulates the beginnings of the 108 successive 3°;20 arcs, four in each *nakṣatra*.

The table appearing in the lower folio, from a manuscript of the *Karaṇakesarī*, enumerates various birth-categories (*yonivicāraḥ*) associated with the 28 *nakṣatras* (including the additional *nakṣatra* Abhijit). In it, Row 1 records the designated animal or *yoni*, Row 2 the order of being (divine, demonic, human), and Row 3 the altitude (high, middle, low). The table heading reads *atha nakṣatrāṇāṃ yonivicāraḥ* "Now, the birth-categories of the *nakṣatras*."

Figure 4.72 shows tables of planetary *aṣṭakavargas* from a fragment of a work entitled *Muktāvalī* (Pingree 1968, p. 33). Starting with the sun and ending with

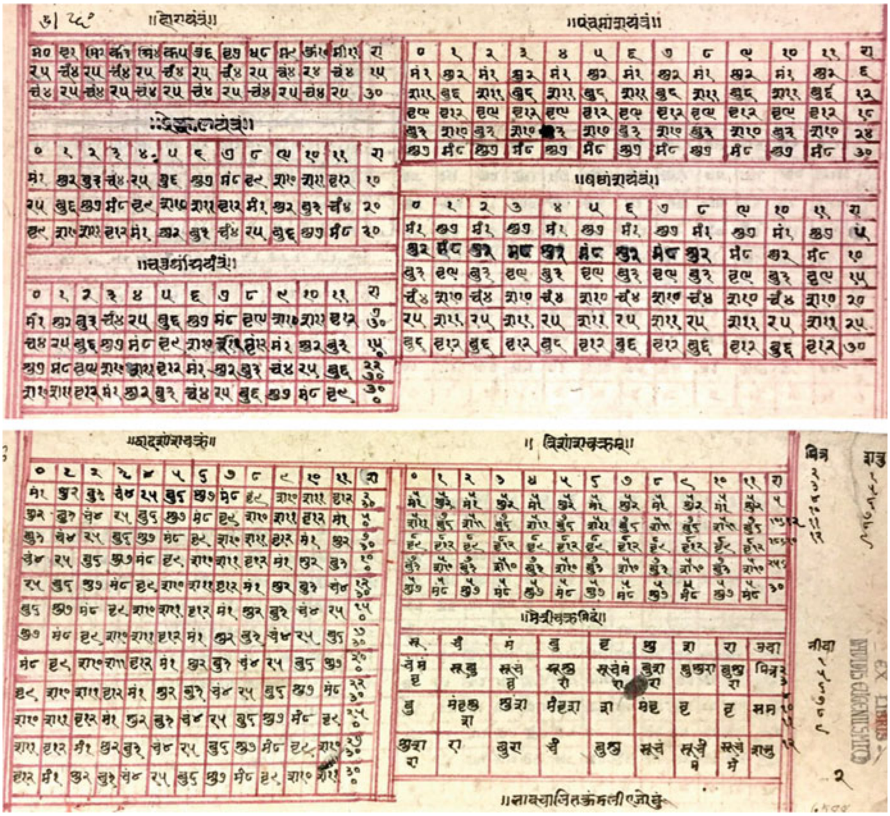

Fig. 4.70 Astrological tables from a fragment of an unidentified astrological manuscript (Smith Indic MB XXXVII). Top (f. 1r): Excerpts from tables for the *vargas* or divisions of zodiacal signs, with divisors 2, 3, and 4 (left side) and 5 and 6 (right side). Bottom (f. 2v): More *varga* tables with divisors 12 and 30 (left and upper right); table of planetary friendships (lower right).

Rāhu, each planet is assigned a table with eight columns, one for each of the seven bright planets and the ascendant. The table entries are the various house-positions 1–12 (relative to the house of the specified object) in which that planet has auspicious effect.[15]

The preceding examples barely scratch the surface of the innumerable quantities that can be tabulated for astrological purposes. For instance, in Figure 4.73 is shown a miscellaneous astrological table from the same manuscript of the *Karaṇakesarī* appearing in Figure 4.71. It tabulates the lords of eclipses (*parveśāḥ*), deities assigned to various intervals in months that separate successive eclipse possibili-

[15]These assignments appear to be based on those given in Varāhamihira's *Bṛhajjātaka* (Jhā 1934, pp. 135–145; Aiyar 1905, pp. 97–99).

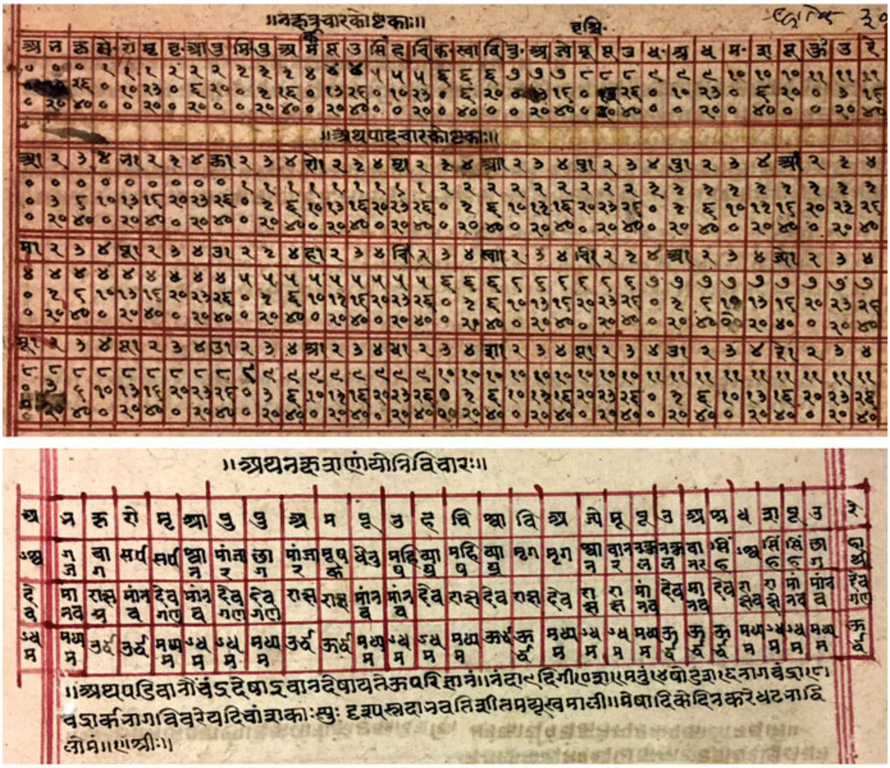

Fig. 4.71 Top: Table of longitudes of the boundaries of the *nakṣatras* and their quarters in an unidentified astrological manuscript (Smith Indic MB XXXII f. 4v). Bottom: Table of birth-categories for the *nakṣatras* in a manuscript of the *Karaṇakesarī* (Smith Indic MB XIV f. 11v).

Fig. 4.72 Tables of planetary *aṣṭakavargas* from a manuscript of the *Muktāvalī* (Smith Indic MB LXXVIII f. 3v).

Fig. 4.73 Table of lords of eclipses from a manuscript of the *Karaṇakesarī* (Smith Indic MB XIV f. 5v).

ties.[16] The table heading reads *atha sapātacaṃdrasūryarāśyuṃpariparveśajñānaṃ rāśināṃ cakraśodhyā vinā* "Now, the knowledge of the lords of eclipses without cycle elimination [i.e., without subtracting integer multiples of 12?]."

[16]The scheme is similar to that given in Varāhamihira's *Bṛhatsaṃhitā* (5.19) (Subrahmanya Sastri and Bhat 1946, p. 50; Kern 1865, p. 26).

Chapter 5
The evolution of the table-text genre

The early development of Sanskrit astronomical tables described in Section 1.5 blossomed by the mid-second millennium into the profuse variety of table-text types categorized in Section 2.3, whose typical components were analyzed in more detail in Chapter 4. As numerical tables became more central to the work of astronomers/astrologers, the mathematical ingenuity of *jyotiṣa* authors to some extent shifted its focus. Algorithms directing users how to compute a desired quantity were supplemented and then partly supplanted by techniques for producing and arranging pre-computed data so that users could simply look up the desired quantity. We argue that at least part of what has long been conventionally described as a "decline" in the innovative development of Sanskrit astronomy after the twelfth century might be more accurately called an "occultation" of it, as the formats of increasingly popular table texts removed a larger share of their authors' mathematical efforts from the direct scrutiny of their readers. The following discussion explores some of the major passages in that long and vigorous growth by surveying several individual *koṣṭhaka* works that seem to us to highlight some of its important characteristics.

5.1 *Rājamṛgāṅka* of Bhojarāja

An astronomical manual in perhaps several hundred verses called *Rājamṛgāṅka* ("Moon of the King") is ascribed to the famous monarch Bhoja or Bhojarāja (d. 1055) of the Rajput Paramāra dynasty, rulers of the Malwa region in central/western India from about the ninth to the thirteenth century.[1] The *Rājamṛgāṅka*'s known manuscripts preserve various widely differing versions, all of which omit many of

[1] Tables from manuscripts of this work are illustrated in Figures 4.1, 4.33, 4.50, 4.62, and 4.63.

C. Montelle, K. Plofker, *Sanskrit Astronomical Tables*, Sources and Studies in the History of Mathematics and Physical Sciences,
https://doi.org/10.1007/978-3-319-97037-0_5

the quoted verses attributed to it in the works of other authors, so an authoritative original text has yet to be established although a provisional critical edition was published in Pingree (1987b). Indeed, the *Rājamṛgāṅka* may possibly have been produced in collaboration with (or even solely by) court scholars, especially considering that Bhoja has been credited with the authorship of at least dozens of other erudite technical works, in addition to his very active political and military career (Raghavan 2006). If so, a definitive original text may never have existed in a unique form. Whatever the details of its origin and textual content, the *Rājamṛgāṅka*'s epoch date is firmly attested as the start of Śaka year 964 (Tuesday 23 February 1042).

In its models and parameters the *Rājamṛgāṅka* adheres to the Brāhmapakṣa school (see Section 1.2.3 and Appendix B.2), besides employing the *bījas* or correction constants typical of second-millennium texts. Its verses largely follow the tradition of earlier Sanskrit astronomical handbooks in prescribing algorithms for the computation of various astronomical quantities beginning with the accumulated days since epoch, the associated mean celestial positions and their correction to true ones, etc. This is consistent with the work's explicit identification with the handbook or *karaṇa* genre, asserted in the second verse of the edition (and in almost identical verses in the other known versions) (Pingree 1987b, p. 4, verse 1.2):

> *vāsanāsārasarvasvaṃ niḥpannaṃ laghukarmavat* ||
> *brūmo rājamṛgāṅkākhyaṃ jyotiṣāṃ karaṇaṃ sphuṭam* || 2 ||

> We state the correct *karaṇa* of the [positions of the] heavenly bodies, named *Rājamṛgāṅka*, the entirety of the epitome of demonstrations effected like an easy task.

The term *karaṇa* is repeated in a subsequent verse (Pingree 1987b, p. 4, verse 1.8), as well as in the text's final verse and colophon (Pingree 1987b, p. 42, verse 8.81):

> *sarvabhūpālavandyena tena śrībhojabhūbhujā* ||
> *mṛgāṅkasaṃjñakaraṇaṃ cakre sarveṣṭasiddhaye* || 81 ||
> *iti mahārājādhirājaśrībhojadevakṛte mṛgāṅkanāmnikaraṇe grahayutyādir adhyāyo 'ṣṭamaḥ* || *samāptaṃ rājamṛgāṅkasaṃjñaṃ karaṇam* ||

> By that king Lord Bhoja who is revered by all kings, the *karaṇa* named *Rājamṛgāṅka* has [been] made for the achievement of all desires.
> Thus in the *karaṇa* named *Mṛgāṅka*, made by the great king of kings His Majesty Lord Bhoja, the eighth chapter beginning with planetary conjunction. The *karaṇa* known as *Rājamṛgāṅka* is complete.

The commentary on the *Khaṇḍakhādyaka* of Brahmagupta composed around 1200 by Āmarāja similarly refers to the *Rājamṛgāṅka* as a *karaṇa* (Misra 1925, p. 75).

Despite its *karaṇa* label, the *Rājamṛgāṅka* significantly departs from the traditional format of the self-contained verse handbook by frequently invoking a separate collection of numerical tables on which many of its computations depend. In fact, separate *karaṇa* and *koṣṭhaka* versions of the work are designated in Pingree (2003, p. 23). For instance, the content of chapter 2, dedicated to computing true planetary positions and velocities, refers several times to *koṣṭhakas* in the sense of table entries, as in the following example of verses treating the *mandaphala* (*manda-*

equation, or first correction to a planet's mean longitude, see Section 2.1.2). These verses instruct the reader to use the planet's first orbital anomaly (*manda-kendra*) to interpolate in the appropriate table of its longitude correction or equation (Pingree 1987b, p. 7, verses 2.20–21):

> *kendrād bhujaṃ vidhāyātha tataḥ sādhyā bhujāṃśakāḥ* ||
> *bhujāṃśasaṃmitaḥ koṣṭho grāhyo mandaphalasya tu* || 20 ||
>
> *sa bhuktakoṣṭhako jñeyas tadeṣyavivarāhatam* ||
> *vikalaṃ ṣaṣṭisaṃbhaktaṃ phalaṃ gataphalānvitam* || 21 ||
>
> Now, determining the arc [reduced to the first quadrant, i.e., the *bhuja*] from the anomaly (*kendra*), the [integer] degrees of the *bhuja* are to be found. The table-entry of the *manda*-equation commensurate with the degrees of the *bhuja* is to be taken.
>
> That is to be understood as the 'elapsed' table-entry. The arcminute [fractional part of the given *bhuja*] is multiplied by the difference of that and the next [table-entry] and divided by 60. The result is added to the [*manda*-]equation [in] the 'elapsed' [table-entry].

A typical *karaṇa* would provide instead a maximum *manda*-equation parameter and an algorithm employing that parameter in some mathematical relation between the anomaly and the equation—or at least a versified table of equation values as part of its textual content (see Sections 5.3.1 and 5.6.1). But the *Rājamṛgāṅka*'s rule reaches outside its text entirely to fetch the requisite data from a *koṣṭhaka* or numerical array of values, rendering the verses incomplete or even meaningless without the tables.

In other cases these two approaches, the algorithmic and the table-based, are prescribed concurrently (and redundantly). For instance, the following group of verses comprises two independent rules for determining lunar latitude. The first specifies a self-contained algorithm approximating the latitude as the maximum latitude $270'$ scaled by the R sin of elongation of the moon from its node. The second requires look-up in a table of latitude values (Pingree 1987b, p. 14, verses 3.34–36)[2]:

> *pātonacandradorjīvāṃ khabhaiḥ saṃguṇayet tataḥ* ||
> *candrakarṇena yallabdhaṃ sa vikṣepo niśāpateḥ* || 34 ||
>
> *pātonacandrasya bhujād bhujāṃśān sādhayet tataḥ* ||
> *tadaṃśasaṃmitaḥ koṣṭhas tadeṣyavivarāhatam* || 35 ||
>
> *vikalaṃ ṣaṣṭibhaktaṃ tu phalaṃ gataphalānvitam* ||
> *evaṃ kṛte vā yaj jātaṃ sa vikṣepo niśāpateḥ* || 36 ||
>
> One should multiply the R sin of the arc [of the longitude] of the moon diminished by [that of] the node by 270. The quotient from [dividing] that by the moon's hypotenuse [? rather than the trigonometric radius R?] is the latitude of the night-lord (moon).

[2] There are many examples in other *jyotiṣa* works of multiple algorithms prescribed for computing the same quantity, or algorithms offered as alternatives to versified lists of pre-computed values. See, for example, the *Karaṇakutūhala*'s list of ecliptic declination-differences followed by an algebraic approximation for finding declination (Mishra 1991, pp. 43–44, verses 3.13–15). But the closest parallel we have found to an instruction calling for look-up in a pre-computed table outside the verse text is Vaṭeśvara's reference in the 905 *Vaṭeśvarasiddhānta* to interpolating within a set of approximate values of planetary equations constructed from scaled multiples of the tabulated R sin-differences (Shukla 1986, vol. 1, p. 121, verse 2.5.2).

> From the arc of the [longitude of the] moon diminished by [that of] the node, one should determine the [integer] degrees of the arc. From that, the ["elapsed"] table-entry commensurate with that degree [is found]. The arcminute [fractional part of the given arc] is multiplied by the difference of that and the next [table-entry] and divided by 60. The result is added to the "elapsed" result [in the table-entry]. When [the computation] is alternatively done thus, what results is the latitude of the night-lord (moon).

Such combinations indicate the hybrid nature of the text, with both trigonometric algorithms and pre-computed tables offered for the user's choice.

The *Rājamṛgāṅka* is clearly an offshoot of the *karaṇa* genre, but not obviously derived from known earlier sources or inspirations within this genre. It does not strongly resemble, for instance, the drastically concise *karaṇa* called *Laghumānasa* from about a century earlier, which contains no graphical-numeric tables and indeed hardly any versified verbal ones (Shukla 1990). Some of its own subsequent impacts are somewhat easier to trace: in particular, the *Grahajñāna* of Āśādhara (see Section 5.2) is largely based on this work. And as a *karaṇa*, the *Rājamṛgāṅka* apparently influenced the *Karaṇakutūhala* composed by Bhāskara II about a century later, in which some of the earlier work's algebraic approximation algorithms for, e.g., gnomon-shadow computations seem to have been adapted and developed (Plofker 2019). Bhāskara, however, did not follow the *Rājamṛgāṅka*'s hybrid *karaṇa/koṣṭhaka* approach in the *Karaṇakutūhala*, firmly maintaining the earlier tradition of the standalone verse handbook.

5.2 *Grahajñāna* of Āśādhara

Almost a century after the *Rājamṛgāṅka*, the *Grahajñāna* ("Knowledge of the planets") was composed by Āśādhara, an astronomer working most probably under Jayasiṃha Siddharāja (r. 1015–1043) of the Chalukya dynasty in the western Deccan.[3] The *Grahajñāna* in its extant form deals almost exclusively with the positions of the five star-planets, omitting the refinements of lunar and solar motion required for timekeeping, calendar maintenance, and computation of eclipses. Its text and accompanying tables constitute the first known *koṣṭhaka* work employing the so-called mean-to-true arrangement (see Section 2.3) that was later widely popularized by the *Mahādevī* of Mahādeva. The exact form of the original text is not conclusively determined but it appears to have contained somewhere around 20–30 verses, now preserved in various recensions in at least seventeen manuscripts. A commentary called the *Āśādharadīpikā*, known from one surviving manuscript, was composed in 1554 (Pingree 1989, pp. 33–36).

The first two verses of the text assert Āśādhara's adherence to the Brāhmapakṣa and establish his epoch as the beginning of Śaka year 1054 (falling in March 1132

---

[3] A table from a manuscript of this work is shown in Figure 4.7. For more information about the work and its author see Pingree (1989), Pingree (1970–94, A1.54, A2.16, A3.16, A4.28), Pingree (1973, pp. 69–72), and Pingree (1981, p. 42).

CE). The opening verse also proclaims the convenience of the *Grahajñāna*'s mean-to-true procedures in eliminating the separate computation of orbital corrections (Pingree 1989, p. 6, verse 1):

brahmeśācyutacandravitsitaravikṣoṇīsutejyārkajān
nakṣatrāṇi sarasvatīṃ gaṇapatiṃ natvā pṛthagbhaktitaḥ ||
śrībrahmoktaparisphuṭopakaraṇair bhaumādayo yādṛśās
tattulyānayanaṃ bravīmi sukhadaṃ mandāṣupātair vinā || 1 ||

Praising Brahman, Śiva, Viṣṇu, the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn, the *nakṣatras*, Sarasvatī [and] Gaṇeśa separately and in succession, I state a pleasant computation equal to whatever [longitudes of the] planets beginning with Mars [are found] with the fully correct techniques (*upakaraṇas*) spoken by Lord Brahman, without the application of *manda* and *śīghra* [equations].

The closing verses include details of Āśādhara's lineage: notably, his father Rihlaka is likely to be the same as the Rīhlīya who is mentioned by Āmarāja in his commentary on Brahmagupta's *Khaṇḍakhādyaka* (Misra 1925, p. 23) around 1200.

5.2.1 Textual algorithms as alternatives to table use

The various versions of the *Grahajñāna* suggest in some respects a continuation of the *karaṇa*/*koṣṭhaka* fluidity observed in parts of the *Rājamṛgāṅka*. This is most strongly marked in their juxtaposition of versified algorithms to be applied by the user with equivalent tables of values pre-computed by the table compiler using these same algorithms. The remainder of this section describes several examples of these parallel approaches.

Annual mean-weekday increment. The determination of the increment in mean weekdays (modulo 7) from the epoch date to the start of a given Śaka year is explained in the second verse (Pingree 1989, p. 6, verse 2):

śāko 'bdhibāṇadaśabhir viyuto hato 'śvi-
bāṇaiḥ pṛthak śaśinavāṣṭanṛpāptahīnaḥ ||
candrābhrayugmavihṛtaḥ phalam abdayuktaṃ
saptoddhṛtaṃ bhavati saṅkramaṇadhruvo 'yam || 2 ||

The Śaka year is diminished by 1054 and [the result] multiplied by 52; [the product] is separately divided by 16891 and diminished [by the quotient, and then] divided by 201. The result is added to the years [elapsed since epoch and] divided by seven; this [quotient] becomes the *dhruva* (fixed amount) at *saṃkrānti* (start of solar year).

That is, the number y of elapsed integer mean solar years since the epoch at the start of Śaka 1054 contains, over and above the y times 52 times 7 days contained in its integer weeks, an additional y integer days plus fractional days roughly equal to $\dfrac{52y - 52y/16891}{201} \approx y \cdot 0;15,31,17,17$. The remainder modulo 7 of this sum is

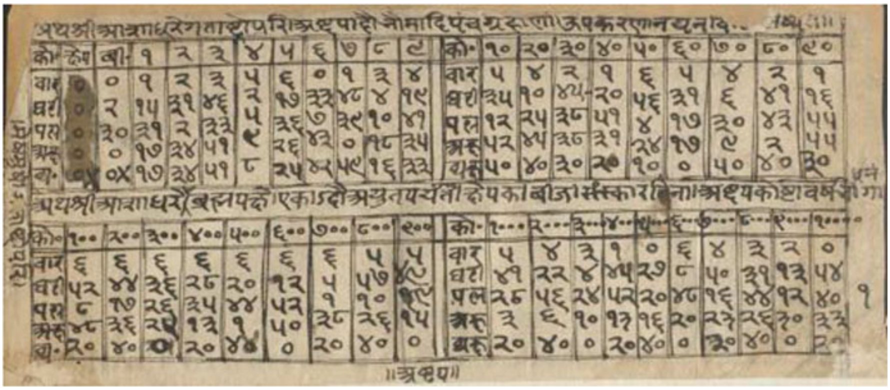

Fig. 5.1 Weekday-increment tables from the *Grahajñāna* (IO 2464c f. 1v).

Table 5.1 Characteristics of the weekday-increment tables in the *Grahajñāna* (IO 2464c f. 1v).

Argument range (years)	Constant difference (weekdays modulo 7)
1–9	1;15,31,17,17
10–90	5;35,12,52,50
100–900	6;52,8,48,20
1000–10,000	5;41,28,3,20

the offset in mean weekdays from the start of the epoch year to the start of the given year.

But we also see among the manuscripts of the *Grahajñāna* an alternative (or supplementary?) approach to this computation, in tabular form. This is illustrated in Figure 5.1 by the tabulated multiples of the abovementioned annual fractional weekday increment that accumulate in periods ranging from single years, decades, and centuries up to 10,000 years (see Table 5.1). Essentially, the table compiler has broken down the number y for almost any conceivable future year-beginning into smaller components, each pre-multiplied by the appropriate multiple of the annual increment. So the user need only choose and add up the particular components that correspond to his value of y, instead of performing the full multiplication as instructed in the above verse.

Planetary mean longitudes. A similar combination of an algorithm to be carried out by the user with an equivalent table from which the user can extract the same result via interpolation is seen in the *Grahajñāna*'s methods for finding the current mean longitudes of the planets. The algorithm includes the requisite multipliers to generate mean longitude increments as well as planetary epoch positions in a somewhat unusual form (Pingree 1989, pp. 6–7, verses 4–7):

bhaumasya vatsaragatir manavaḥ kudasrā
nāgendavo yamayugāni nabho bhapūrvāḥ ||

sthāneṣu pañcasu budhasya yugānyathāṅgā-
ny agnyaśvino 'kṣapavanāḥ khayugānyadho 'dhaḥ[4] || 4 ||

jīvasya netre 'ṅgabhuvo 'bdhirāmā
nandeṣavo nāgabhuvaḥ krameṇa ||
śukrasya bhūpās triśarās tridasrā
dantās tathāṅkā ganakair niruktāḥ || 5 ||

śaner viyadvedaśarā nagākṣāḥ
kumārgaṇā vahniyamā gatābdaiḥ ||
nighnāḥ svakīyair dhruvakair vihīnā
bhavanti saurābdamukhe grahendrāḥ || 6 ||

rudrāḥ saptaguṇā yamau guṇaguṇā bhaumadhruvo mārgaṇās
tattvāny aśvaguṇā nṛpāḥ śaśisutasyāto guror niścalāḥ[5] ||
rāmāḥ ṣaḍbhujagābdhayo rasaguṇāḥ śukrasya rūpaṃ radās
trīṇy aṅkāś ca śaner daśa dviyamalā rāmā radāḥ kīrtitāḥ || 7 ||

The yearly velocity of Mars is 14, 21, 18, 42, 0 in five [sexagesimal] places beginning with [integer] *nakṣatras*; moreover, [that] of [the *śīghra*-apogee of] Mercury is 4, 6, 23, 55, 40 one below another; [that] of Jupiter 2, 16, 34, 59, 18 in order; [that] of [the *śīghra*-apogee of] Venus is proclaimed by calculators [to be] 16, 53, 23, 32, 9; [that] of Saturn 0, 54, 57, 51, 23. [These] are multiplied by the elapsed years [and] diminished by their own *dhruvas*; [they] become [the mean longitudes of] the planets [in units of *nakṣatras*] at the start of the solar year.

11, 37, 2, 33 is Mars's *dhruva*, 5, 25, 37, 16 [that of the *śīghra*-apogee] of the Moon's son [Mercury], then the *dhruva* of Jupiter is 3, 6, 48, 36, of [the *śīghra*-apogee] of Venus 1, 32, 3, 9, [and] of Saturn 10, 22, 3, 32 is stated.

The multiplication of the yearly velocity or annual mean longitude increment (given in arc-units of 13°;20 *nakṣatras*) by the number of elapsed years since epoch produces an integer number of completed revolutions plus a fractional revolution or mean longitude at the start of the current year. (The mean longitude increments for the lunar node are also tabulated, in this case for 1 to 27 periods of 27 years, 1 to 27 single-year periods, and 1 to 27 *avadhis*.)

According to the verse, the user's accumulated mean longitude increments for the planets in arc-units of *nakṣatras* are to be *diminished* rather than increased by the planetary *dhruvas* or epoch positions to get the desired mean longitudes for the start of the current year. This appears to be because the epoch *dhruvas* are in fact "complementary" longitudes: i.e., 27 *nakṣatras* or one full circle minus the actual epoch mean longitudes.

Besides this algorithmic approach, the *Grahajñāna* supplies pre-computed tables of such mean longitude increments also in *nakṣatras* (modulo complete revolutions). The tables cover Mars, Mercury's *śīghra*-apogee, Jupiter, Venus's *śīghra*-apogee, and Saturn for time intervals of 1 to 9 single years, 1 to 9 decades, 1 to 9 centuries, and 1 to 10 millennia. Figure 5.2 illustrates their structure with the mean longitudinal displacement tables for Mars.

[4] The edition reads *svayugānyadho* at the end of this verse.

[5] The edition reads *aśvi* "2" for *aśva* "7" in this line.

Fig. 5.2 Mean longitudinal displacement tables from the *Grahajñāna* (IO 2464c f. 2r, Mars) including the epoch adjustment (*kṣepa*; second column, first row) and the *bīja* (third column, first row).

Synodic phenomena. The periods in days of retrogradation and direct motion for each star-planet are specified in the following verse (Pingree 1989, p. 8, verse 14):

> *vakrāḥ sāgaraparvatās trinayanā dvyarkā navāmbhodhayaś*
> *candrendrā ṛṇabhuktijā dhanagatau ṣaḍvyomaśailāḥ kramāt* ||
> *tryaṅkāḥ saptanagāśvino 'kṣahutabhugbāṇā nagatryaśvino*
> *bhaumāder grahamaṇḍalasya divasā mārgāditaḥ kīrtitāḥ* || 14 ||

> Retrogradations: 73, 23, 122, 49, 141, with negative velocity; in positive velocity 706, 93, 277, 535, 237 respectively are the declared days of [each] planetary cycle beginning with Mars from the start of [its] direct motion.

Figure 5.3 illustrates a crude tabular approximation of this data in the tables of true motion for Mercury. The velocity value in the second row of the entry for a true longitude when the planet is retrograde is marked negative with a $\times$ symbol, while the corresponding velocity value after the next reversal of the planet's direction is marked positive.

5.2.2 Textual instructions for table use

The core of the *Grahajñāna* as a table text is its extensive set of tables of true planetary longitudes. Each planet has $360/13;20 = 27$ such tables, one for each successive *nakṣatra* of mean longitude (although the only known manuscript witness to these tables omits the ones for Venus and Saturn). The user enters them with a given mean longitude (in integer and fractional *nakṣatras*) of a desired planet for a desired time and interpolates in two dimensions: between successive tables depending on the position of the planet, and between successive entries within a table depending on the mean position of the sun.

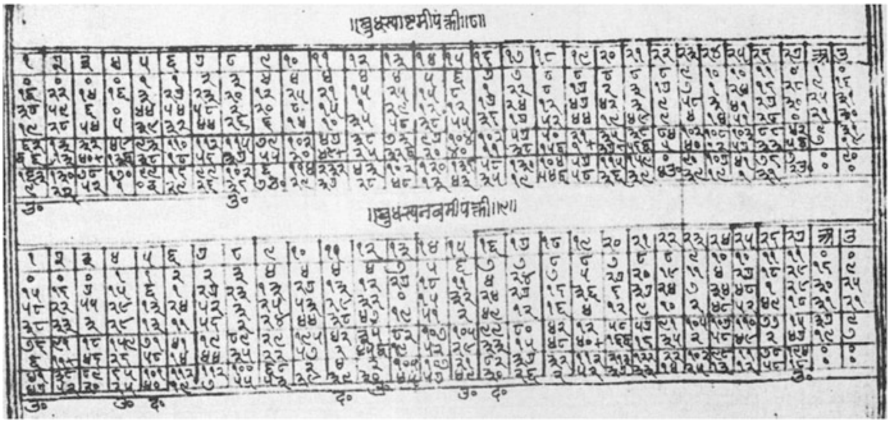

Fig. 5.3 True longitude tables from the *Grahajñāna* (RAS Tod 36c f. 1r, Mercury, *paṅkti* 8 and 9) indicating changes between negative and positive true velocity (i.e., retrograde and direct motion) with sign markers in the second row of the tables.

For instance, consider the two successive tables (*paṅktis*) of Mercury's true longitude for 7 and 8 completed *nakṣatras*, respectively, of mean longitude in Figure 5.3. Suppose a user wishes to compute Mercury's true longitude corresponding to its mean longitude of 7;20 *nakṣatras* at a time when the mean sun's position is 4;30 *nakṣatras*. The first step would be to find the midpoint between the fifth and sixth true longitude entries in each table (in accordance with the fractional part of the sun's position), after which one would interpolate one-third of the way between those two results (in accordance with the fractional part of Mercury's position) to arrive at the desired true longitude.

This procedure is described by Āśādhara in the following two verses, emphasizing the interpolation between successive tables (Pingree 1989, p. 7, verses 8–9):

yāvanti bhuktāni bhavanty uḍūni
kujādikair abdamukhe grahendraiḥ ||
tattulyapaṅktyaṃśaśarāḥ sphuṭās te
sūrye 'śvinīmukhagate krameṇa || 8 ||

However many [integer] *nakṣatras* [of mean longitude have been] completed at the beginning of the year by the planets beginning with Mars, [their] true [positions] are the degrees [of longitude and] latitude in the [true-longitude] table (*paṅkti*) commensurate with that, when the sun is in [the *nakṣatras*] beginning with Aśvinī, in order.

gataiṣyapaṅktyudbhavakhetayoś ca
pṛthag lavādyantaram eva kuryāt ||
svarṇaṃ ghaṭīghnaṃ kharasāptam eṣyān
yūnādhike bhuktanabhaścarendre || 9 ||

And one should make the difference in degrees etc. of the two [true longitudes of the] planet in the elapsed and next [true-longitude] tables separately. [That difference] multiplied by the sixtieths [i.e., the fractional part of the planet's mean longitude], divided by 60, [is applied] positively or negatively when the past [longitude of the] planet is less or greater than the future one [respectively].

Here the textual content is devoted exclusively to specifying the techniques for table use, rather than expounding algorithmic methods as alternatives to tables. It should be reiterated that the incoherence of the surviving versions of the work makes it very difficult to determine exactly how its author originally structured it, and how its structure was altered by later scribes. But it seems likely that no verse algorithms for independently reproducing the tabulated true longitudes ever formed part of the *Grahajñāna*: such computational schemes remained buried in the table data.

5.2.3 Influences and impacts

The textual and scribal history of the *Grahajñāna*, as far as we know it, places it unambiguously in western India and in the tradition of the *Rājamṛgāṅka*. The explicit connection between the two works is limited to one line quoted without attribution from the earlier work in a commentary on the later one, namely the *Āśādharadīpikā* composed in 1554.[6] But the anonymous commentator's inference about their relationship seems plausible. More far-reaching is Pingree's hypothesis that the fundamental form of the *Grahajñāna*'s mean-to-true structure originated in the seventh-century *Dhyānagrahopadeśādhyāya*, a supplementary chapter to Brahmagupta's seminal *siddhānta* work *Brāhmasphuṭasiddhānta* of 628 (Dvivedī 1901–1902). (The *Dhyānagrahopadeśādhyāya* does employ *nakṣatra* arc-units but prescribes planetary equations in typical mean-with-equation form.)

Almost all of the *Grahajñāna*'s attested text was subsequently incorporated into the 120 verses of a *karaṇa* composed in around 1580, the *Gaṇitacūḍāmaṇi* of Harihara. Near the end of the text, Harihara explicitly invokes the works of Āśādhara and Bhojarāja, as well as the *Karaṇakutūhala* of Bhāskara and the *Laghukhecarasiddhi* derived from it (both discussed in the following section) and a *Pañcāṅgapattra* (Pingree 1989, p. 32):[7]

kutūhalād rājamṛgāṅkād vā
āśādharāt khecarasiddhitas tu ||
pañcāṅgapatrād vidadhīta khetān
spaṣṭān sukhārthaṃ punar atra samyak || 114 ||

From the [*Karaṇa*-]*kutūhala* or the *Rājamṛgāṅka*, from Āśādhara and the *Khecarasiddhi*, [or] from the *Pañcāṅgapatra*, one could establish [the positions of] the true planets; however, for the sake of simplicity, here [in this work it is] correctly [done].

[6] See, for this and the following references to Harihara, the combined edition of the *Grahajñāna* and the *Gaṇitacūḍāmaṇi* published in Pingree (1989).

[7] This *Pañcāṅgapattra* may refer to the approximately contemporary *Candrārkī* of Dinakara, who introduces the work as a *patra* "leaflet" called *pañcāṅga* "calendar/almanac", see Kolachana et al. (2018).

5.3 *Brahmatulyasāraṇī* and *Laghukhecarasiddhi*

5.3.1 The *Karaṇakutūhala* of Bhāskara

The two table texts treated in this section are adaptations of the renowned handbook *Karaṇakutūhala* ("Wonder of *karaṇas*") composed by Bhāskara II with epoch date Śaka 1105 (23 February 1183), also known as the *Brahmatulya* ("equal to the Brāhmapakṣa"). It is the first identifiable *karaṇa* text to have been systematically recast by later authors in table-text form. Some of its characteristic formulas that they relied on are discussed below; see Sections 2.1.2 and 2.1.5 for related details.

***manda*-equation.** For the planetary longitude equation μ or correction derived from the *manda*-anomaly, the *Karaṇakutūhala* prescribes the following approximation algorithm (Mishra 1991, p. 23, verses 2.9–10ab):

> *sūryādikānāṃ mṛdukendradorjyā*
> *digghnī vibhājyā ca svapañcabānaiḥ* ||
> *nāgāgnidasrair giripūrṇacandrai-*
> *rvasvaṅkabhūmirvasunetranetraiḥ* || 9 ||
> *yugāṣṭaśailair munipañcacandraiḥ*
> *phalaṃ lavāḥ kendravaśād dhanarṇam* ||

> The Sines of the *manda*-anomalies [of the planets] beginning with the sun are to be multiplied by 10 and divided by 550, 238, 107, 198, 228, 784, [and] 157. The result in degrees [is] positive or negative according to [the value of] the anomaly.

This rule is equivalent to the following expression:

$$\mu \approx 120 \sin \kappa_M \cdot \frac{10}{\textit{manda}\text{-divisor}}$$

where the "*manda*-divisor" is a parameter customized for each planet, as shown in column 2 of Table 5.2. The resulting *manda*-equation or μ values are to be applied positively or negatively depending on what quadrant the anomaly falls in.

***manda* velocity correction.** The planetary velocity correction $\overset{M}{\Delta}_v$ due to the *manda*-anomaly is determined in the *Karaṇakutūhala* as follows (Mishra 1991, p. 24, verses 2.11cd–12):

Table 5.2 Planetary parameters for *manda*-corrections according to the *Karaṇakutūhala*'s approximations. Column 2: Divisors for computing the *manda*-equation. Column 3: Scale factors for the velocity correction.

Planet	*Manda*-divisor	Velocity-correction factor
Sun	550	1/9
Moon	238	13/4
Mars	107	2/7
Mercury	198	2/7
Jupiter	228	1/50
Venus	784	1/12
Saturn	157	1/120

svabhogyakhaṇḍaṃ navahṛtkharāṃśor
viśvāhataṃ vedahṛtaṃ himāśoḥ || 11 ||
dvighnaṃ nagāptaṃ kujasaumyayośca
khākṣair inaiḥ khārkamitaiś ca bhaktam ||
jīvādikānāṃ ca gateḥ phalaṃ tat
svarṇaṃ kramāt karkamṛgādikendre || 12 ||

Its own current [R sin-]difference [of the *manda*-anomaly] is divided by 9 in [the case of] the sun; multiplied by 13 [and] divided by 4 in [the case of] the moon; multiplied by 2 [and] divided by 7 in [the case of] Mars and Mercury; divided by 50, 12 and 120 [respectively in the case] of [the remaining planets] beginning with Jupiter. That is the correction of the velocity, [applied] positively [or] negatively according as the anomaly is in [the six signs] beginning with Cancer [or] Capricorn.

This algorithm approximates the *Siddhāntaśiromaṇi* formula

$$\overset{M}{\Delta}_v = (\bar{v} - v_{A_M}) \cdot \cos \kappa_M \cdot (r_M / R)$$

by

$$\overset{M}{\Delta}_v \approx \Delta R \sin \kappa_M \cdot \text{scale factor}$$

where $\Delta R \sin \kappa_M$ is the value in the *Karaṇakutūhala*'s R sin-difference table corresponding to the 10° interval within which the anomaly κ_M falls. (See Mishra (1991, p. 21, verses 2.6–7) and column 3 of Table 5.3 for these R sin-differences.) This R sin-difference is a fairly crude substitute for the $R \cos \kappa_M$ in the previous formula; it is then simply multiplied by a scale factor tailored to the given planet, as shown in column 3 of Table 5.2 (Montelle and Plofker 2015, pp. 15–16).

Sample values resulting from these approximations for solar anomaly values at 10° intervals are listed in Table 5.3.

Table 5.3 The solar *manda*-equation and corresponding velocity-correction (*gatiphala*) reconstructed from the *Karaṇakutūhala*'s approximation formulas for each multiple of 10° of anomaly in the first quadrant. Column 5 is computed by multiplying the solar scale factor 1/9 by the successive *Karaṇakutūhala* R sin-differences in column 3.

Anomaly (°)	R sin	ΔR sin	*manda*-equation (°)	*gatiphala* (′)
10	21	21	0;22,54,...	2;20,0
20	41	20	0;44,43, ...	2;13,20
30	60	19	1;5,27,...	2;6,40
40	77	17	1;24	1;53,20
50	92	15	1;40,21,...	1;40,0
60	104	12	1;53,27,...	1;20,0
70	113	9	2;3,16,...	1;0,0
80	118	5	2;8,43,...	0;33,20
90	120	2	2;10,54,...	0;13,20

***manda*-correction for Mars.** The *Karaṇakutūhala* additionally defines a special *manda*-correction for Mars as dependent on the *śīghra*-anomaly or elongation between Mars and the mean sun (Mishra 1991, p. 20, verse 2.5)[8]:

bhaumāśukendre padayātagamya-
svalpasya liptāḥ khakhavedabhaktāḥ ||
labdhāṃśakaiḥ karkamṛgādikendre
hīnānvitaṃ spaṣṭam asṛṅmṛdūccam || 5 ||

In the case of Mars's *śīghra*-anomaly, the minutes of [whichever is] the lesser of [the part] past and [the part] to come of [its] quadrant are divided by 400. Depending on whether the anomaly is in [the semicircle] beginning with Cancer or Capricorn, [the longitude of] the *manda*-apogee of Mars diminished or increased [respectively] by the degrees of the quotient is correct.

That is, for every degree of synodic elongation between Mars and its nearest point of conjunction, opposition, or quadrature, the longitude of its *manda*-apogee changes by 9 arcminutes. If the planet is within 90° of opposition, the apogee longitude is increased by this amount, or decreased by it if Mars is within 90° of conjunction.

Right ascensions. The *Karaṇakutūhala* states the standard three rising-times at Laṅkā (see Figure 4.21) as follows (Mishra 1991, p. 35, verse 3.1):

laṅkodayā nāgaturaṅgadasrā |
goṅkāśvino rāmaradā vināḍyaḥ ||
kramotkramasthāś carakhaṇḍakaiḥ svaiḥ |
kramotkramasthaiś ca vihīnayuktāḥ || 1 ||

The rising times at Laṅkā [for the first three zodiacal signs are] 278, 299, and 323, [measured in] *vināḍīs*. [These right ascensions] in order and reversed are [respectively] decreased and increased by [one's] own ascensional differences [to produce oblique ascensions].

The two *koṣṭhaka* versions of the *Karaṇakutūhala* discussed here, the anonymous *Brahmatulyasāraṇī* and the *Laghukhecarasiddhi* of Śrīdhara, illustrate how differently the same handbook can be regarded for the purpose of adaptation into tables.[9]

5.3.2 *Brahmatulyasāraṇī*: Overview

Other than its title and a brief reference in one verse to an algorithm "stated in the *karaṇa*," this *koṣṭhaka* makes no direct claim about its relationship to the *Karaṇakutūhala/Brahmatulya* of Bhāskara. In fact, it contains no statement at all concerning its authorship, date of composition, or other identifying information.[10] Furthermore,

[8]This rule follows Bhāskara's own *Siddhāntaśiromaṇi* (Śāstrī 1989, p. 46, verses 2.24–25). For details see Montelle and Plofker (2015, pp. 26–28) and Plofker (Forthcoming).

[9]The textual content of the *Brahmatulyasāraṇī* has been edited with a translation and study in Montelle and Plofker (2015), which, however, omits to mention some of its extant manuscripts. The *Laghukhecarasiddhi* is edited and discussed in Pingree (1976).

[10]There exists a work entitled *Karaṇakutūhala-gata-sāraṇī* composed by one Nāgadatta (Pingree 1970–94, A5.166), but the relation, if any, between this work and the *Brahmatulyasāraṇī* remains

only two of its eleven known manuscripts are explicitly titled *Brahmatulyasāraṇī*, although similarities of content establish their common identity. But its adoption of the *karaṇa*'s 1183 epoch date and the evident influence of the *Karaṇakutūhala*'s text on both its numerical data and its verses confirm its derivation from Bhāskara's handbook, although various *bījas* and other corrections appended to the tables suggest that it was in use several centuries later than Bhāskara's date (Montelle and Plofker 2015, p. 3).

The *Brahmatulyasāraṇī*'s verses, numbering around ten and appearing only in a minority of its known manuscripts, contain primarily instructions for using its tables. The introductory verse sets out the aims of the work:

natvā vallabhanandanaṃ tadanugopālāṃhripadmadvayaṃ
jñātvā śrīguruvākyato hy aharniśaṃ [dṛṣṭvā] dyum evādhunā ||
siddhānteṣu yathoktakhecaravidhi[bhya]ḥ spaṣṭakoṣṭaṃ muhur
madhyaspaṣṭavibhāgato grahagaṇāt kurve dinaughād aham || 1 ||

Saluting Vallabhanandana [?] and after him the two lotus feet of Viṣṇu, having learned from the word of the revered teacher and having observed the heavens themselves by day and night, now I shall compute an accurate set of tables from the rules of the planets as spoken in the *siddhāntas*, separately for mean and true [quantities], for the various planets, from the accumulated days.

5.3.3 *Laghukhecarasiddhi*: Overview

The *Laghukhecarasiddhi* ("Easy determination of the planets") was composed for epoch Śaka 1149 (Saturday 20 March 1227) by the astronomer Śrīdhara.[11] It contains a set of planetary tables with an accompanying text of twenty-one verses. The distribution of the surviving manuscripts (three which have been studied, and a few at present unknown except by title) suggests that Śrīdhara was located in western India.[12] The opening and closing verses and the colophon contain all the available details concerning its composition:

nārāyaṇaṃ śrīdharapādapadmaṃ
pārāyaṇaṃ puṇyavatāṃ praṇamya ||
śrībrahmasiddhāntasamāṃ karomi

to be investigated. Tables from the *Brahmatulyasāraṇī* are shown in the manuscript images in Figures 2.15, 3.3, 4.2, 4.9, 4.13, and 4.28.

[11]The scribe of Manuscript L (Pingree 1976) notes that this is 16,096 days from 23 February 1183 epoch of Bhāskara II's *Karaṇakutūhala*. This number is equivalent to 4 × 4016, the latter number being a fundamental time period used in Gaṇeśa's *Grahalāghava* (see Section 4.1). Various tables from the *Laghukhecarasiddhi* are shown in the manuscript images in Figures 2.14, 2.16, 3.6, and 4.18.

[12]There is some evidence that he may have been working in Khāndeśa, in the territory of the Yādavas, where perhaps he encountered two of Bhāskara II's relatives who were astrologers at the court of Siṅghaṇa (ca. 1210–1246), the Yādava monarch of Devagiri (Pingree 1976, p. 2).

śrīśrīdharaḥ khecarasiddhim alpām || 1 ||
nandābdhirudronaśako [. . .]

Having bowed to Nārāyaṇa, the lotus-footed Viṣṇu, [and] the totality of the auspicious ones [?], I, the eminent Śrīdhara, construct [this] brief *Khecarasiddhi* equal to the sacred *Brahmasiddhānta*. The Śaka [year] diminished by 1149 [. . .]

pāṭīkuṭṭakabījagolasahitān gaṇitān paitāmahādīn vayaṃ
siddhāntān api manmahe pratidinaṃ kheṭān api prasphuṭān ||
ity ākarṇya vidāṃ vacāṃsi kṛpayā śrīśrīdharaḥ prasphuṭāṃ
cakre khecarasiddhim indudhavalāṃ satkīrtivallīm iva || 21 ||

iti śrīgaṇakacakracūḍāmaṇiśrīśrīdharācāryaviracitā laghukhecarasiddhiḥ samāptā ||

"We consider the mathematics treatises beginning with the *Paitāmaha*, combined with arithmetic and indeterminate equations and algebra and spherics, as well as the accurate daily [longitudes of the] planets": having thus heard the statements of the wise, out of compassion I, the eminent Śrīdhara, have made the accurate moon-lovely *Khecarasiddhi* like a vine [?] of good reputation.

Thus the *Laghukhecarasiddhi* composed by the eminent teacher Śrīdhara, the crest-jewel of the tribe of excellent mathematicians, is complete.

It remains unclear whether Śrīdhara considered his composition an abridgement of some existing larger work called *Khecarasiddhi*, or whether the designation *laghu* was added to its title by later copyists or users.

5.3.4 Adapting *karaṇa* algorithms to *koṣṭhaka* tables

Tables 5.4 and 5.5 summarize the treatment of various portions of the *Karaṇakutūhala*'s content in these two *koṣṭhaka* texts. The *Karaṇakutūhala* also contains much additional material on the standard "ten chapters" *jyotiṣa* topics that is not treated in these *koṣṭhakas*, to wit (Plofker 2016):

Chapter 3: 16 verses on time and locality
Chapter 4: 23 verses on lunar eclipses
Chapter 5: 9 verses on solar eclipses
Chapter 6: 15 verses on synodic phenomena of the planets (some of which were adapted into the *Laghukhecarasiddhi*, as summarized in Table 5.5)
Chapter 7: 4 verses on the lunar crescent
Chapter 8: 6 verses on planetary conjunctions
Chapter 9: 15 verses on the *pātas*
Chapter 10: 4 verses on syzygies

The *Brahmatulyasāraṇī* and *Laghukhecarasiddhi* provide illuminating (and, as far as is currently known, unprecedented) glimpses of procedures by which a handbook, or rather a heavily abridged version of a handbook, was recast as a table text. The two works sometimes differ widely in how they employ and present their source material. An example of this is seen in Figures 5.4 and 5.5, which contain

Table 5.4 Comparison of *Karaṇakutūhala* content and corresponding material in *Brahmatulyasāraṇī* and *Laghukhecarasiddhi*, I.

Karaṇakutūhala verse(s)	*Brahmatulyasāraṇī*	*Laghukhecarasiddhi*
Computation of *ahargaṇa*, 1.2–3	Invokes *Karaṇakutūhala* algorithm, 2	Algorithm similar to *Karaṇakutūhala*'s, 2–3
Planetary epoch mean longitudes, 1.4–6	Incorporated into mean longitude increment table	Appended to mean longitude increment table; instructions for combining with mean longitude increments, 4
Algorithms for mean longitude computation, 1.7–12	Tables of mean longitude increments	Tables of mean longitude increments
Mean daily velocities, 1.13		
Algorithm for local longitude-difference correction, 1.14–15	Invokes local longitude-difference correction and *rāmabījas*, 3	Algorithm for local longitude-difference correction, 5
Corrections for more accurate mean longitudes, 1.16		
Planetary apogee longitudes, 2.1	Sometimes stated in equation tables paratext	
Parameters of *śīghra* epicycles, 2.2	Incorporated into *śīghra*-equation tables (sometimes stated in equation tables paratext)	Incorporated into *śīghra*-equation tables
	Table interpolation procedure, 4	
Determining argument and sign of equation, 2.3–4	Determining argument and sign of equation with interpolation in tables, 5–7	Determining argument of equation, 6
Algorithms for special corrections for parameters of Mars, 2.5–6	Invokes special table for Mars *manda*-apogee correction, 10	Same algorithm as *Karaṇakutūhala* for Mars *manda*-apogee correction, 7

excerpts from the lunar mean longitude tables of the *Laghukhecarasiddhi* and the *Brahmatulyasāraṇī*, respectively. The former tabulates the longitude increments for 1, 10, and 100 365-day years and for 1, 10, and 100 days, while the latter computes them separately for 1–30 days, 1–12 30-day "months," 1–20 360-day "years," and 1–30 20-"year" periods (Montelle and Plofker 2015, pp. 7–9).

The *Karaṇakutūhala*'s trigonometric techniques for correcting mean to true longitudes were also handled in significantly different ways by Śrīdhara and the author of the *Brahmatulyasāraṇī*. The *Brahmatulyasāraṇī*'s table entries for individual degrees within the 10° intervals have evidently been produced by linearly interpolating between results thus computed, as we can see from the table excerpt in Figure 5.6. But its verse instructions confine themselves to explaining the mechanics

Table 5.5 Comparison of *Karaṇakutūhala* content and corresponding material in *Brahmatulyasāraṇī* and *Laghukhecarasiddhi*, II.

Karaṇakutūhala verse(s)	*Brahmatulyasāraṇī*	*Laghukhecarasiddhi*
Computation of sine and arc values, 2.6–8		
Computation of *manda*-equation, 2.9–10	Incorporated into *manda*-equation tables; table interpolation procedure, 4	Incorporated into *manda*-equation tables; determining argument and sign of equation with interpolation in tables, 8
Equation of time correction for moon, 2.11		
Velocity correction due to *manda*, 2.11–12	Incorporated into *manda*-equation tables; table interpolation procedure, 5	Table interpolation procedure, 9–10
Computation of *śīghra*-equation, 2.13–14	Incorporated into *śīghra*-equation tables; instructions for table use, 6–7	Incorporated into *śīghra*-equation tables; instructions for table use, 14–15
Iteration of equation corrections, 2.14	Iteration of equation corrections, 7	Iteration of equation corrections, 16
Velocity correction due to *śīghra*, 2.15–16	Velocity correction from table entries based on method in *Siddhāntaśiromaṇi*, 8–9	Determination of velocity correction due to *śīghra*, with interpolation, 17–18
Equation of time correction, precession 2.16–17	Tables of the equation of time correction	Precession, 10
Computation of ascensional differences, 2.18–19		Computation of ascensional differences and half-equation of daylight, 11–12
Longitude correction due to ascensional differences, 2.19		Longitude correction due to ascensional differences, 13–14
Conversion to different time-units, 2.20–21		Conversion to different time-units, 19
Planetary synodic phenomena, 6.5–7		Table of planetary synodic phenomena
Conversion of synodic intervals from degrees to days, 6.8		Conversion of synodic intervals from degrees to days, 20
Longitudes of planetary nodes, 6.9		Table of planetary node longitudes

of interpolation when using the table rather than identifying the algorithm on which it was based (Montelle and Plofker 2015, pp. 11–16, verses 4–5):

kendrasya doraṃśamitiś ca koṣṭe
bhuktaṃ tadagraṃ parabhogyakaṃ ca ||
kalādikaṃ tadvivarāhataṃ tu
ṣaṣṭyuddhṛtaṃ bhuktakamānakena || 4 ||

Fig. 5.4 Mean longitudinal displacement tables from the *Laghukhecarasiddhi* (IO 2408b f. 5r, Moon) with marginal corrections.

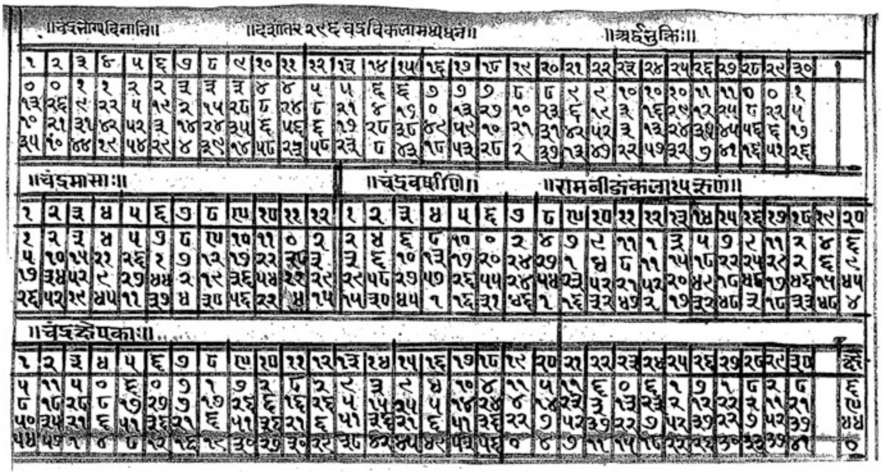

Fig. 5.5 Mean longitudinal displacement tables from the *Brahmatulyasāraṇī* (Smith Indic MB LVIII f. 1v, Moon).

Fig. 5.6 *Manda*-equation tables (excerpt) from the *Brahmatulyasāraṇī* (Smith Indic 45 f. 6v, Sun) for arguments 70–90.

> The amount in degrees of the arc of the [desired] anomaly (*kendra*) is [entered] in the table. [The table entry for the degree] before that is the "elapsed" (*bhukta*) and [then] the following "future" (*bhogya*). The minutes etc. [of the argument] are multiplied by the difference of those [i.e., the two table entries] and divided by sixty, [and the result increased] by the amount of the "elapsed".

yuktaṃ bhaven mandaphalaṃ grahāṇāṃ
svarṇaṃ kramān meṣatulādikendre ||
grahasya bhuktir vivarāhataṃ ca
ṣaṣṭyuddhṛtaṃ kendravaśād dhanarṇam || 5 ||

The *manda*-equation (*mandaphala*) of the planets should be applied positively or negatively [to the mean longitude of the planet] when the anomaly (*kendra*) is in [the semicircle] beginning with Aries or Libra respectively (i.e., when the anomaly is between 0 and 180 or between 180 and 360 degrees). The velocity of a planet [is modified by the *manda*-correction as follows: the fractional part of the desired value of anomaly], multiplied by the difference [between successive entries in the table of velocity-correction (*gatiphala*)] and divided by sixty, [is the increment to the appropriate tabulated *gatiphala* entry. The resulting *gatiphala* is applied to the mean daily velocity] positively or negatively according to [whether] the anomaly [is in quadrants II and III or quadrants IV and I respectively].

Śrīdhara's approach to this material in the *Laghukhecarasiddhi*, on the other hand, follows the *Karaṇakutūhala* in computing only nine *manda*-equation values, one for each 10° in the first quadrant of anomaly (Pingree 1976, p. 5, verse 8):

bhujāṃśakāḥ khenduhṛtās tu mānda-
koṣṭhās tadaiṣyāntaranighnaseṣāt ||
digāptayuṅmandaphalaṃ graheṣu
svarṇaṃ svakendre 'jatulādige syāt || 8 ||

The degrees of the arc divided by 10 [are the arguments for] the table-entries of the *manda*-[equation]. From the [fractional] remainder (*śeṣa*, in arcminutes and so on) multiplied by the difference between that [elapsed table-entry] and the next [table-entry], the [elapsed] *manda*-equation [table-entry] joined to [that] divided by 10 [produces the correct *manda*-equation. This] should be applied positively or negatively [to the mean longitude], as its anomaly (*kendra*) is in [the semi-circle beginning with] Aries or Libra [respectively].

But as the *Laghukhecarasiddhi* solar equation table in Figure 5.7 reveals, Śrīdhara appears to be using both a different value for the maximum equation and a different procedure for deriving the other tabulated values of it. In fact, our reconstruction of the table compilation (see Montelle and Plofker forthcoming) suggests that he abandoned the *Karaṇakutūhala*'s linear approximation technique altogether in favor of the theoretically exact trigonometric correction model outlined in Section 2.1.2.

The two *koṣṭhaka* works likewise differ in their treatment of the *Karaṇakutūhala*'s special *manda*-correction for Mars. The *Laghukhecarasiddhi* prescribes for this adjustment a verse algorithm identical in meaning, and strikingly similar in form, to the *Karaṇakutūhala*'s (Pingree 1976, p. 5, verse 7):

Fig. 5.7 Table of solar *manda*-equation from the *Laghukhecarasiddhi* (IO 2408b f. 6r, Sun).

Fig. 5.8 Table of corrections to Mars's apogee from the *Brahmatulyasāraṇī* (Smith Indic 45 f. 7v).

bhaumāśukendrasya padaiṣyayāta-
svalpasya liptāḥ khakhavedabhaktāḥ ||
āptāṃśahīnāḍhyam asṛṅmṛdūccaṃ
sphuṭaṃ bhavet karkimṛgādikendre || 7 ||

The *Brahmatulyasāraṇī* invokes instead the pre-compiled table shown in Figure 5.8. Each degree of argument from 1 to 45 (the maximum elongation from a quadrant boundary) is just multiplied by 0;9 to produce the appropriate entry for the correction in arcminutes to the *manda*-apogee longitude. The accompanying rule for retrieval and application of these tabulated values is a slightly modified version of Bhāskara's (Montelle and Plofker 2015, pp. 26–28, verse 10):

bhaumāśukendrasya padasya jāta-
gamyasya bhāgāḥ phalavat phalaṃ ca ||
kulīra[*na*]*krādigate svakendre*
hīnādhikaṃ spaṣṭam asṛṅmṛdūccam || 10 ||

The degrees of the past [or] future [part, whichever is smaller,] of the quadrant of the *śīghra*-anomaly of Mars are like [the argument of an] equation [in the table of *manda*-apogee correction for Mars]. And the [corresponding] equation, when its own [*śīghra*-]anomaly is in Cancer or Capricorn, is [respectively] subtracted or added [to make] the *manda*-apogee of Mars accurate.

Although the *Brahmatulyasāraṇī*'s author likewise closely follows Bhāskara's phrasing, he has transferred even the trivial task of division by 400 (or equivalently multiplication by 9/3600) from the user of the work to the compiler of its tables.

A similar comparison of the two *koṣṭhakas*' approaches to the *Karaṇakutūhala*'s *śīghra*-correction techniques yields some other interesting divergences. The trigonometric procedures for finding the *śīghra*-equation summarized in Section 2.1.3 are converted in the *Laghukhecarasiddhi* to a set of 18 tabulated values per planet as illustrated in Figure 5.9, accompanied by the following instructions for interpolating within their 10° intervals (Pingree 1976, p. 6, verses 14–15):

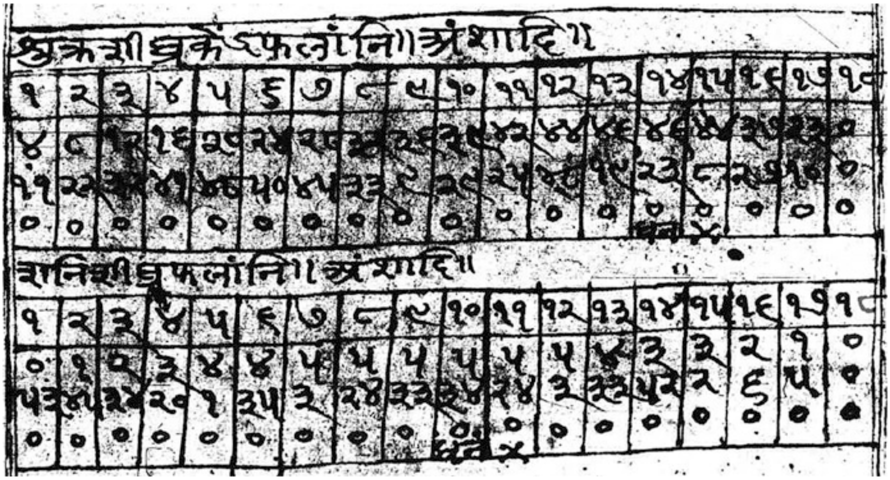

Fig. 5.9 *Śīghra*-equation tables from the *Laghukhecarasiddhi* (IO 2408b f. 8r, Venus (top) Saturn (bottom)).

grahonaśīghraṃ calakendram eta-
ñcakrād viśodhyaṃ rasabhādhikaṃ cet || 14 ||

tadaṃśakādho daśabhir hṛtaḥ syāt
koṣṭhas tadaiṣyāntaranighnaśeṣāt ||
digāptahīno gatakoṣṭha eṣye
hīnaṃ phalaṃ syād adhike tu yuktam || 15 ||

The [longitude of the] *śīghra*[-apogee] diminished by [that of] the planet is the *śīghra*-anomaly; it is to be subtracted from a [full] circle if greater than six signs. The [integer result of] the degrees of that divided by ten should be the [number of the] table entry. The tenth part from the remainder [from dividing the anomaly degrees by ten] multiplied by the difference [between] that [entry] and the next [should be] the result, subtracted from the past entry if the next [entry] is smaller [than the past], added if [it is] larger.

The *Brahmatulyasāraṇī*, on the other hand, provides not only a more detailed set of tables listing values of the *śīghra*-equation, their successive differences, and the corresponding orbital hypotenuse distance (as illustrated in Figure 5.10), but also a more complicated procedure for extracting and using these data (Montelle and Plofker 2015, pp. 16–26, verses 6–8):

grahoṇam uccaṃ ca phalaṃ rasādhikaṃ
cet sūryataḥ śodhya lavādikaṃ kṛtam ||
bhāgāṅkasaṃkhyāgatakoṣṭakaṃ tayoḥ
kalādikaṃ śeṣaṃ vivarāhataṃ tat || 6 ||

ṣaṣṭyā vibhaktaṃ svam ṛṇaṃ ca bhogyāt
kāryaṃ vihīnādhi[ka]tatkrameṇa ||
ādau hi mandārdha [x] kena tasmāt
samagraṃ [x x x] punaḥ punaś ca || 7 ||

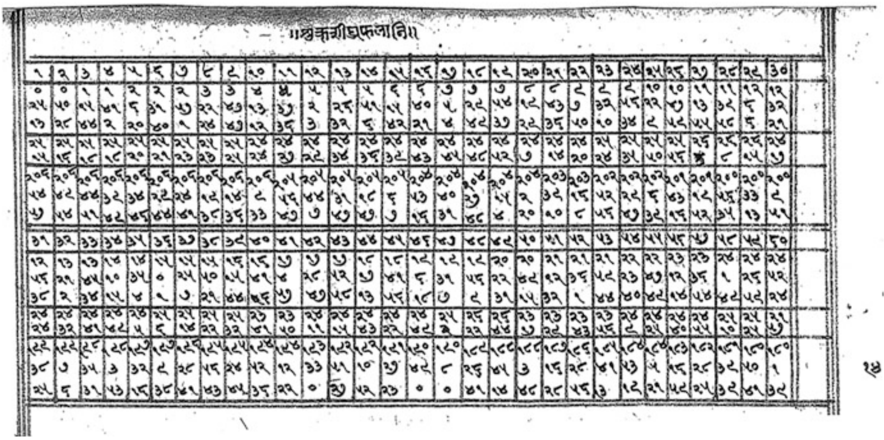

Fig. 5.10 A page from the *Brahmatulyasāraṇī* (Smith Indic 45 f. 14v) showing the first 60 entries of Venus' *śīghra*-equation table.

drākkendrabhuktir vivareṇa nighnā
ṣaṣṭyuddhṛtaṃ svaṃ ca phalasya vṛddhau ||
hrāsa ṛṇaṃ mandagater grahāṇāṃ
kṛtām iti syāt sphuṭakheṭabhuktiḥ || 8 ||

[The longitude of] the [*śīghra*-]apogee is diminished by [the longitude of] the planet. Having subtracted the result from 12 [signs] if it is greater than 6, it is made into degrees etc. [Subtract from this reduced *śīghra*-anomaly] the previous table entry for the number [equal to its] number of degrees; the remainder [from the subtraction] of those two is in arcminutes and so on. That is multiplied by the difference [between the previous and the next table entries, and] divided by sixty. [The result is] applied [to the previous entry] positively or negatively, according as that is respectively less or greater than the next entry. At first, [the mean longitude is corrected] with half the *manda*-equation and afterwards with the whole, repeatedly.

The velocity of the *śīghra*-anomaly is multiplied by the difference [between successive *śīghra*-equation values corresponding to that *śīghra*-anomaly] and the quotient with sixty [is applied] positively with respect to the *manda*[-corrected] velocity of the computed planets when there is increase of the equation [in successive tabulated values], negatively when there is decrease. Thus the velocity of the true planet should be [computed].

These instructions commence with rules similar to those in the *Laghukhecara-siddhi* for computing and reducing the *śīghra*-anomaly and applying the resulting equation. But the rule for the velocity correction, as discussed in Montelle and Plofker (2015, pp. 16–26), is significantly different from that in the *Karaṇakutūhala* (and from the *Laghukhecarasiddhi* which doesn't address this correction in any way). It follows instead Bhāskara's treatise *Siddhāntaśiromaṇi* in using the change between two values of the *śīghra*-equation to compute the resulting correction in the planet's velocity. Ingeniously, the author of the *Brahmatulyasāraṇī* has taken advantage of his tabular format to interpret the change in the *śīghra*-equation as simply the recorded difference between two successive entries in the table.

To sum up, these two adaptations of Bhāskara's *Karaṇakutūhala* illustrate how the same highly regarded handbook, disseminated since the late twelfth century in the traditional strictly verse *karaṇa* format with no separate numerical tables, was rendered into *koṣṭhaka* versions in very different ways. As we have seen, both Śrīdhara and the author of the *Brahmatulyasāraṇī* clearly borrowed from Bhāskara in composing their verses. Their quite distinct approaches to structuring their tables may be due to individual innovations, or to imitation of earlier conversions from verse-only *karaṇas* to table-focused *koṣṭhakas*, or some combination of both.

It is likewise unclear exactly how the later tradition of table texts was influenced by these two *koṣṭhaka* versions of the *Karaṇakutūhala* (especially since we do not know exactly what "later" means with respect to the imprecisely dated *Brahmatulyasāraṇī*). The uncertainty is compounded by the possibility of imperfectly identified additional works or recensions bearing some form of the name *Karaṇakutūhalasāriṇī*, see, for instance, MS BORI 501/1895–1902 ff. 2–28. Other than the brief allusion by Harihara already discussed (Section 5.2.3), we know of no subsequent author who cites these tables, nor any who followed their example of combining the mean-with-equation format with Brāhmapakṣa parameters.

5.4 *Mahādevī* of Mahādeva

The *Mahādevī* (epoch Śaka 1238/1316 CE), named after its author Mahādeva, became the standard planetary *koṣṭhaka* in mean-to-true format within the Brāhmapakṣa school. In the numerous surviving manuscripts of the *Mahādevī*, its extensive set of tables is frequently accompanied by a text of around 43 verses entitled *Grahasiddhi* which contains some details about Mahādeva's lineage as well as the usual instructions and data for using the tables.[13] Its many commentators include Nṛsiṃha (1528) of Nandipura, Dhanarāja (1635) of Padmāvati in Mārwār, and Mādhava (Pingree 1968, p. 37).

In his introductory verse, Mahādeva attributes the origin of this seminal work to another author, Cakreśvara (Pingree 1970–94, A4.374, 376):

siddhiṃ karotīśajakendrabhorvī-
nādīn śivau kṣetrapavāggurūṃś ca ||
cakreśvarārabdhanabhaścarāśu-
siddher mahādeva ṛṣīṃś ca natvā || 1 ||

[13]Tables from manuscripts of this work are shown in Figures 3.7, 3.17, and 4.8. For more details on the work and its author, see Pingree (1968, pp. 37–39), Neugebauer and Pingree (1967), Pingree (1973, p. 82), and Pingree (2003, pp. 51–54). Aspects of the *Mahādevī*'s tables were investigated in Neugebauer and Pingree (1967), the first detailed analysis of a mean-to-true *koṣṭhaka*. Given the distribution of its extant manuscripts, it seems likely Mahādeva was working in Gujarāt or Rājasthān. However, the genealogy in the closing verses (41–43) of the *Grahasiddhi* identifies his father as Parāśurāma, son of Padmanābha, son of Mādhava, son of Bhogadeva of the Gautama *gotra*, all of whom were astrologers (*daivajña*), a dweller on the Godāvarī river, presumably in Mahārāṣṭra (Pingree 1981, p. 42; Pingree 1968, p. 37).

> Mahādeva, having honored Gaṇeśa Brahman, Indra, the *nakṣatras*, the earth [and the planets] beginning with the sun, Śiva and Pārvatī, the guardian deity, Vāc, and the teachers as well as the sages, makes the fulfillment/completion of the rapid determination of the planets begun by Cakreśvara.

Some commentators have enlarged slightly on this attribution: *cakreśvaraḥ graṃthakarttuḥ*, "Cakreśvara [is the name of] the maker of the book," according to Mādhava (MS LDI 7671 f. 1v). And Dhanarāja remarks *māhādevanāmādhijaḥ cakreśvaranāmācāryeṇārabdhāyā nabhaścarāṇāṃ grahāṇāṃ āśusiddiḥ śīghrasiddhiḥ sā aparipūrṇā sthitā tasyāṃ siddhiṃ pūrṇatāṃ karoti* "Of the [work] begun by the teacher named Cakreśvara, [i.e.,] the rapid determination [or] *Śīghrasiddhi* of the sky-goers [or] planets, remaining uncompleted [by him]: of that [work], the *brāhmaṇa* named Mahādeva makes the fulfillment, [i.e.,] completion" (MS BORI 497 f. 1v). Nothing else is known of this author (Pingree 1970–94, A4.88), who presumably might have been Mahādeva's own teacher.

The subsequent verses of the *Grahasiddhi* make Mahādeva's case for the superiority of his tables to the cumbersome computational procedures of treatises and handbooks (MS RAS Tod 24 f. 1v):

> *paitāmahāryabhaṭajiṣṇujabhāskarādi-*
> *siddhāntabhedakaraṇair nitarām agādhe* ||
> *saṃkhyārṇave khacarakarmajale nimagna-*
> *jyotirvidāṃ prataraṇāya dṛḍhorunāvaḥ* || 2 ||
>
> *vinā dyuvṛndāśumṛdukriyādyaiḥ*
> *sabījamadhyārkasamāsamāpteḥ* ||
> *caturdaśāhāntaritā bhasaṅkhyā*
> *ghaṭīsv ihābhrartuṣu siddhakheṭāḥ* || 3 ||
>
> *yathopayogyā asakṛtparisphuṭāḥ*
> *siddhāntamadhyasphuṭakarmaṇā kṛtāḥ* ||
> *yātrāvivāhādiṣu jātake grahāḥ*
> *sphur [syur?] atra tadval laghukarmaṇā sphuṭāḥ* || 4 ||
>
> [The *Mahādevī* is] a staunch large ship for [giving] passage to the astronomers entirely plunged into the waters of planetary calculation in the unfathomable ocean of number by the *karaṇas* [derived from] the different *siddhāntas*: *Paitāmaha* [and those of] Āryabhaṭa, the son of Jiṣṇu [Brahmagupta], Bhāskara, etc.
>
> Without the procedures etc. [for finding] *ahargaṇa*, *śīghra* and *manda* [equations], here the true [longitudes of the] planets [are found] from the correct determination of the mean sun with [its] *bīja*. [They are] 27 in number, separated by 14 days, in [units of] sixtieths [of a circle] in [each of] sixty [separate tables].
>
> In the same way that the repeatedly corrected [longitudes of the] planets made by the mean and true method of *siddhāntas* are to be used in horoscopy for travel, marriage, etc., the true [longitudes produced] in this [work] by an easy method are [to be used] like that.

The classical "ten chapters" anatomy of treatises and handbooks, with true planetary positions laboriously determined by repeated computations of correction terms, here gives way to a more direct (although far less concise) mapping from mean positions of the five star-planets to true ones. The 300+ tables of the *Mahādevī* include single-year and 60-year mean longitudinal displacement tables for the epact, the lord of the year, the moon, the lunar node, and the five planets, along with a *bīja* for each of these entities. The bulk of its content, however, consists of the sixty annual true longitude tables given for each of the five planets, each table containing 27 longitudes at intervals of 14 days, as specified in the verse.

5.4.1 The first "canonical" *koṣṭhaka*

The known existence of over a hundred manuscripts of the *Mahādevī*, far more numerous than the surviving witnesses of any earlier *koṣṭhaka*, suggests that many thousands of copies of it were written and read over the centuries. Knowledge of the manuscript corpus of Sanskrit astronomical tables is still far too incomplete to permit any definite conclusions about the causes of the *Mahādevī*'s apparently unprecedented popularity. But we speculate that the following features may have been significant factors in the history of its reception and its rivals.

Mean-to-true structure with purely sexagesimal units. As in the earlier *Grahajñāna*, the pre-computation of true longitude values in the *Mahādevī* tables makes the cumbersome calculation of *manda* and *śīghra* equations for a known mean longitude unnecessary. But the *Mahādevī*'s planetary true longitudes are expressed in 6° arc-units or sixtieths of a circle, here called "sexagesimoria," and their sexagesimal fractions. These seem to have been more appealing to users than the 13°;20 *nakṣatra* units employed in the *Grahajñāna*, possibly because they are more easily converted to or from the standard measures of zodiacal signs and degrees. The *Grahasiddhi* helpfully, although not entirely clearly, explains the use of the units (MS RAS Tod 24 f. 1v):

syāt palaṃ vipalaṣaṣṭi tair nabhā-
ṅgair ghaṭīti tadanu grahabhramaḥ ||
tair nabhastri bhir agair bhramaḥ kramāt
svāṃśaśuddhiśanivārapūrvayāiḥ || 5 ||

vīthīnāgvaikyajaikaikā
śuddhyabdeśaguṇadhruvau ||
tithivārādikāv uktāv
anye nāḍyādikāḥ pṛthak || 6 ||

krameṇāvadhayo 'bdāder
manubhir manubhir dinaiḥ ||
tatpaṅktiṣu grahāḥ spaṣṭāḥ
santi dhruvaghaṭībhavāḥ || 7 ||

A *pala* should be 60 *vipalas*; a *ghaṭī* [is measured] by 60 [of] those [*palas*]; thus a planet's cycle [is made] analogous to that [i.e., purely sexagesimal]. With those thirty [*tithis* or] with seven [weekdays] a cycle [is made] [for] epact [or lord of the year] beginning with Saturday [respectively, each with] its own fractions.

[There is an] individual row [column, entry?] produced from the sum [of corrections for each body from] the lord [of the year? or sun?] [to] Rāhu. The multiplier and initial mean longitude of the epact and lord of the year are stated beginning with *tithis* and weekdays [respectively]; other [quantities] separately [are stated] in sixtieths (i.e., sexagesimoria) and so forth.

Fig. 5.11 Tables of epoch adjustments and annual multipliers from the *Mahādevī* (RAS Tod 24 f. 2v).

> The *avadhis* in order from the beginning of the year [are measured] with 14 days each. In the rows of those [*avadhis*, i.e., the individual tables], the true [longitudes of the] planets are produced for the sexagesimoria of their mean longitudes.

Abundance of tables and data. The use of the smaller sexagesimoria units as opposed to *nakṣatras* also provides 60 rather than 27 true longitude tables for each planet, and consequently smaller interpolation intervals. In addition, these tables contain a large amount of compactly organized data per argument value. Each entry for true longitude is accompanied by the tabulated difference between that entry and the corresponding one in the next table, simplifying the task of interpolation between tables. The true velocity and its inter-tabular differences are similarly tabulated. Finally, occurrences of synodic phenomena are noted next to the longitude entries where they occur.

Reduction and simplification of text: reinforcement of crucial parameter values. With its total verse text amounting to less than 50 verses, the *Mahādevī* avoids duplication of computational techniques. No algorithms are prescribed as (potentially confusing) alternatives to use of the tables.

In fact, nearly a fifth of the total verses, beginning with the two quoted below (MS RAS Tod 24 f. 1v), merely restate verbally the mean longitude parameters listed in the tables. Figure 5.11, reproduced from a manuscript of the *Mahādevī*, displays the tabulated annual mean longitude increments enumerated in these verses, while Figure 5.12 (from a different *Mahādevī* manuscript) illustrates the inclusion of extensive additional tables containing successive multiples of these annual parameters. The fundamental data needed to produce the mean longitudes for the desired time that are essential for the use of the tables are thus reinforced by a variety of different formats.

> *guṇaḥ śuddheḥ śivās 11 trīṇi 3*
> *tryakṣā 53 dvyakṣīṇi 22 khābdhayaḥ 40*
> *vārasyaikaḥ śarorvy āḍi-*[14]
> *rāmā vājīndavo 17 dvidhā* || 13 ||
>
> *kujasyelāgnayo 31 'bdhyakṣā 54*
> *rūpe devā 33 mahīyamāḥ 21*[15]

[14] MS has *śarorvyāṭu*.

[15] MS has *12*.

Fig. 5.12 Table of mean longitudinal displacements from the *Mahādevī* (Smith Indic 80 f. 1r).

> *budhasyāṅkā 9 nagā 7 lekhā 33*
> *kakubhā 10 netra 22 bāhavaḥ* || 14 ||
>
> The multiplier of the *śuddhi* [is] 11, 3, 53, 22; of the [week-]day, 1, 15, 31, putting down 17 twice.
>
> [The multiplier] of Mars [is] 31, 45, 1, 33, 21; of Mercury, 9, 7, 33, 10, 22.

5.4.2 *Laghumahādevī*

An abbreviated recension of the *Mahādevī*, containing a condensed and rearranged version of the planetary true longitude tables along with introductory tables of parameters and mean motions as well as a very short text, was constructed for epoch date Śaka 1500 (1578 CE) by an unidentified author. Currently known from only

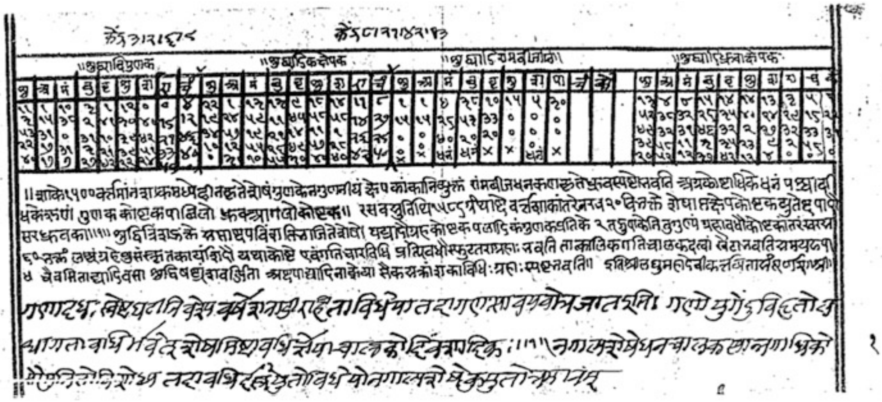

Fig. 5.13 The first page of the unique manuscript of the *Laghumahādevī* (JVS 147.2479 f. 1v) showing the text and, from left to right, tables of multipliers for mean displacement (*guṇaka*), epoch mean values for Śaka 1500 (*kṣepaka*), *rāmabīja*, and epoch mean values for Śaka 1586 (*dhruvakṣepaka*).

one manuscript, this work is an interesting though obscure witness to the varied evolution of established *koṣṭhakas*.

The *Laghumahādevī*'s known text in its entirety together with the first four tables is reproduced in the manuscript page shown in Figure 5.13. The beginning of the initial portion in the first scribal hand reads as follows:

śāke 1500 varttamāṃnaśākamadhye hīnakṛte śeṣaṃ guṇakena guṇanīyaṃ kṣepakāṃkāniyuktaṃ rāṃmabījadhana ṛṇakṛte dhruva spaṣṭo bhavati |

agrakoṣṭhādhike dhanaṃ paścād adhike ṛṇaṃ guṇakakoṣṭhakapābilo dhruva āgalokoṣṭhaka ||

When the Śaka [year] 1500 is subtracted from the current Śaka [year], the remainder is to be multiplied by the multiplier [and] added to the digits of the epoch mean longitude. When [the sum is] positively or negatively made [adjusted] by the *rāmabīja*, the initial mean longitude becomes accurate.

When [a table entry] is greater than the previous table entry, positive; when greater than the following [table entry], negative. The multipliers table is first, the epoch-values table is next [? vernacular *pābilo* and *āgalo*?].

rasavasutithi 1586 graṃthābde varttaśākāṃtare nakha 20 vibhakte śeṣāptakṣepakoṣṭhakayute bdapādau saradhruvakā || 1 ||

śuddhitriṃśadbhakte saptābdapaviṃśatibhājite śeṣo ghaṭyādau ...

When the difference of the current Śaka year in [comparison to] the text [epoch] year 1586 is divided by 20 [and] when the the lord of the year etc. [is made by] adding the epoch mean position table entries [corresponding to] the remainder and the quotient, [those are] the current initial mean values.

When [that sum is] divided by thirty [in the case of] the epact, [or] divided by seven [in the case of] the lord of the year [or] by twenty [otherwise], the remainder is in [the form of] sixtieths etc. [...]

This rather peculiar combination of epoch years 1500 and 1586 (see Figure 5.13) seems to echo the multiple versions of determining desired mean positions presented in the original *Mahādevī*. That is, the user may multiply the difference between the current year and 1500 by the given multiplier for the desired quantity and add the product (after removing any completed cycles) to that quantity's epoch mean value for 1500. Or alternatively, the difference between the current year and 1586 may be broken down into a sum of multiples of 20 years and of single years, whose corresponding mean increments are to be picked out of the tables illustrated in Figure 5.14 and added together along with the epoch mean value for 1586.

Besides the highly condensed text, there are several other ways in which the *Laghumahādevī* achieves a more compact version of Mahādeva's original work. Most significantly, it tabulates the true longitudes using 18° arc-units instead of 6° ones, copying every third value of the *Mahādevī* tables, so that only 20 true longitude entries instead of 60 are needed to complete a planetary cycle. At the same time, the author of the *Laghumahādevī* has reversed the *Mahādevī*'s tabular relationship of the mean longitude intervals within a cycle to the *avadhis* in a year.

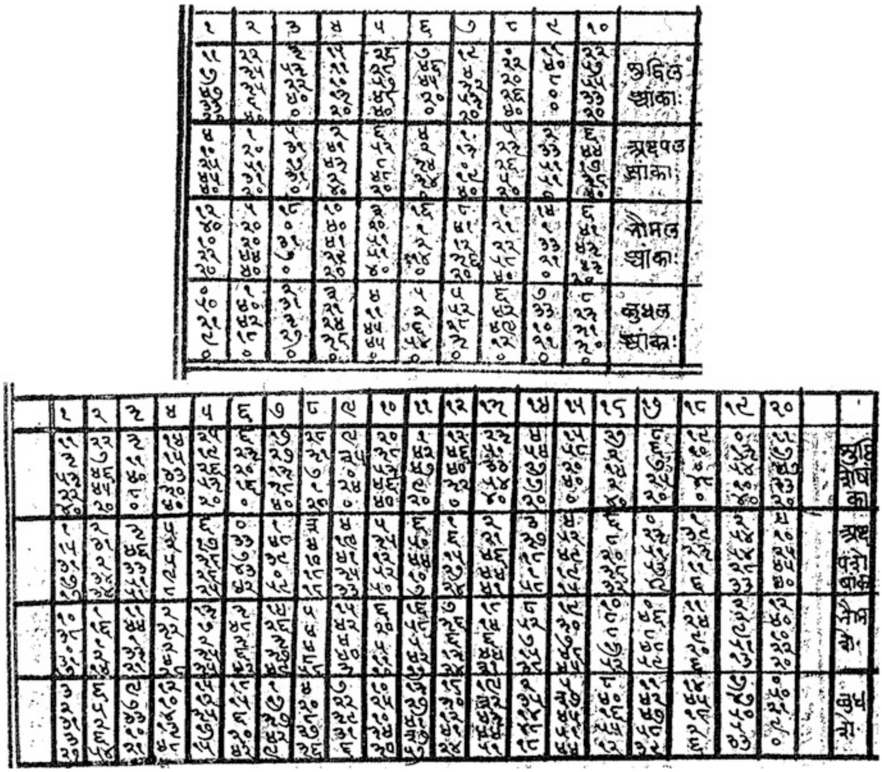

Fig. 5.14 Tables of mean increments from the *Laghumahādevī* (JVS 147.2479 f. 2r–v) for 1 to 10 20-year intervals (above) and 1 to 20 single years (below) for the epact, the lord of the year, Mars, and Mercury.

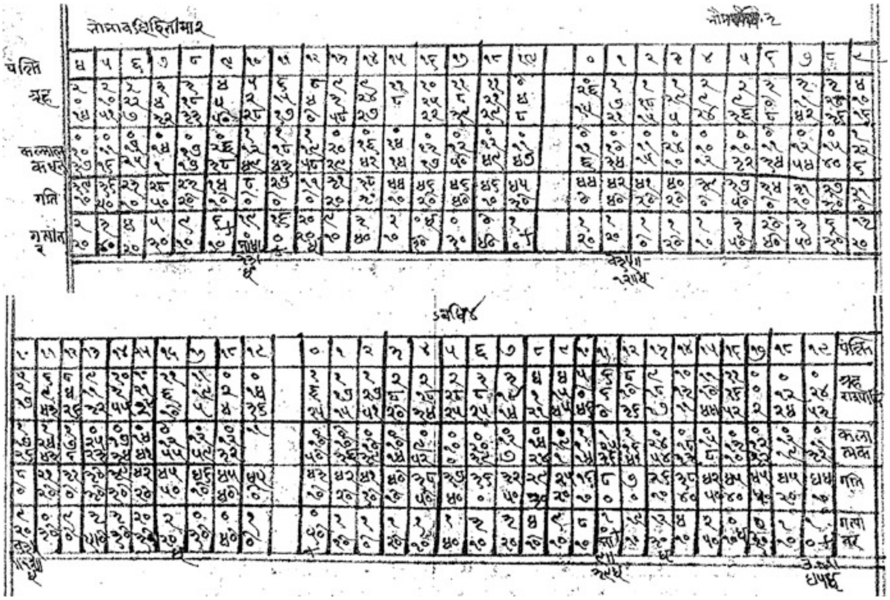

Fig. 5.15 Two pages from the *Laghumahādevī* (JVS 147.2479 f. 4r–v) showing Mars's true longitude tables for mean longitude intervals 0–19 of *avadhis* 3 and 4 (part of the table for *avadhi* 2 also appears). Synodic phenomena occurrences are noted in the bottom row of the appropriate columns.

Namely, for each planet the *Mahādevī* lists true longitudes for 27 successive 14-day *avadhis* in each of 60 separate tables, one for each successive 6° interval of mean longitude. The *Laghumahādevī*, on the other hand, lists true longitudes for 20 successive 18° intervals of mean longitude in each of 27 separate tables, one for each *avadhi*. This reversal can be seen more clearly by comparing the entries in the *Mahādevī*'s and *Laghumahādevī*'s true longitude tables for Mars shown in Figures 4.16 and 5.15, respectively. For example, the true longitude entry for argument value *avadhi* 3 of table 0 in the former is 0;26,15, the same as the true longitude entry for argument value 0 of table *avadhi* 3 in the latter.

5.4.3 Characteristics of *laghu*/condensed versions in general

Compositions taking the form of an abridged (*laghu*) or extended (*bṛhat* or *mahā*) recension of some earlier work long pre-date this period in Sanskrit astronomy, usually in the context of authors tinkering with their own previous productions. Instances include the *Mahābhāskarīya* and *Laghubhāskarīya* of Bhāskara in the seventh century, the *karaṇas Bṛhanmānasa* and *Laghumānasa* of Muñjāla or Mañjāla in the tenth, and the so-called *uttara* chapters of handbooks and treatises (such as Brahmagupta's *Khaṇḍakhādyaka*) in which topics covered earlier in the text are discussed in greater detail. The *Laghukhecarasiddhi* (Section 5.3.3) and the *Tithicintāmaṇi* (Section 5.6.3) may represent other examples of such an abridgement process involving *koṣṭhakas*. The *Laghumahādevī* illustrates the extent of the liberties a redactor might take with such a table text, even a well-established classic of the genre. It should be borne in mind, however, that the *laghu* prefix on a title taken from an existing work does not always reliably indicate dependence of the *laghu* composition on that work. For example, the seventeenth-century astronomer Murāri (Pingree 1970–94, A4.441–442) named one of his table texts *Tithidarpaṇa* and the other *Laghutithidarpaṇa*, despite the fact that the latter is based on the *Makaranda* (Section 5.5) rather than on the former.

5.5 *Makaranda* of Makaranda

Despite (or perhaps partly because of) their streamlined procedures, mean-to-true table texts did not oust the more familiar mean-with-equations format in the genre of Sanskrit astronomical tables. One of the most popular *koṣṭhakas* ever composed, the *Makaranda* or *Makaranda-sāraṇī* written at Kāśī/Varanasi with epoch Śaka 1400 (1478 CE) by Makaranda, retained this earlier format. Like other notable astronomical works from this region, the *Makaranda* follows the Saurapakṣa.[16] Unusually in terms of the *koṣṭhakas* we have examined so far, this work has no accompanying set of verses except for the following invocatory stanza:

śrīsūryasiddhāntamatena samyag
viśvopakārāya gurūpadeśāt ||
tithyādipattraṃ vitanoti kāśyām
ānandakando makarandanāmā ||

Thoroughly [informed] by the opinion of the excellent *Sūryasiddhānta*, for the use of all, from the instruction of the teachers, Ānandakanda, called Makaranda, unfolds in Kāśī a leaflet on the *tithi* and so forth.

Presumably the *Makaranda* relies on the user's familiarity with the standard mean longitude correction procedures of Indian astronomy to ensure the correct

[16] Numerous examples of tables in the *Makaranda*'s manuscripts are shown in Figures 2.18, 3.2, 3.10, 3.11, 3.18, 4.6, 4.48, 4.51, 4.52, 4.53, 4.54, 4.55, 4.59, and 4.60. This work has been extensively treated in, e.g., Pingree (1968, pp. 39–46), Pingree (1973, p. 92), Pingree (1981, p. 42), Pingree (1970–94, A4.341–343), Pingree (2003, pp. 54–59). As discussed therein and in Sarma (1997), the *Makaranda* has been edited multiple times, e.g., Makaranda (1923); Miśra (1982). Note that there is no textual attestation of any epoch date in the work itself, although various manuscripts containing commentaries and notes mention that the year of the text is Śaka 1400. Additionally, the earliest Śaka year occurring as a table argument in some of the manuscripts is 1400, whose beginning is assigned to *ghaṭī* 30;57 of weekday 6 in *tithi* 24. For the regional Saurapakṣa tradition see also works of Munīśvara (Pingree 1970–94, A4.436–441) and Kamalākara (Pingree 1970–94, A2.21–23).

application of the table entries to given data for desired times and phenomena, despite its paucity of explicit verbal instructions. Another distinctive feature that seems likely to have contributed to this *koṣṭhaka*'s popularity is its comprehensiveness: as illustrated by the topics found in some sample recensions enumerated in Table 5.6, its tables embrace almost any standard astronomical quantity that a user might want to calculate. These include the calendar fundamentals of dates and times marking the starting moments of *tithis*, *nakṣatras*, *yogas*, and *saṃkrāntis*, as well as planetary mean motions, orbital equations, and synodic phenomena. More exceptional events are also provided in tables for calculating characteristics of eclipses, such as parallax and shadow diameter.

5.5.1 Some noteworthy features of the *Makaranda*

Like the *Mahādevī* before it, the *Makaranda* records and manipulates celestial position data in a strictly sexagesimal numerical format whose integer unit is the abovementioned sexagesamorion or 6° arc-unit. (As illustrated in the worked example quoted in Section 5.5.2, this notation in a mean-with-equations table requires the user to carry out the conversion of the desired true longitude from sexagesimoria units to the standard signs, degrees, etc.) Sample values in tables shown in Figures 4.6 and 4.51 in the previous chapter exemplify the use of this convention in Makaranda's work.

Unique to the *Makaranda*, as far as we can tell, is the whimsical but memorable set of vegetation-themed Sanskrit technical terms for its various tables—perhaps inspired by the author's personal names Makaranda ("honey," "nectar," and "bee") and Ānandakanda ("root of joy")?—summarized in Table 5.7. No obvious rationale seems to guide the assignment of words meaning literally "garden," "blossom," etc., to particular table types. Although earlier Sanskrit mathematical texts sometimes employed words meaning "vine" or "creeper" for certain computational techniques that produce an undulating line of figures (Rangacarya 1912, pp. 80, 117), (Sastri 1957, p. 77), this figurative sense does not seem to apply to Makaranda's use of *vallī* for tables. Unlike other fanciful systems of technical nomenclature such as the application of metaphorical names like "door-hinge" or "cow's urine" to Indian multiplication techniques (Datta and Singh 1935/38, pp. 134ff.), or quark flavors in modern particle physics, Makaranda's terminology appears to have remained confined to the text in which it originated.

The *Makaranda* also employs what appear to be some rather idiosyncratic units of measurement. For example, in the table of lunar eclipse half-duration shown in Figure 5.16, the accompanying instructions read in part *grāso bānāgni 35 nighnaḥ sphuṭaśaśidvadviṃśakāḥ syuś ca tebhyaḥ sthityarddhaṃ* ("The obscuration [is] multiplied by 35, and [those] are the 22nd parts [?] of the true moon [disk]; to them [corresponds] the half-duration [in the table entries]"). The accompanying half-duration entries range from 0 to the typical maximum value of 4;40 *ghaṭīs*,

Table 5.6 Contents of the *Makaranda* in different manuscripts.

	Topic	BORI 446	BORI 546	Baroda 3225	Nepal 4-1796	RORI 5498
Calendar	*tithi*	1544–1816	1400–1768	1480–1624	1448–1800	1444–1832
	nakṣatra	1520–1808	1400–1736	1448–1616	1448–1808	1448–1880
	yoga	1568–1832	1400–1664	1448–1592	1448–1808	1448–1880
	saṃkrānti	1520–1688	1496–1664	1448–1716	1448–1808	1592–1760
Planetary positions	Accumulated civil days	1400–1799		1471–1685	1400–1913	1457–1571
	Mean longitudes	Complete		Complete	Complete	Complete
	Equations	Complete		Complete	Complete	Complete
Eclipses	Half-durations	Incomplete	Complete	Complete	Complete	Complete
	Solar declination		Complete	Complete	Complete	Complete
	Lunar latitude		Complete	Complete	Complete	Complete
	Parallax		Complete			
	Diameters		Complete	Complete	Complete	Complete
	Solar altitude		Complete			
Planetary phases	Planetary mean long. displacements (days)	1,7,13–17	1–10, 20, 30,. . .,100	1, 13–17	1, 13–17	1, 13–17
	Synodic phenomena	Complete		Complete	Complete	Complete
Astrological topics	*Nakṣatra*-quarter divisions	Complete			Complete	Complete
	Ascendant			Complete	Different	Complete
	12×12 unidentified table		Complete	Complete	Complete	Complete
Other		Two 5-entry tables: argument values 34,35,0,1,2 and 16–20, minimum entry 0, maximum entry 680,0.	Table entitled *dinamāma ayanāṃśa sekaḥ* (argument values 1–60), and *chāyāghaṭikākasyaṃ*, and *turyachāyā* (double argument 0–17, horizontal and 0, 13–17, vertical).	Additional tables with headings *unnata*, *chāyā*, *karṇa* (argument values 1–14 and 1–17).	A table entitled *dinamāma ayanāṃśa sekaḥ* (argument values 1–60); a special *śīghra* table for Venus and Mercury.	

Table 5.7 Botanical-themed terms used in the *Makaranda*.

Sanskrit term	Literal meaning	Significance in *Makaranda*
kanda	Bulb, root	Calendar day/time for mean time-unit
guccha	Blossom, flower; shrub	Calendar day/time for mean time-unit
vallī	Creeper, vine	Increments to linearly increasing quantities such as civil days or lunar anomaly
vāṭikā	Garden, plantation	Mean position increments
saurabha	Fragrance, perfume; saffron	Corrections due to *vallī* values

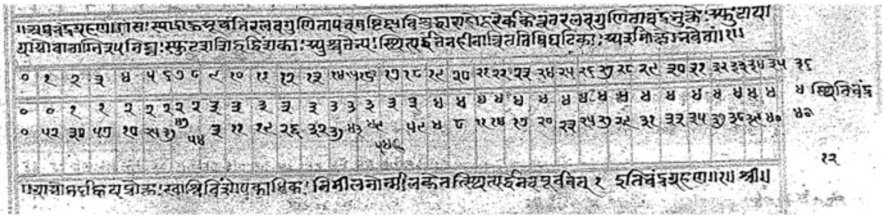

Fig. 5.16 Table of lunar eclipse half-duration from the *Makaranda* (BORI 546 f. 12v).

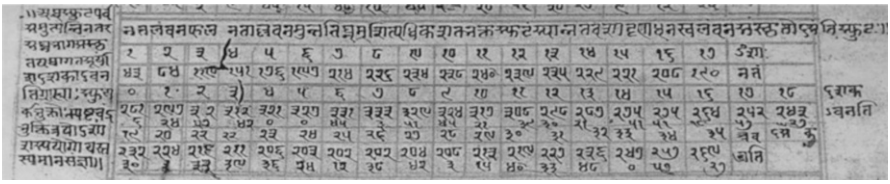

Fig. 5.17 Top: Table of parallax in longitude, for 1–17 sexagesimoria of sun-nonagesimal elongation, from the *Makaranda* (BORI 546 f. 13r).

but it is not clear exactly what quantity the argument numbers 0–36 are supposed to represent.[17]

5.5.2 The *Makaranda* and the Saurapakṣa canon

The general identification of the *Makaranda* with Saurapakṣa parameters (see Section 1.2.3 and Appendix B.2) is all we currently know about its textual heritage. Occasional hints, however, connect it with earlier sources. For example, its table of longitudinal lunar parallax for each 6° arc-unit of longitudinal elongation between the sun and the nonagesimal, reproduced at the top of Figure 5.17, recalls the

[17] See Makaranda (1923, f. 28r). Pingree (1968, p. 45) interpreted them as follows: “This is a table of lunar eclipse half-durations in *ghaṭīs* for 1–36 units of 0;3°. Since one digit equals 0;5°, a *viṃśopaka* equals 0;36 digits and 36 *viṃśopakas* equal 21;36 digits.”

Fig. 5.18 Solar mean longitudinal displacement table from the *Makaranda* (BORI 446 f. 14v).

versified table of this quantity embedded in the *Karaṇakutūhala* of Bhāskara (Plofker Forthcoming).

The *Makaranda*'s offspring in the *jyotiṣa* canon are more clearly delineated than its ancestors. Most importantly, its canonical position was further established or enhanced by the circulation of a number of detailed commentaries expounding its use, sometimes with elaborate worked examples like the one quoted below from the *Makarandodāharaṇa* of Viśvanātha Daivajña, composed in Śaka 1540. This excerpt describes how to determine the mean position of the sun for a given date by assembling from the table reproduced in Figure 5.18 the separate mean longitude increments which correspond to the successive digits of the *vallī* or accumulated civil days since epoch at that date (Miśra 1982, pp. 71–72; Makaranda 1923, f. 39r):

> *udāharaṇaṃ śake 1534 vaiśākhaśukla 15 vallī 7 | 58 | 11 | 40 ravivāṭikāyāṃ catvāriṃśatkoṣṭhakād adhasthāṃkaḥ 6 | 34 | 14 | 27 ekādaśakoṣṭhakasthāṅkādyaṅkaṃ vihāya catvāriṃkāḥ 48 | 24 | 58 | 39 aṣṭapaṃcāśatkoṣṭhakād adhasthāṅkaḥ 38 | 58 | 20 | 34 saptamakoṣṭhakād adhasthāṃkaḥ 31 | 52 | 8 | 14 caturṇāṃ yoge jātāravivallī 125 | 49 | 41 | 54 ūrddhvāṃke ṣaṣṭitaṣṭe śeṣaṃ 5 | 49 | 41 | 54 idaṃ ṣaṭguṇaṃ jātā aṃśā 34 | 58 | 11 | 24 aṃśās triśadbhaktāḥ labdhaṃ rāśayaḥ evaṃ rāśyādyo raviḥ 1 | 4 | 58 | 11*
>
> Example. In Śaka 1534, Vaiśākha, bright fortnight, 15[th *tithi*], the *vallī* is 7 | 58 | 11 | 40. In the solar *vāṭikā* (mean motion) [table], the number placed below the 40th [argument] cell is 6 | 34 | 14 | 27. Disregarding the first number of the numbers placed [below] the 11th [argument] cell, the [next] four numbers are 48 | 24 | 58 | 39. [Similarly], the number placed below the 58th [argument] cell is 38 | 58 | 20 | 34; the number placed below the 7th [argument] cell, 31 | 52 | 8 | 14. When the sum of the four [numbers is computed], the *vallī* of the sun is produced: 125 | 49 | 41 | 54. When the upper [most significant] number is reduced with respect to sixties (i.e., expressed modulo 60), the remainder is 5 | 49 | 41 | 54. This multiplied by 6 produces degrees: 34 | 58 | 11 | 24. The degrees are divided by 30. The result is in zodiacal signs. In this way, the [longitude of] the sun in zodiacal signs [and so on] is 1 | 4 | 58 | 11.

Moreover, as indicated in Table 5.6, the highly popular *Makaranda* branched into numerous versions and recensions containing various subsets of its tables, sometimes with additional material that may not have originated with Makaranda. Like the earlier *Handy Tables* of Ptolemy, it seems to have become a family of comprehensive astronomical tables, distinctive by virtue of its terminology and correction procedures but permitting customized modifications to suit its users' different needs.

5.6 Major works of the Gaṇeśapakṣa

Probably the most renowned astronomer-mathematician of northern India after Bhāskara II was Gaṇeśa Daivajña son of Keśava, born in 1507 in Nandigrāma on the Mahārāṣṭra coast (Sarma 2010). Gaṇeśa's fame rests primarily on his immensely popular astronomy manuals, a *karaṇa* called *Grahalāghava* or *Siddhāntarahasya* ("Facility in planet[-computation]" or "Secret of *siddhāntas*," with epoch date Śaka 1442 = 1520 CE) and related calendric table texts, discussed in Section 5.6.3. But he is also known for several other compositions including descriptions of observational instruments, a table text on the computation of the *mahāpātas* (see Section 4.11.1), and a commentary on the premier Sanskrit arithmetic treatise of the period, Bhāskara's *Līlāvatī*. Moreover, he originated a modified version of Saurapakṣa parameters that became known as an astronomical school in its own right, the Gaṇeśapakṣa.

The best-known commentary on the *Grahalāghava* was composed apparently in the early seventeenth century by the Viśvanātha mentioned in Section 5.5.2, who was the son of a pupil of Gaṇeśa named Divākara. It enumerates Gaṇeśa's output as follows (Jośī 1994, p. 3):

kṛtvādau grahalāghavaṃ laghubṛhattithyādicintāmaṇī
satsiddhāntaśiromaṇeś ca vivṛtiṃ līlāvatīvyākṛtim ||
śrīvṛndāvanaṭīkikāṃ ca vivṛtiṃ muhūrttatattvasya vai
sacchrāddhādivinirṇayaṃ suvivṛtiṃ chandorṇavākhyasya vai ‖ 1 ‖

sudhīrañjanaṃ tarjaniyantrakaṃ ca sukṛṣṇāṣṭamīnirṇayaṃ holikāyāḥ ||
laghūpāyayātas tathānyān apūrvān gaṇeśo gurur brahmaṇirvāṇam āgāt ‖ 2 ‖

Having first made the *Grahalāghava*, the small and the great *Tithyādicintāmaṇi*, and a commentary on the true *Siddhāntaśiromaṇi*, [namely] an exposition of the *Līlāvatī*, a commentary on the excellent [*Vivāha*]-*vṛndāvana* [of Keśavārka], also a commentary on the *Muhūrtatattva* [of Keśava], a true determination of *śrāddha* etc. [the *Śrāddhavidhi*], and a good commentary on the *Chandorṇava*; and [the texts] *Sudhīrañjana* and *Tarjaniyantra* [on instruments] and the excellent determination of the *Kṛṣṇāṣṭamī* for the Holi [rites], by means of easy methods, [and] then likewise other incomparable [works], the teacher Gaṇeśa attained liberation [from life].

The relative abundance of surviving works and biographical information gives us quite a detailed picture of Gaṇeśa's overall place in *jyotiṣa*, at least compared to the little we know about most earlier *koṣṭhaka* authors.

5.6.1 *Grahalāghava/Siddhāntarahasya* of Gaṇeśa

This handbook, the "Ease of planet-[computations]" or "Secret of *siddhāntas*," is arguably the best known and certainly the most published Sanskrit astronomical manual of the late second millennium (Pingree 1970–94, A2.99–100, A5.72). Its most remarkable feature (besides the fact that according to the textual tradition, Gaṇeśa composed it at the age of 13!) is its systematic substitution of algebraic approximation algorithms and tabular interpolation for trigonometric procedures requiring tabulated sine values. Gaṇeśa's introductory and closing verses reflect his pride in this achievement (Jośī 1994, pp. 5–7, 342):

paribhagnasamaurvikeśacāpaṃ
dṛḍhaguṇahāralasat suvṛttabāhuṃ ||
suphalapradam āttanṛprabhaṃ tat
smara rāmaṃ karaṇaṃ ca viṣṇurūpam ‖ 1.2 ‖

yady apy akārṣur uravaḥ karaṇāni dhīrās
teṣu jyakādhanur apāsya na siddhir asmāt ||
jyācāpakarmarahitaṃ sulaghuprakāraṃ
kartuṃ grahaprakaraṇaṃ sphuṭam udyato 'smi ‖ 1.3 ‖ [. . .]

pūrve prauḍhatarāḥ kvacit kim api yac cakrur dhanurjye vinā
te tenaiva mahātigarvakubhṛducchṛṅge 'dhirohanti hi ||
siddhāntoktam (edition has *oktabh*) *ihākhilaṃ laghukṛtaṃ hitvā dhanurjye mayā*
tadgarvo mayi māstu kiṃ na yad ahaṃ tac chāstrato vṛddhadhīḥ ‖ 16.4 ‖

Consider Rāma, the form of Viṣṇu, making the bow of Śiva together with [its] string entirely destroyed, showing the destruction of the firm bowstring, with well-rounded arms, granting good results, appearing in human form. [Equivalently:] Consider this pleasing *karaṇa*, lovely as Viṣṇu [??], having the arc together with sine and versine entirely eliminated, showing the removal of the cumulative sine [values], with good circles and arcs giving accurate equations, with determined gnomon-shadows.

Even when the great [and] wise make *karaṇas*, in them there is no accomplishment [of the goal] when the sine-arc [combination] is excluded. Because of this, I am undertaking to make a very easy method omitting sine and arc computation, an accurate method for the planets. [. . .]

The former [and] greater [? full-grown, more mature?] [authors] sometimes to some extent accomplished without the arc and sine some [results] by means of which they ascended to the topmost mountain-peak of great pride. Here, what is stated in *siddhāntas* is simplified by me, entirely eliminating the arc and sine. Let there be no pride in that for me, since from their treatises I [gained] wisdom [??].

To accomplish his aim of eliminating all explicitly trigonometric computations, Gaṇeśa included versified tables of scaled values of *manda* and *śīghra* equations for the star-planets at every fifteen degrees of anomaly (see Sections 4.2.1 and 4.2.2), along with rational approximations to the *manda*-equation formulas for the sun and moon. All the standard astronomical tasks of *karaṇa* works are similarly carried out by such ingeniously adjusted formulas. Gaṇeśa's *manda*-equation list is represented as a numerical table by a *Grahalāghava* scribe in Figure 5.19 (compare Table 5.8).

Fig. 5.19 A numerical version of the versified scaled *manda*-equation table in a manuscript of the *Grahalāghava* (UPenn 390 1822 f. 8r).

Table 5.8 Tables of planetary *manda*-coefficients (left) and velocity-correction factors (right) from *Grahalāghava* 3.7–8, 11.

Planet	15° interval of anomaly κ_M						Velocity correction
	1	2	3	4	5	6	factor, *manda*
Mars	29	57	85	109	124	130	1/75
Mercury	12	21	28	33	35	36	1/5
Jupiter	14	27	39	48	55	57	1/30
Venus	6	11	13	14	15	15	2/5
Saturn	19	40	60	77	89	93	2/5

The *Grahalāghava*'s *manda*-equation algorithm for the sun (Jośī 1994, pp. 50–56, verse 2.2), based on an algebraic approximation to the sine function,[18] is equivalent to the following expression:

$$\mu = \frac{\left(20 - \frac{\kappa_M}{9}\right)\frac{\kappa_M}{9}}{57 - \frac{1}{9}\left(\left(20 - \frac{\kappa_M}{9}\right)\frac{\kappa_M}{9}\right)}$$

A somewhat similar rule is prescribed for computing the sun's *manda-gatiphala* or velocity correction, where κ_M is the arc of *manda*-anomaly reduced to the first quadrant (Jośī 1994, pp. 58–60, verse 2.4):

$$\overset{M}{\Delta}_v = \frac{\left(11 - \frac{90-\kappa_M}{20}\right)\left(\frac{90-\kappa_M}{20}\right)}{13}$$

[18]For this approximation in general see Section 2.1.8; the trigonometric rationales underlying the *manda*-correction procedures for longitude and velocity are discussed in Section 2.1.2. For the transformation of the algebraic sine approximation into the given μ-rule and the following velocity-correction rule, see Rao and Uma (2007, pp. S56–60, S65–68).

Another innovation in the *Grahalāghava*, reminiscent of techniques used in the table-text genre, is Gaṇeśa's introduction of a unit called the *cakra* "cycle" of 4016 days or approximately 11 years (see Section 4.1). Instead of computing mean longitude increments by multiplying mean daily velocities by the total *ahargaṇa* or elapsed days since the epoch date of the text, the user breaks down the elapsed interval into a number of completed *cakras* plus a number of days since the end of the last full *cakra*:

dvyabdhīndronitaśaka īśahṛt phalaṃ syāc
cakrākhyaṃ ravihataśeṣakaṃ tu yuktam ||
caitrādyaiḥ pṛthagamutaḥ sadṛgghnacakrād
digyuktād amaraphalādhimāsayuktam ‖ 4 ‖

The Śaka year diminished by 1442 is divided by 11; the result should be the so-called cycle. And the remainder multiplied by 12 is increased by [the months] beginning with Caitra, [and] increased by the intercalary months [produced by] the quotient with 33 from that cycle[-remainder] separately multiplied by 2 [and] added to 10.

For each planet, a number called its *dhruva* represents the 360°-remainder of its mean longitudinal increment during a *cakra*. Depending on the number of elapsed *cakras* since epoch, the appropriate multiple of the *dhruva* would be subtracted from the planet's specified epoch mean longitude or *kṣepaka*. Then the number of elapsed days in the current *cakra* multiplied by the daily mean longitudinal increment is added to the result to obtain the mean longitude at the desired time. Figure 5.20 depicts a *Grahalāghava* scribe's numerical tabulation of these quantities for Gaṇeśa's 4016-day *cakras*.

The *Grahalāghava*'s practical utility is further enhanced by chapters on simplified eclipse prediction employing calendar (*pañcāṅga*) data and a method for extending its use to desired dates preceding its own epoch of Śaka 1520. Overall, the work reflects Gaṇeśa's fascination with ingenious computational techniques, combining simplicity and efficiency for the user with sophisticated and often innovative constructions on the part of the compiler.

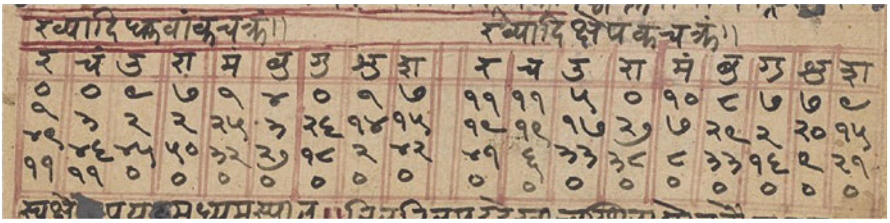

Fig. 5.20 Numerical versions of the versified tables of the planets' mean longitude multipliers in a manuscript of the *Grahalāghava* (UPenn 390 1822 f. 3r). Left: Planetary *dhruvas* or 360°-remainders of their mean longitudinal increments for 4016-day *cakras*. Right: Planetary *kṣepakas* or epoch mean longitudes.

Table 5.9 Table of planetary *śīghra*-coefficients from the *Grahalāghava* (Jośī 1994, p. 73, verses 3.1–5).

Planet	Number of 15° steps of *śīghra*-anomaly κ_S										
	1	2	3	4	5	6	7	8	9	10	11
Mars	58	117	174	228	279	325	365	393	400	368	249
Mercury	41	81	117	150	178	199	212	212	195	155	89
Jupiter	25	47	68	85	98	106	108	102	89	66	36
Venus	63	126	186	246	302	354	402	440	461	443	326
Saturn	15	28	39	48	54	57	57	53	45	33	18

Table 5.10 Scale factors for computing the *śīghra* velocity correction for each planet according to the *Grahalāghava* approximation (Jośī 1994, p. 90, verse 3.12).

Planet	Velocity-correction factor, *śīghra*
Mars	1/5
Mercury	6/5
Jupiter	1/3
Venus	1/4
Saturn	2/5

5.6.2 Table texts called *Grahalāghavasāriṇī*

The many works by various authors (generally anonymous) entitled *Grahalāghavasāriṇī*, as the name indicates, are attempts to cast Gaṇeśa's *karaṇa Grahalāghava* into table-text form for carrying out the planetary calculations not included in his strictly calendric *Tithicintāmaṇi* tables (see Section 5.6.3). They mostly consist of tables of mean longitude increments and orbital equations, arranged according to some pattern of the author's choice (see the example in Figure 2.13).

Table 5.11 compares the arrangements used for the mean longitude increment tables in various *Grahalāghavasāriṇī* manuscripts, which have been discussed in more detail in Section 4.1; samples of them are shown in Figures 4.3 and 4.5. Most of them employ Gaṇeśa's time-units of 4016-day *cakras* as well as days and years. Other manuscripts, such as the one partially reproduced in Table 4.4, may tabulate these increments for even shorter time-units.

Grahalāghavasāriṇī tables of the planetary *manda*- and *śīghra*-equations, on the other hand, generally just expand the *Grahalāghava*'s own short lists of scaled equation values (reproduced in Tables 5.8 and 5.9). This is done by reducing the listed values to standard units of degrees and interpolating between them for every degree of argument. Examples discussed in the previous chapter include the manuscripts shown in Figures 4.10 and 4.15. Some versions of the *Grahalāghavasāriṇī* interpolate these equation values more coarsely for argument intervals greater than one degree, as illustrated in Figures 4.11, 4.12, and 4.14.

In addition to these fairly standard data for finding mean and true planetary positions, *Grahalāghavasāriṇī* manuscripts may sometimes tabulate other quantities such as the *Grahalāghava*'s oblique ascensions (e.g., in Figure 4.22) or astrological

Table 5.11 Mean longitude increment table arrangements in various *Grahalāghavasāriṇī* manuscripts, with starting dates and numbered *cakra* periods (beginning with *cakra* 0 at the *Grahalāghava*'s own epoch date) for which they provide initial position offsets or *dhruvas*.

Units	Combination of units	Manuscripts employing this combination of units
	1–9 single days	CSS d. 774g; 17 March 1531, *cakra* 1
	1–9 10-day intervals	IO 2083b; 13 March 1553, *cakra* 3
	1–9 100-day intervals	RAS Tod 36a; 27 Feb 1641, *cakra* 11
	1–4 1000-day intervals	Plofker 25; 13 March 1553, *cakras* 3–20
	1–30 4016-day *cakras*	
Years/days/*cakra*	1–30 single days	RAS Tod 57; 18 March 1520, *cakra* 0
	1–11 "ideal" months	Plofker 26; 18 March 1520, *cakra* 0
	1–11 "ideal" years	RAS Tod 36d; 13 March 1553, *cakra* 3
	0–30 4016-day *cakras*	CSS d. 774g
	1–67 single days	UPenn 390 1824; 29 March 1707, *cakras* 17–33
	1–17 4016-day *cakras*	
Years/days	1–11 10-year intervals	Smith Indic 26; 2 April 1754
	1–10 single years	
	1–3 100-day intervals	
	1–9 10-day intervals	
	1–9 single days	

phenomena (e.g., in MS Smith Indic 26 ff. 5v–6v). Other closely related table texts following the *Grahalāghava*, usually by named authors, are known by other titles differing slightly from the general designation *Grahalāghavasāriṇī*: e.g., the *Grahaprabodhasāriṇi* of Yādava (1663) and the *Grahasāraṇī* of Gaṅgādhara (1630) (see Appendix A).

5.6.3 *Tithicintāmaṇi* and *Bṛhattithicintāmaṇi* of Gaṇeśa

As noted by Viśvanātha in the commentary quoted above, after the *Grahalāghava* Gaṇeśa composed two table texts for use in constructing *pañcāṅgas*. The shorter and far more popular one was entitled *Tithicintāmaṇi* (sometimes called *Laghutithicintāmaṇi* or *Tithyādicintāmaṇi*) and the other *Bṛhattithicintāmaṇi*: i.e., the lesser and greater "jewels of thought about the *tithi* (etc.)."[19] Each work consists of a brief verse text (largely identical, as the comparison of their content in Table 5.12 makes clear) accompanied by tables. Their epoch dates, enunciated in the initial verses quoted and translated in Table 5.13, are Śaka 1447 = 1525 CE for the *Tithicintā-*

[19] Examples of tables from the former are shown in Figures 4.56, 4.57, and 4.58, and from the latter in Figures 3.4 and 3.14. See Pingree (1970–94, A2.100–103, 104–105) and Ikeyama and Plofker (2001).

Table 5.12 The content and ordering of the verses in Gaṇeśa's *Bṛhattithicintāmaṇi* (BTC) and *Tithicintāmaṇi* (TC). The equals sign "=" signifies that the two designated verses are entirely or very nearly identical in their wording.

Topic	*Bṛhattithicintāmaṇi* verse number		*Tithicintāmaṇi* verse number	
Invocation/introduction	1		1	
Increments for current year	2			
Lord of the year	3	(= TC 2)	2	(= BTC 3)
Longitude-difference correction	4	(= TC 3)	3	(= BTC 4)
Epact	5	(= TC 4)	4	(= BTC 5)
Anomalistic time offsets	6		6	
Synodic time offsets	7	(= TC 5)	5	(= BTC 7)
Corrections to anomalistic time offsets	8		7	
Weekday time offsets	9	(= TC 8)	8	(= BTC 9)
Negligible differences in results	10			
Increments for current year	11	(= BTC 2)		
Modified weekday time offset			9	
Use of weekday tables	12	(= TC 10)	10	(= BTC 12)
Use of correction tables	13–14		11–13	
Excess and deficiency of day	15	(= TC 14)	14	(= BTC 15)
Determining *nakṣatra* and *yoga*	16	(= TC 15)	15	(= BTC 16)
Solar entrance into signs	17	(= TC 16)	16	(= BTC 17)
Intercalary/omitted months	18	(= TC 17)	17	(= BTC 18)
Parameters of *pakṣas*	19	(= TC 18)	18	(= BTC 19)
Accurate *nakṣatra* computation	20			
Time corrections	21–22			
Use of the divisor	23–24			
Omitted and additional *tithis*	25–27			

maṇi and Śaka 1474 = 1552 CE for the *Bṛhattithicintāmaṇi* (which, however, also employs the 1447 date for some of its calculations).

Chronology and commentaries. The internal development of Gaṇeśa's adaptation of astronomical computation to the *koṣṭhaka* format in these two works remains somewhat obscure. Judging from their epoch dates it appears most probable, and is generally asserted (Pingree 1970–94, A2.100, 104), that the *Tithicintāmaṇi* was composed before the *Bṛhattithicintāmaṇi*. Yet Gaṇeśa clearly implies in the first verse of the former that he had previously produced a "thought-jewel" relating to the *tithi* etc., whose extent intimidated some unfortunate users, for whose sake he is now compiling the *Tithicintāmaṇi* instead. This is the explanation offered by the abovementioned *Grahalāghava* commentator Viśvanātha who also composed one of the many surviving commentaries on the *Tithicintāmaṇi*, using in his *udāharaṇa* or worked examples the year Śaka 1556 = 1634 CE (Āpaṭe 1942b, p. 1):

Table 5.13 Opening verses of Gaṇeśa's *Tithicintāmaṇi* and *Bṛhattithicintāmaṇi*.

Tithicintāmaṇi	*Bṛhattithicintāmaṇi*
yaś cintāmaṇir aṅkalekhyabahulo 'tyalpakriyo matkṛtas	*natvā brahmaharīśvareśvarasutāryārkādikheṭān dvijo*
tithyādyāvagamaprado 'sya sukhino ye lekhane bhīravaḥ ||	*'hno 'rdhenābdadinādisiddhidam ahaṃ tithyādicintāmaṇim* ||
tatprītyai laghum alpakṛtyam amalaṃ tithyādicintāmaṇiṃ	*kurve 'tyalpakṛtīṃ vidhāya bahulaṃ yatnaṃ gaṇeśaḥ kṛtī*
vighneśārkamukhān praṇamya kurute śrīmad gaṇeśaḥ kṛtī ‖1‖	*pūrvābhyo 'ticamatkṛtiṃ tithikṛtiṃ paśyantu sujñā iha* ‖1‖
A jewel of thought which [was] made by me [is] abundant in numbers and figures, very small in operations [and] giving comprehension of the *tithi* and so on. To satisfy those [people] who are timid about the writing of this pleasant [work], having saluted [the deity] Gaṇeśa and [the planets] beginning with the sun, the skillful Gaṇeśa makes a jewel of thought about the *tithi* etc., which is easy, simple, and without errors.	Having honored Brahman, Viṣṇu, Śiva, Śiva's son [Gaṇeśa], and the planets beginning with the lord sun, I, a *brāhmaṇa*, make a jewel of thought [about] the *tithi* etc. determining the year, day, etc. in half a day. The skillful Gaṇeśa having made great efforts [to render it] very small in operations, let the knowledgeable ones view the determination of the *tithi* here as much more remarkable than previous [methods].
vyagayugamanuśākaḥ syāt samaugho hato 'yaṃ	*śāko 'bdhyagendrarahito rasavedabhaktaḥ*
svarakhakhakubhir āpto nāgaśatyābdapaḥ syāt ||	*śeṣe 'bdapādaya ime phalapaṅktigāḍhyāḥ* ||
dyumukha iha paleṣu tryabdhihṛdvarṣayuktaḥ	*prākpaścime vicaraṇāntarayojanāni*
śrutibhir iṣusamudrair bhair yutaḥ saptataṣṭaḥ ‖2‖	*bhogābdapeṣu ca palāni kuru svamasvam* ‖2‖
The Śaka year minus 1447 should be the number of years [since epoch]. This multiplied by 1007, divided by 800, should be the year-lord beginning with days, [when] increased in its *palas* by the year-[number] divided by 43, added to 4;45,27, and divided by 7.	The Śaka year minus 1474 [is] divided by 46. The year-lord etc. in [table entries for] the remainder are augmented by [entries] in tables for the quotient. And make the *yojanas* of displacement-interval east or west [into] *palas* [applied] +/− to the weekday-offset and year-lord [etc.?].
	vyagayugamanuśākaḥ syāt samaugho hato 'yaṃ
	svarakhakhakubhir āpto nāgaśatyābdapaḥ syāt ||
	dyumukha iha paleṣu tryabdhihṛdvarṣayuktaḥ
	śrutibhir iṣusamudrair bhair yutaḥ saptataṣṭaḥ ‖ 3 ‖
	[same as *Tithicintāmaṇi* 2]

ya iti | *śrīmadgaṇeśas tithyādicintāmaṇiṃ kuruta ity anvayaḥ* | *tithir ādir yeṣāṃ te tithyādayaḥ* | *ādiśabdena nakṣatrayogādayaḥ* | *teṣāṃ cintā* | *tasyā maṇir iva maṇir yasya saḥ* | [...]

tithyādyāvagamapradas tithyādijñānapradaś cintāmaṇir bṛhaccintāmaṇir iti yāvat | nanv asyāpi cintāmaṇer mahattvāl lekhane bhīrutvalakṣaṇo doṣas tadavastha evety ata āha [*laghum iti*] | *laghutvāt kaṭhino bhaviṣyatīty ato viśinaṣṭi | alpakṛtyam iti | svalpeti kartavyatākam ity arthaḥ || etadānītasya tithyādeś ca saṃvādān nātra kaścid doṣo 'stīty āśayenāha amalam iti | na ca laghunā svalpakriyaitattithicintāmaṇinaiva samīhitatithyādijñānalābhād bṛhaccintāmaṇer evānarthakyam | etadapekṣayā svalpakriyātvena tasyāpi sārthakyāt | tasmāl lekhanaśaktānām arthe 'tyalpakriyo bṛhaccintāmaṇir nirmitaḥ | lekhanāśaktānāṃ hitāya svalpakriyo laghucintāmaṇir iti dvayor api sārthakyam iti yāvat* ||

'Which…': The learned Gaṇeśa makes the *Tithyādicintāmaṇi*: this is the word-sequence. The *tithi* is the first of those which are 'the *tithi* and so on'. By the word 'first' the *nakṣatra*, *yoga*, etc. [adi again!] [are meant]. 'Thought' of them: which [is] its jewel, [i.e.,] jewel-like […]

'Giving comprehension of the *tithi* and so on': giving knowledge about the *tithi* and so on; 'a jewel of thought': as much as [to say], the 'great jewel of thought' [*Bṛhattithicintāmaṇi*]. Indeed, the characteristic of timidity in [regard to] writing on account of the great [size] of this *cintāmaṇi* is a fault; in [respect to] this very situation hence he says ['easy']. Thus from [the concept of] easiness he clarifies that it [i.e., the underlying work?] will be difficult. 'Simple', i.e., little in [tasks] to be done: this is the meaning. And because of the correctness of the *tithi* etc. calculated by this [work], here there is no fault at all: with this intention he says 'without errors'. And [yet] there is not, because of the acquisition of the desired knowledge of the *tithi* etc. by means of just this easy simple *Tithicintāmaṇi*, uselessness of the *Bṛhattithicintāmaṇi*. With regard to this: Because of the usefulness of that [the *Bṛhattithicintāmaṇi*] too, by means of [its] simplicity. Therefore, for the sake of [those] competent in writing, the exceedingly simple *Bṛhattithicintāmaṇi* is constructed, [and] for the benefit of [those] incompetent in writing, the very simple *Laghutithicintāmaṇi*: as much as [to say], thus there is usefulness of both.

Viśvanātha's brother Viṣṇu Daivajña, who produced the only known commentary (date unknown) on the more obscure *Bṛhattithicintāmaṇi*, seems to corroborate the claim of its primacy. His commentary on its first verse includes the following remarks, whose wording in places is strongly reminiscent of Viśvanātha's (Āpaṭe 1942a, pp. 1–2):

[…] *śrīmatkeśavadaivajñasuto gaṇeśanāmā gaṇitaskandhaikabhūtaṃ grahalāghavākhyaṃ karaṇam ādau kṛtvedānīṃ pañcāṅgasādhakam apūrvam atyalpakriyaṃ bṛhaccintāmaṇisaṃjñaṃ granthaṃ cikīrṣur*

[…]

natveti | tatra kurva ity anvayaḥ | kaḥ | ahaṃ gaṇeśaḥ | kam | tithyādicintāmaṇim | tithir ādir yeṣāṃ te tithyādayaḥ | ādiśabdena nakṣatrayogādayaḥ | teṣāṃ cintā | tasyāṃ maṇir iva maṇiḥ | […]

nanvanyatithisādhaneṣu satsu kimartham ayaṃ granthaḥ kṛta iti granthasyāsya vaiyarthyaprasaṅgāpattir ityāśaṃkyāha | […]

anyagranthebhyaḥ pañcāṅgaṃ kartum upaviṣṭeṣu gaṇakeṣu dinatrayeṇāpi pañcāṅgasiddhir na syāt | yatas tatra mahatī kriyā | teṣu mandadhiyo 'tikleśamāpnuvanti | ataḥ kṛpālur ācāryo 'muṃ kṛtavān |

[…]

[…] Keśava's son, Gaṇeśa by name, having first made the *karaṇa* called *Grahalāghava* [having] the varieties of calculation [united] in one form, now desiring to make a book called *Bṛhaccintāmaṇi* [for] producing *pañcāṅgas*, unprecedented [and] exceedingly simple […]

'Having honored. . .', then 'I make. . .'; this is the word-sequence. Who? I, Gaṇeśa. What? The *Tithyādicintāmaṇi*. The *tithi* is the first of those which are 'the *tithi* and so on'. By the word 'first' the *nakṣatra*, *yoga*, etc. [are meant]. 'Thought' of them, in it [there is] a jewel, [i.e.,] jewel-like. [. . .]

Indeed, in [light of the fact that there are] other true determinations of the *tithi* what is the purpose [that] this book is made? Thus the fault of possibility of uselessness of this book. Having thus supposed [a possible objection], he states [. . .]

In [the case of] astronomers undertaking to make a *pañcāṅga* from other books, even in three days there's no completion of the *pañcāṅga*, because great labor [is required] there. The slow-witted among them undergo extreme trouble. Hence, the compassionate teacher was making this [book]. [. . .]

On the other hand, if the *Bṛhattithicintāmaṇi* was indeed composed first, it is unclear why the *Tithicintāmaṇi* was assigned an earlier epoch date and why the *Bṛhattithicintāmaṇi* uses both Śaka 1447 and Śaka 1474 in different algorithms. The discussion in the following section sheds a little more light on their interconnections; however, a conclusive resolution of this issue will probably have to await a thorough comparative analysis of the textual and commentarial traditions of the two works.

Computational methods and the *upakaraṇas*. Both of Gaṇeśa's table texts share the fundamental goal of enabling *pañcāṅga*-makers to determine the exact day and time of the initial moment of any one of the time-units *tithi*, *nakṣatra* and *yoga* over the course of a given year. As indicated in Table 5.12, the bulk of the verse instructions are exactly the same in the two works, but some interesting differences appear.

The *Tithicintāmaṇi* directs the user to compute various offsets in time (called *kendra*) from the start of solar, synodic, and lunar anomalistic cycles to pinpoint the initial moments of the first time-units in the year (verses 2–9). Then these offsets are manipulated to produce the arguments with which to look up in the accompanying tables the approximate weekday/time and correction terms for any desired subsequent time-units during that year (verses 10–13). The last five of the text's 18 verses contain general advice for *pañcāṅga*-makers on allowing for intercalation, omitted days, etc.

The algorithms for computing the initial offsets for a given year are based on the parameters of the relevant astronomical school or *pakṣa*. Gaṇeśa explains in *Tithicintāmaṇi* 18 (=*Bṛhattithicintāmaṇi* 19) how he combined and modified different *pakṣa* parameters for best agreement with observation, and notes that results in accordance with other *pakṣas* could be produced by changing the offsets: *yadi cālitopakaraṇais tatpakṣajā syāt tithiḥ* "when [computed] with modified *upakaraṇas*, the *tithi* should be appropriate for that [other] *pakṣa*."

This word *upakaraṇa* appears to be Gaṇeśa's technical term for a time offset pertaining to the initial time-unit in a year.[20] Viśvanātha's and Viṣṇu's introductions to the second verses of their respective base-texts echo Gaṇeśa's use of it:

Viśvanātha on *Tithicintāmaṇi* 2:

atha pañcāṅgakaraṇībhūtopakaraṇasādhanaṃ āha | tatrādāv abdapasādhanam āha |

[20] Ikeyama and Plofker (2001) somewhat inaccurately interpret this term as "auxiliary tables"; compare the discussion of the nearly identical term *upakarṇa* in Montelle and Plofker (2013, p. 14).

Now he states the determination of the *upakaraṇas* obtained for the construction of *pañcāṅgas*. There he first states the determination of the lord of the year.

Viṣṇu on *Bṛhattithicintāmaṇi* 2 Āpaṭe (1942a, p. 2):

atha pañcāṅgakāraṇībhūtopakaraṇajñānaṃ tatsaṃskārajñānaṃ ca siṃhoddhatayā vakṣyati |

Now he will say the knowledge of the *upakaraṇas* obtained for the construction of *pañcāṅgas* and the knowledge of their correction, with a *siṃhoddhatā* [verse].

The components of the *Bṛhattithicintāmaṇi* are generally less clear to us because as of this writing we have not seen its accompanying tables. But based on published and manuscript material, we venture to describe its overall construction as follows.

The times of true *tithis*, *nakṣatras* and *yogas* are tabulated for 27 cycles of approximately 1 year in length. (Manuscript catalogue descriptions suggest that these tables typically take up around 30–40 folios, which substantiates Gaṇeśa's and Viśvanātha's remarks about the intimidating extent of the writing required to generate a copy of this work!) The user need only find the position of the desired year in a 46-year cycle, look up the year-initial offsets and adjust them for the local distance east or west of the prime meridian as per *Bṛhattithicintāmaṇi* 2/11, and go into the tables with the desired time-units as per *Bṛhattithicintāmaṇi* 12–14. The subsequent five verses contain the same general techniques for calendar adjustment given in *Tithicintāmaṇi* 14–18, and the final eight verses specify various corrections and refinements.

But the seven intervening verses in *Bṛhattithicintāmaṇi* 3–9, equivalent to *Tithicintāmaṇi* 2–8, are apparently not directly employed in obtaining the desired results. They seem to be explained by Viṣṇu as supplementary material for better understanding the *upakaraṇa* tables (Āpaṭe 1942a, p. 3):

upakartuṃ svīyakartuṃ yogyāni upakaraṇāni pañcāṅgasādhanāyeti śeṣaḥ | *aho atrācāryeṇa vedādrīndratulyam iṣṭaśākaṃ prakalpya vakṣyamāṇaprakāreṇa sukhārtham upakaraṇāni kṛtāni* | *tadupapattijñānaṃ vinā katham etāni kāryāṇi* | *atas tadupapattijñā-nārthaṃ ślokāṣṭakam ādau vyākhyāyate* | *nanv evaṃ cet tarhi granthakṛtā kim etādṛśaṃ kṛtam iti cet tad ucyate* | *pañcāṅgaṃ kartum upaviṣṭānāṃ gaṇakānām upakaraṇānām evāpekṣā kim upapattyā* | *ataḥ siddhāny upakaraṇāni grāhyāṇīty ādāv uktam* | *bhavati/u? etāny evā santu* | *kiṃ taiḥ prayojanam iti cet* | *etāni yadā kadācid bhraṣṭāni tadā kathaṃ kāryāṇy atas tāny apyuktāni* | *ataḥ siddhopakaraṇopapattijñānārthaṃ vakṣyamāṇopakaraṇopapattis tāvaj jñāyate* |

The *upakaraṇas* are useful to accomplish, to make one's own; that is, for the sake of determining the *pañcāṅga*. Now by the teacher, having determined the desired Śaka [year] equal to 1474, the *upakaraṇas* are made for convenience' sake by a method to be stated. Without knowledge of the rationale of that, how are these [*upakaraṇas*] to be made? Hence for the sake of the knowledge of its rationale, eight verses are first explained. If [you think] 'Indeed if [it is] thus, then what of this sort is done by the book author?', it is explained. [?] [There is] need of *upakaraṇas* indeed [on the part] of astronomers undertaking to make a *pañcāṅga*, [but] what [is served] by a rationale? Hence correct *upakaraṇas* are to be obtained: thus in the beginning [it] is stated. Let just these be [?]. If [you think] 'What purpose [is served] by them?': If at some time these [*upakaraṇa* tables] are lost, then how are they hence to be made [as] stated? Hence, for the sake of knowing the rationale of the correct *upakaraṇas*, the forthcoming [? to be stated] rationale of the *upakaraṇas* is now known.

Then *Bṛhattithicintāmaṇi* 10, the final one in the set of these "eight verses" giving the algorithms underlying the *upakaraṇas*, is glossed by Viṣṇu as follows (Āpaṭe 1942a, p. 11):

atha pūrvakṛtopakaraṇānāṃ kadācit svalpam antaraṃ bhavet tathāpi tithyādisāmyam evety anuṣṭubhāha |

> *pūrvopakaraṇebhyo 'tra kadācit kiṃcid anyathā* |
> *tathāpyebhyo 'pi tithyāditulyam evāgamiṣyati* ‖10‖

spaṣṭārthaṃ | *evam anena prakāreṇopakaraṇasya sādhanaṃ kṛtvācāryeṇa sukhārthaṃ mandānāṃ kleśaparihārārthaṃ ca vedādrīndra 1474 tulyam iṣṭaṃ śākaṃ prakalpya viracayya likhitāni* | *tatkaraṇaṃ pūrvam evoktam* |

Now, there could be a small difference at some time from the earlier-made *upakaraṇas*. Nevertheless, there *is* agreement in the *tithi* etc.; thus he says with an *anuṣṭubh* [verse]:

> Here sometimes there is something different with respect to the earlier *upakaraṇas*.
> Nonetheless, [the user] will obtain equality of the *tithi* etc. with respect to them.

The meaning is clear. By the teacher, having made thus with this method the determination of the *upakaraṇa*, [then] having considered [and] constructed the desired Śaka year equal to 1474, for the sake of easiness and for alleviating the toil of the slow-[witted], [the *upakaraṇa* values are] written [in tables]. The procedure for that is stated just previously.

It is perhaps not surprising that this somewhat confusing combination of techniques in the *Bṛhattithicintāmaṇi* proved less attractive to users than the more compact *Tithicintāmaṇi*, regardless of which was composed first. Viṣṇu's commentary is interesting here not only because it illuminates some aspects of these works and their users' expected reactions to them, but for its arguments in favor of "rationales" for table texts, emphasizing and explaining techniques for constructing tables as well as recipes for using them.

5.7 *Jagadbhūṣaṇa* of Haridatta

During the reign (1628–1652 CE) and evidently under the patronage of Jagatsiṃha I of Mewar in Rājasthān, Haridatta son of Harajī compiled the first known set of Sanskrit astronomical tables in the "cyclic" form (see Section 2.3.2). This work, entitled *Jagadbhūṣaṇa* "Ornament of the world/of Jagat[siṃha]" in compliment to his patron, comprises several hundred tables and an accompanying text sometimes referred to as the *Jagadbhūṣaṇa-prabandha*, in five chapters consisting of about 130 verses and several short prose passages.[21] It begins (MS BORI 399 f. 1r):

[21] See, e.g., Pingree (1968, pp. 55–9), Pingree (1973, pp. 141–142), Montelle (2014), and Pingree (2003, pp. 83–87). Examples of the *Jagadbhūṣaṇa*'s tables appear in Figures 3.3, 3.13, 4.17, 4.19, and 4.31. Numerical evidence gleaned from tabular data in some of the manuscripts suggests that the tables were computed for a latitude of $\phi = 24°$, roughly corresponding to Ujjain in Madhya Pradesh (see Figure 4.31).

praṇamyādidevaṃ vibhuṃ nārasiṃhaṃ
grahān bhāskarādyān gaṇeśaṃ bhavānīm ||
graharkṣādititiyādipañcāṅgasiddhyai
karomi prabandhaṃ jagadbhūṣaṇākhyam || 1 ||

Having honored Viṣṇu the all-pervading, first among the deities, the planets beginning with the sun, Gaṇeśa, [and the goddess] Bhavānī, I make a composition called *Jagadbhūṣaṇa* for establishing [the positions of] planets and stars etc. [as well as] a *pañcāṅga* [for] the *tithi* etc.

After about a dozen verses describing and lauding the lineage and qualities of Jagatsiṃha, Haridatta reintroduces his own work, tying its epoch to his patron's reign (MS BORI 399 f. 1r–v):

rājan śrīsabalābhidhānanṛpatir māṃ proktavāṃś cintyatāṃ
pṛthvyāṃ yena bhavet prabhoḥ sakalagā kīrtīḥ sadāsthāyinī ||
tenāhaṃ haribhaṭṭaharidattākhyas tvadīye śake
paṃcāṃgagrahaṇādilabdhisahitāṃ siddhiṃ khagānāṃ bruve || 16 ||

śālivāhanamahīpatiśāke pūrṇabāṇaviṣayendusamāne
śrījagannarapatir nijaśākaṃ bhūridānajanitaṃ kurute sma || 17 ||

śrīmadvikramasaṃjñakasya dharaṇībhartuḥ śarāgnīndubhiḥ
hīne syāt kila śālivāhanadharādhīśasya śākaḥ sphuṭam ||
tasmin pūrṇaśarendriyeṃdurahite śrīmajjagatsiṃhabhū-
bhartuḥ śākavaraḥ khacandrarahito 'bdānāṃ gaṇo granthajaḥ || 18 ||

O king, the ruler called Lord Sabala has declared [to?] me: let [him] be thought of, by whom on earth the splendor of the king should be complete [and] everlasting; through him I, Haribhaṭṭa-Haridatta by name, state the determination of the planets along with finding the calendar and eclipses and so on, for your Śaka year.

When the Śaka year of the ruler Śālivāhana [is] equal to 1550, the ruler Lord Jagatsiṃha makes his own Śaka [era] produced by cutting off the greater [Śaka year].

When [the desired Saṃvat year] of the protector of the world called Lord Vikrama is diminished by 135, [that] should be correctly the Śaka year of the world-lord Śālivāhana. When that [Śaka year] is diminished by 1550, [it is] the supreme [regnal] Śaka year of the protector of the world, Lord Jagatsiṃha. [That regnal year] diminished by 10 is the accumulation of years pertaining to [the epoch of this] book.

This elaborate praise of Jagatsiṃha is reinforced by a similar encomium repeated in every chapter colophon of the *Jagadbhūṣaṇa*:

iti śrīgaṇakacakracūḍāmaṇibhaṭṭaśrīharajītatputranūjaharadattaviracite nikhilarāja-maṇḍalīmaulimaṃḍanapratāpanikaravidhvastaripumaṃḍalavividhavidyāvinodarasika-mahārājādhirājamahārājaśrījagatsiṃhanāmāṃkite jagadbhūṣaṇe [. . .]

Thus in the *Jagadbhūṣaṇa* named after the ruler of supreme rulers Lord Jagatsiṃha, the crest-jewel of the entire body of kings, whose adversaries are all destroyed, the essence of brilliance [who is] devoted to various branches of learning and arts; [which was] composed by Haridatta, son of the learned revered Harajī, the crest-jewel of the circle of excellent mathematicians [. . .]

The astronomical material constituting most of the *Jagadbhūṣaṇaprabandha* is summarized in Table 5.14, and its accompanying table content in Table 5.15. As these overviews indicate, the *Jagadbhūṣaṇa* partakes to some extent of the hybrid

Table 5.14 Contents of the *Jagadbhūṣaṇa-prabandha*.

Chapter	Title	Number of verses	Contents
1. Sun and moon	*sūryacandrasphuṭī-karaṇa* "true sun and moon"	1–36	Invocation/dedication: epoch date Using tables to find initial offsets Civil days, *avadhis*, solar/lunar positions The *tithis*, *nakṣatras*, *yogas* Time corrections
2. Planets	*kujādisphuṭatā* "true planets"	1–8, prose passages	Cycle numbers Using tables to find true velocity Direct/retrograde motion Lunar node Computations before/after epoch
3. Locality-based quantities: time, direction, place	*vimiśrasphuṭatā* "correct miscellaneous [computations]	1–23	Ascensions, shadows Verse table of sine differences Verse table of solar declination Local latitude Appearances of Canopus
4. Eclipses	*ravīndvor grahaṇa* "eclipses of sun/moon"	1–33	Importance of eclipses Eclipse limits, lunar latitude, disk size Eclipse magnitude, duration, deflection, parallax, colors Graphical projection; techniques for observing eclipses
5. Calendar	*dinādisphuṭatā* "true day etc."	1–25	Annual offsets for *tithis*, *nakṣatras*, *yogas* Constructing the *pañcāṅga* Sixty-year cycle Astrological predictions

karaṇa/*sāraṇī* nature seen in some earlier works such as the *Rājamṛgāṅka*. Versified algorithms, and in some cases short versified lists of values of functions such as sines or declinations, are as essential to its use as its enormous numerical tables are.

The primary innovative feature of the *Jagadbhūṣaṇa* is the cyclic arrangement of its planetary true longitude tables. As discussed in Section 2.3.2, cyclic tables enumerate these true positions depending only on the time since the start of some recurring cycle, rather than on a combination of the planets' mean and synodic positions as in mean-to-true tables. Each planet has a cycle of different length, based on its own period relation of some integer number n of revolutions in true longitude completed in some integer number y of years. The advantages of this arrangement include the (theoretically) eternal applicability of the tables, as the planets return (almost exactly) to the same true positions after the same constant time increments,

Table 5.15 Tables of the *Jagadbhūṣaṇa*.

Tabulated quantity	Tables in cycle	Argument
True longitude and velocity; synodic phenomena		
Mars	0–78	1–27 *avadhis*
Mercury	0–45	1–27 *avadhis*
Jupiter	0–82	1–27 *avadhis*
Venus	0–226	1–27 *avadhis*
Saturn	0–58	1–27 *avadhis*
Annual lord of the year increment (weekday/time)		0–88 years
Annual epact increment (*tithis*)		0–121 years
Mean longitudinal displacement		
Moon		0–121 years
		1–27 *avadhis*
		1–13 days
		1–60 *ghaṭīs*
Lunar anomaly		0–42 years
		1–27 *avadhis*
		1–13 days
		1–60 *ghaṭīs*
Lunar equation and velocity correction		0–30 3° arc-units of anomaly
Mean time correction functions		1–27 *avadhis*
Solar true longitude and velocity		1–27 *avadhis*
Length of daylight		1–27 *avadhis*
Mean longitudinal displacement		
Lunar node		0–92 years
		1–27 *avadhis*
Solar entries into zodiacal signs		12 zodiacal signs, 27 *nakṣatras*

Table 5.16 The lengths of Haridatta's cycles for the planets.

Planet	Cyclic period (years)	Revolutions per period
Mars	79	42
Mercury	46	46
Jupiter	83	7
Venus	227	227
Saturn	59	2

and the convenient replacement of the double-argument combination of planetary and solar mean longitudes with the single input of time since epoch (modulo cycle length).

The period relations employed by Haridatta are listed in Table 5.16. He calls these periods *cakras*, adopting the terminology used by, e.g., Gaṇeśa for time cycles longer than a year. In fact, as illustrated by the examples shown in Table 5.17, much of the *Jagadbhūṣaṇa-prabandha* is adapted or borrowed verbatim from

Table 5.17 Appropriations from verses of other texts identified in the *Jagadbhūṣaṇa*.

Chapter and verse	Topic	Source
1.20	Civil days	*Candrārkī* 11
1.24–26	True moon	*Candrārkī* 16–17
1.27	Mean motion of moon/anomaly	*Candrārkī* 14
1.28	Reduction of arcs to first quadrant	*Karaṇakutūhala* 2.4
1.33	*nakṣatra* and *yoga*	*Candrārkī* 23
3.2–3	Right ascensions	*Karaṇakutūhala* 3.1–2
3.4–5	Noon equinoctial shadow	*Grahalāghava* 2.5
3.6	Ascendant at given time	*Karaṇakutūhala* 3.3
3.7	Divisions of ecliptic	*Grahalāghava* 4.6
3.9	Half-length of daylight; hour angle	*Karaṇakutūhala* 3.7
3.11–16	Time for given ascendant	*Karaṇakutūhala* 3.3–6
3.21–22	Terrestrial latitude	*Karaṇakutūhala* 3.16
3.23	Heliacal rising/setting of Canopus	*Karaṇakutūhala* 6.15
4.3	Latitude of the moon	*Grahalāghava* 5.2
4.4–5	Angular diameters	*Karaṇakutūhala* 4.7–8
4.6	Magnitude	*Karaṇakutūhala* 4.9
4.7	Half-duration	*Karaṇakutūhala* 4.10
4.8–10	Phases of eclipse	*Karaṇakutūhala* 4.11–13
4.13–15	Deflection	*Karaṇakutūhala* 4.14–16
4.16	True lunar latitude at impact/release	*Karaṇakutūhala* 4.17
4.18	Zenith distance	*Grahalāghava* 6.1
4.19–21	Longitudinal parallax	*Grahalāghava* 6.2–3
4.23–25	Corrected eclipse phases	*Grahalāghava* 5.5
4.25	Visibility of an eclipse	*Karaṇakutūhala* 5.9
4.27	Colors	*Grahalāghava* 5.6
4.28–32	Graphical projection	*Karaṇakutūhala* 4.18–23
5.4	*dhruva* of *tithi*, *nakṣatra*, and *yoga*	*Tithicintāmaṇi* 5
5.5–6	*bhoga* of *tithi*, *nakṣatra*, and *yoga*	*Tithicintāmaṇi* 8

these predecessors, primarily Bhāskara II, Dinakara and Gaṇeśa. Moreover, his parameters for the lord of year, lunar mean motion and lunar anomaly are those of the *Mahādevī* (Melnad Forthcoming).

None of these earlier Sanskrit works, however, is directly responsible for the fundamental and ingenious modification to the mean-to-true table structure represented by the cyclic arrangement. It is plausible that Haridatta learned of the planetary period relations underlying his *cakras* from some source ultimately derived from the Greco-Islamic astronomy tradition. In particular, both the *Jagadbhūṣaṇa* and the *zīj* of al-Zarqālī (epoch 1088 CE) use 83 years as the cycle length for Jupiter, as opposed to the 71 years of the Ptolemaic model. It is also not impossible that the south Indian *vākya* system of versified repeating patterns of true longitude played a role in Haridatta's innovation, but a thorough analysis of the text will be required to sort out all the potential sources of his inspiration (Melnad Forthcoming).

The abovementioned slight inexactitude in the planetary period relations means that the tables are not in fact purely cyclic: that is, computing positions and synodic phenomena for any time more than one *cakra* beyond the epoch date requires small, mostly linear, additional corrections (see Figure 4.17). Unusually, Haridatta explains these neither in verse nor in tables, but rather in a series of prose instructions appended to the verses of his second chapter, as in the following excerpts (MS BORI 399 f. 2r–2v:

atha cakrabhrame gadyena saṃskāro likhyate bhaumādīnām ‖ [. . .]

budhasya cakrabhrame saṃskāro nāsty eva ‖

atha guror cakrabhrame yasyā paṅkter guruḥ sādhyate tatpaṅktimadhye pratikoṣṭhakaṃ kalāś catvāriṃśa 40 ṛṇaṃ bhavanti ‖ *dvitryādicakrabhrama etā dvitryādiguṇitā ṛṇaṃ kāryā* ‖ *iti gurucakrabhrame saṃskāraḥ* ‖ [. . .]

atha granthaśakāt pūrvā ced grahāḥ sādhyante tadā pūrvoktaṃ viparītaṃ kāryaṃ ‖ *iti bhaumādipañcakamadhye cakrasaṃskāraḥ* ‖

bhaumasya cakrabhrama udayāstavakramārgeṣu ṣaṇṇavati ghaṭikā 96 dhanaṃ bhavanti ‖ *budhasya cakrabhrama udayāstavakramārgeṣu caturviṃśadghaṭikā 24 ṛṇaṃ bhavanti* ‖ *guroś cakrabhrama udayāstavakramārgeṣu aṣṭacatvāriṃśad 48 ghaṭikā ṛṇaṃ bhavanti* ‖ [. . .]

dvitryādicakrabhrama etā dvitryādiguṇāḥ saṃskāryāḥ ‖ *iti bhaumādīnāṃ cakrabhrama udayāvakramārgeṣu saṃskāraḥ* ‖

Now, the correction of Mars etc. when [there is] a completed cycle is written with a prose [section]. [. . .]

When [there is] a completed cycle of Mercury, there is no correction at all.

Now, when [there is] a completed cycle of Jupiter, whatever table [(*paṅkti*) the true longitude of] Jupiter is determined with respect to, 40 arcminutes are to be subtracted in that table [from] every table-entry. When there [are] two, three, etc., completed cycles, those [computed arcminutes] multiplied by two, three, etc., are subtracted. Thus the correction when [there are] completed cycles of Jupiter. [. . .]

Now, if the planets are determined before the Śaka [year] of [the epoch of] the book, the previously stated [procedure] is to be done in reverse. Thus the cycle-correction is applied in the five [planets] beginning with Mars.

When [there is] a completed cycle of Mars, to the rise, set, retrograde and direct-motion [times] 96 *ghaṭīs* are added. When [there is] a completed cycle of Mercury, to the rise, set, retrograde and direct-motion [times] 24 *ghaṭīs* are subtracted. When [there is] a completed cycle of Jupiter, to the rise, set, retrograde and direct-motion [times] 48 *ghaṭīs* are subtracted. [. . .] When there [are] two, three, etc., completed cycles, those corrections are multiplied by two, three, etc. Thus the correction to the rise, set, retrograde and direct-motion [times] when [there is] a completed cycle of Mars etc.

5.8 Summary

We have seen that several Sanskrit *sāraṇī*/*koṣṭhaka* works were ultimately derived from Sanskrit astronomical handbooks or *karaṇas*, and indeed are sometimes referred to as *karaṇas* themselves. This close connection between the genres persisted in, e.g., the Saurapakṣa works of Rāma writing in Kāśī at the end of the sixteenth century. His handbook and table text, identically entitled *Rāmavinoda* "sport of Rāma," are sometimes copied as a single work with the verses of the latter treated as the initial chapter of the former (Pingree 1970–94, A5.426–428).

But the close relation between these astronomical genres does not mean that every astronomical table is merely a record of mechanically repeated applications of some algorithm prescribed by a *karaṇa*. A table compiler might combine, simplify, or otherwise modify algorithms from various sources to produce his tabulated values, or might develop an innovative approach of his own like the various time-offsets devised in, e.g., the *Makaranda* and the *Tithicintāmaṇi*. And interpolation techniques were constantly invoked to ease the labor of filling in intermediate values within a table once certain key entries had been computed by more complicated methods.

The specific interconnections between individual works will require a great deal of further scrutiny, analysis, and detailed textual scholarship to elucidate fully. We have noted several examples of the most easily identified forms of intertextuality in the table-text genre (besides the explicit dependence of the commentary/*udāharaṇa* on a selected base-text): namely, direct copying from an earlier work, and creating a revision/abridgement of one, frequently with the word *laghu* prefixed to its title. But the foregoing discussion has barely scratched the surface of such issues; there are, at a conservative estimate, literally thousands of significant questions remaining to be investigated about how table entries were computed and how their accompanying texts were constructed.

Chapter 6
Further explorations

The previous overview reinforces our original impression that the proliferation of table texts was one of the most important developments in Sanskrit astronomy in the second millennium. The flexible and innovative construction of *sāraṇīs* increased computational convenience and efficiency for their users as well as affording new outlets for the creative ingenuity of their authors. Just in terms of sheer volume, we can tentatively infer from inventories of manuscript collections that tables account for over half of extant Sanskrit scientific manuscripts. It is all the more necessary to reiterate that this study remains very incomplete as a survey and analysis of these important texts. We conclude it with some supplementary details on the Indian table-text genre and some proposals for its further investigation.

6.1 Interactions with other table-text traditions

In addition to the vast ocean of Sanskrit astronomical tables derived from earlier *jyotiṣa* works, there is a far smaller but nonetheless informative corpus of Sanskrit tables directly derived from Islamic and European ones. Overwhelmingly, these tables form part of Sanskrit translations or adaptations of foreign astronomical works, usually produced in a metropolitan court hosting many scholars and scientists from different traditions. The results are interesting for their hybrid of traditional Sanskrit and "Sanskritized" foreign features, as illustrated by the following sample instances. (See also specific topics and tables in such works discussed in Sections 4.5 and 4.12.)

C. Montelle, K. Plofker, *Sanskrit Astronomical Tables*, Sources and Studies in the History of Mathematics and Physical Sciences,
https://doi.org/10.1007/978-3-319-97037-0_6

Fig. 6.1 Table of solar declination with $\varepsilon = 1415' = 23°;35$ from a manuscript of the *Yantrarāja* (JVS 57.831 f. 5r–v).

6.1.1 *Yantrarāja* of Mahendra: Manual on astrolabes, Delhi, 1370

The earliest known Sanskrit direct translation of Islamic astronomical material is the *Yantrarāja* ("Astrolabe," literally "king of instruments") composed by Mahendra Sūri in Śaka 1292 (= 1370 CE).[1] Commissioned by Sultan Fīrūz Shāh of Delhi, this guide to constructing and using astrolabes derives from an Islamic source or sources as yet unidentified (or perhaps partly or primarily on consultations with Muslim colleagues explaining such sources).

Its accompanying tables follow typical *zīj* conventions such as adopting the trigonometric radius value $R = 3600 = 1, 0, 0$ and the ecliptic obliquity value $\varepsilon = 23°;35$ (see Figure 6.1).[2] Attempts are made in other ways, however, to integrate

[1] For extensive scholarship on Mahendra and the Indian astronomical instrumentation tradition in general, see Sarma (2008) and the works cited therein.

[2] For more details on the use of these values of R and ε in the *zījes* of, e.g., al-Khāzinī (ca. 1130), Ḥabash al-Ḥāsib (ca. 850), al-Battānī (ca. 900), Kūshyār (ca. 1000), and al-Bīrūnī, see Kennedy (1956, pp. 151–159).

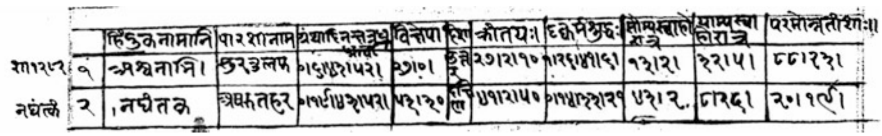

Fig. 6.2 First two entries from the star-list table in a manuscript of the *Yantrarāja* (JVS 57.831 f. 7v).

their content into the Sanskrit tradition, such as including in the geographical tables latitudes and ascensional differences for cities within as well as outside India (Raikva 1936, pp. 19–25). Moreover, the *Yantrarāja*'s table of star coordinates (itself very typical of the *zīj* genre) lists both the *hiṃduka-nāmāni* or Indian (Sanskrit) names and the *nāgarī*-transliterated *pāraśī-nāma* or Persian names in its columns (see Figure 6.2).

6.1.2 *Siddhāntasindhu* of Nityānanda: Translated *zīj*, Delhi, 1628

Nityānanda, an astronomer at the Mughal imperial court of Shāh Jahān, was commissioned by the emperor's father-in-law Āsaf Khān to produce a translation of the Persian *Zīj-i Shāh Jahānī* composed by Farīd al-Dīn Ibrāhīm al-Dihlawī. This *zīj* in turn was based on another set of astronomical tables, the Sulṭānī *zīj* of Ulug Beg (Samarkand, ca. 1440) (Misra 2016). Nityānanda named his composition *Siddhāntasindhu* ("river/ocean of *siddhāntas*," also a play on the name of the Sindh region) and, following the author of the Persian original, took as its epoch 1628, the year of Shāh Jahān's accession.[3] Comparing the two versions reveals some of the accommodations Sanskrit authors had to make when translating tabular data from a *zīj* for audiences more accustomed to the Sanskrit *koṣṭhaka* format.

Most immediately obvious is the reversal of the Arabic/Persian right-to-left presentation of data and tables into the left-to-right orientation normal to Sanskrit, illustrated in Figures 6.3 and 6.4. The former shows the *zīj*'s shadow/cotangent table for 1 to 90° of solar zenith distance, entitled *jadwal al-ẓill al-thānī wa-yusmā al-ẓill al-mustawī* "Table of the second shadow, [which] is called the right [i.e., vertical] shadow." These shadow lengths (cast by a horizontal gnomon, thus equivalent to the cotangent of the solar zenith distance) are tabulated in Arabic alphanumeric *abjad* numerals for three different standard gnomon measures: 60 (*as-sittīnī* "sixties"), 12 (*al-aṣābiʿ* "digits"), and 7 (*al-aqdām* "feet"). The table begins in the top right corner and the numerical entries in order of increasing argument are read from right to left and from top to bottom.

[3] Examples of its tables are shown in Figures 4.26, 4.30, and 4.65.

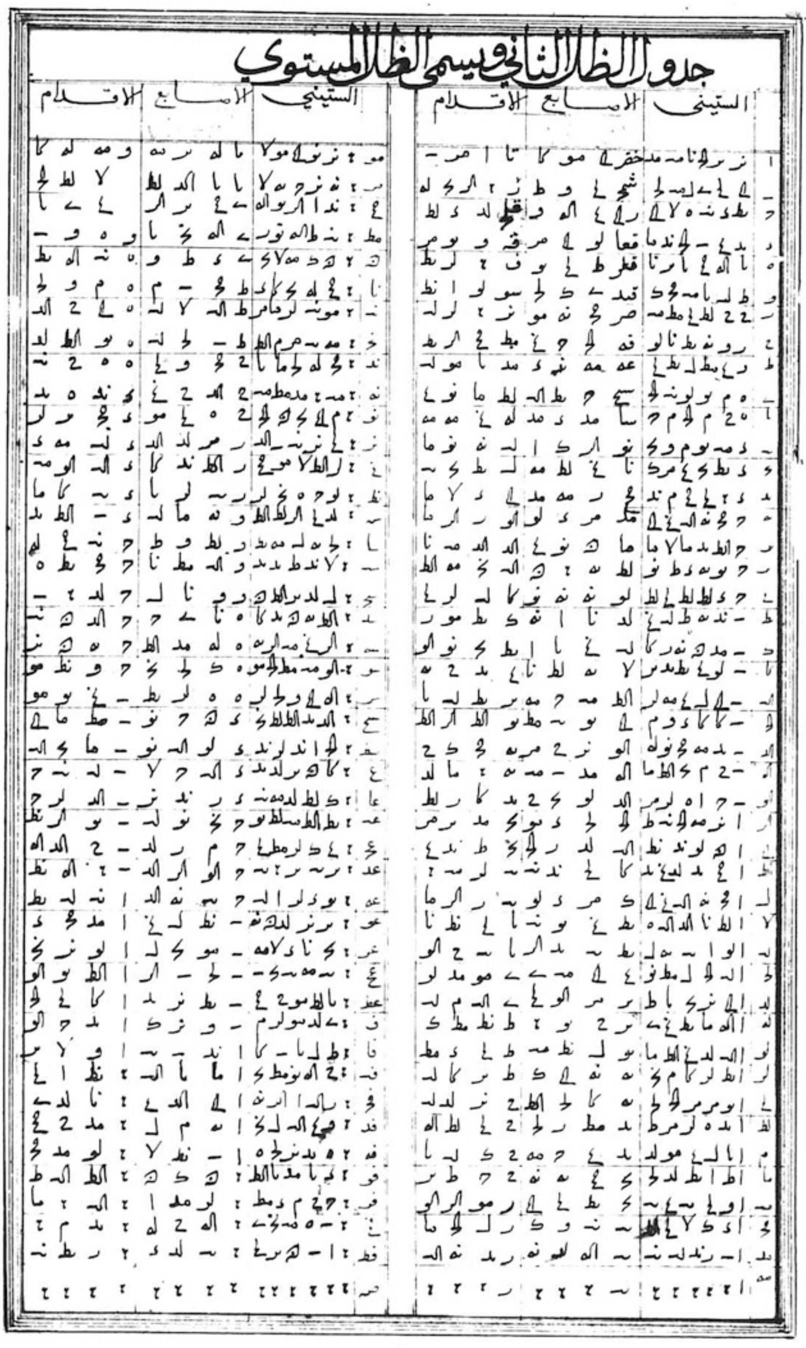

Fig. 6.3 Table of vertical shadow/cotangent of solar zenith distance, from a manuscript of the *Zīj-i Shāh Jahānī* (British Library OR 372 f. 39v). Shadow lengths in *abjad* numerals for three different horizontal gnomon measures for 1–45° of argument are tabulated top to bottom in the three columns of the right half, and for 46–90° of argument in the left half.

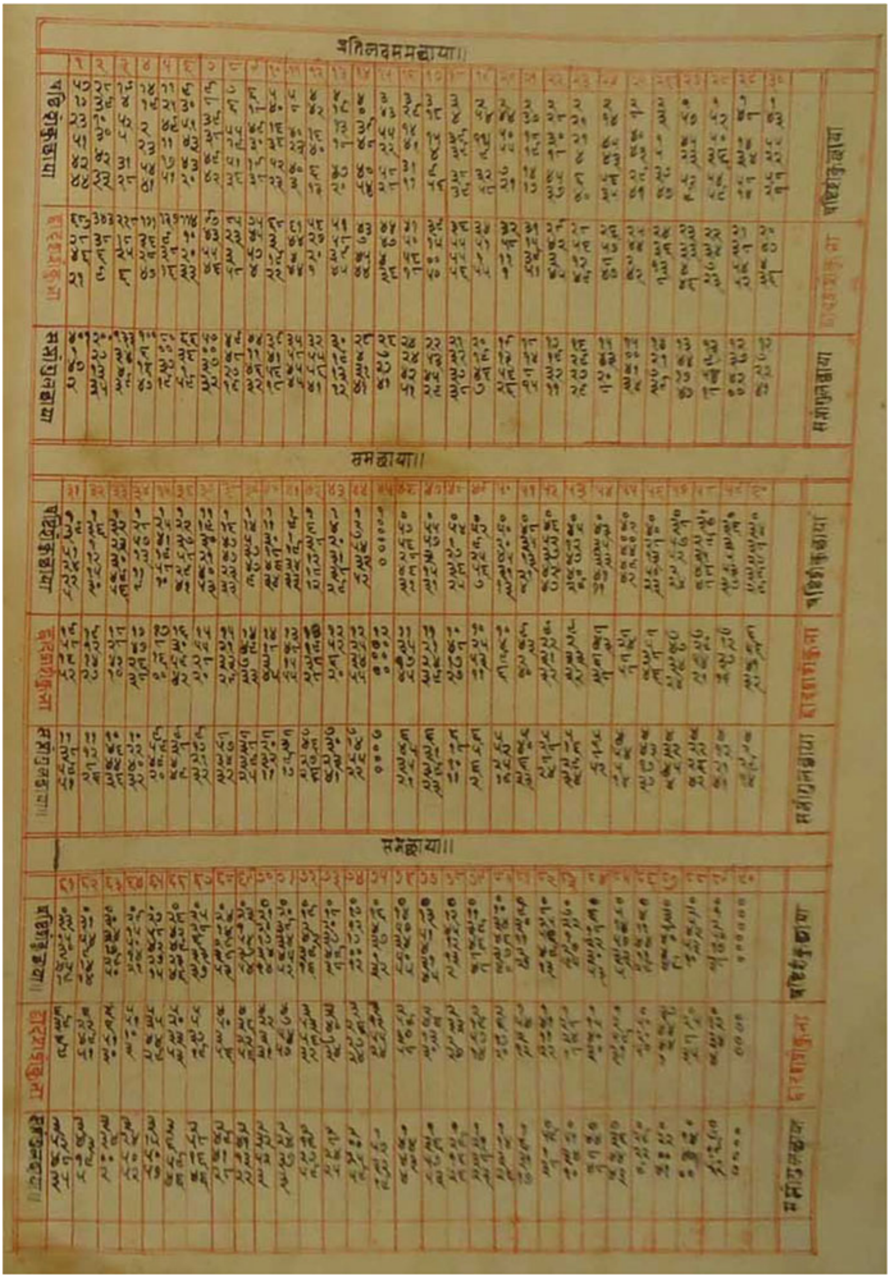

Fig. 6.4 Sanskrit version of the above *zīj* table of vertical shadow/cotangent of solar zenith distance, from a manuscript of the *Siddhāntasindhu* (Jaipur Khasmohor 4962 f. 47r).

Figure 6.4 shows Nityānanda's version of the same shadow table. Entitled *pratilavasamachāyā* "Right [i.e., vertical] shadow per degree," it lists the three gnomon measures as 60 (*ṣaṣṭiśaṃkuchāyā*, "shadow of a 60[-unit] gnomon"), 12 (*dvādaśaśaṃkubhā*, "shadow of a 12[-unit] gnomon"), and 7 (*saptāṃgulachāyā*, "shadow of a 7-digit [gnomon]" presumably in error for "7-foot"?). In its reversed "Sanskritized" layout the original columns for the three gnomon measures become rows, arranged in three horizontal blocks for 30° intervals of argument instead of in two vertical blocks for 45° intervals, and the sexagesimal digits of the individual table entries are stacked vertically instead of horizontally.

6.1.3 *Phiraṅgicandracchedyopayogika*: European lunar theory, Jaipur, 1734

The process of transmitting and adapting foreign tables was especially active at the court of Mahārāja Jayasiṃha of Amber (r. 1700–1743) under the Mughal emperor Muḥammad Shāh (Pingree 1999, 2002, 1987a). Renowned for his patronage of the astral sciences, Jayasiṃha acquired not only a number of *zījes* in Arabic and Persian, but also (from a resident Portuguese astronomer, Pedro da Silva) a copy of the 1727 edition of a Latin treatise on Copernican astronomy entitled *Tabulae astronomicae* by Philippe de La Hire. With the help of da Silva and subsequently of some French Jesuits better schooled in de La Hire's work, the *jyotiṣīs* at Jayasiṃha's court in Jaipur produced various translations and adaptations of the *Tabulae*, along with Sanskrit versions of its numerical tables.

One of the Sanskrit works inspired by de La Hire's treatise was entitled *Phiraṅgicandracchedyopayogika* ("Concerning the use of the European lunar diagram"), composed around 1734. Figure 6.5 shows de La Hire's Latin table of the correction to Saturn's longitude due to the so-called *reductio* or projection of its motion along its orbit to the ecliptic, as well as the corresponding Sanskrit version in the *Phiraṅgicandracchedyopayogika*. The table's argument, as indicated by the column header *arg. latit.* (*argumentum latitudinis*), runs from 0 to 30° (*gradus*) of each zodiacal sign of Saturn's argument of latitude: namely, its elongation along its orbit from its orbital node. This is rendered in the Sanskrit version by the table caption *śara-phala*, literally "equation of latitude."

The translated table is structurally and numerically identical to the Latin original, with the entries read top to bottom for the specified zodiacal signs in the first and third quadrants, and bottom to top for the signs in the second and fourth quadrants. The column headers/footers of the translation write out the Sanskrit names of the relevant zodiacal signs in place of their Latin numerals 0–XI, and add the arithmetic sign indicators *ṛ* (for *ṛṇa*) "negative" and *dhana* "positive" at top and bottom, respectively, to convey the sense of the Latin *Reductio Subtrahenda descendendo* "the reduction [correction] is to be subtracted when going down [the table]" and *Addenda ascendendo* "added when going up." The *nāgarī* numerals include *daṇḍas* to separate the sexagesimal digits of the entries.

Tabula XXXIII. Reductio ♄.

Reductio Subtrahenda descendendo.

	S. O.		S. I.		S. II.		
Arg. Latit.	S. VI.		S. VII.		S. VIII.		
G.	M.	S.	M.	S.	M.	S.	
0	0	0	1	27	1	27	30
1	0	4	1	29	1	25	29
2	0	8	1	30	1	23	28
3	0	11	1	32	1	21	27
4	0	15	1	34	1	19	26
5	0	18	1	35	1	17	25
6	0	22	1	36	1	15	24
7	0	25	1	37	1	13	23
8	0	28	1	38	1	11	22
9	0	32	1	38	1	8	21
10	0	35	1	39	1	5	20
11	0	38	1	39	1	3	19
12	0	42	1	40	1	0	18
13	0	45	1	40	0	57	17
14	0	48	1	41	0	54	16
15	0	51	1	41	0	51	15
16	0	54	1	41	0	48	14
17	0	57	1	40	0	45	13
18	1	0	1	40	0	42	12
19	1	3	1	39	0	38	11
20	1	5	1	39	0	35	10
21	1	8	1	38	0	32	9
22	1	11	1	38	0	28	8
23	1	13	1	37	0	25	7
24	1	15	1	36	0	22	6
25	1	17	1	35	0	18	5
26	1	19	1	34	0	15	4
27	1	21	1	32	0	11	3
28	1	23	1	30	0	8	2
29	1	25	1	29	0	4	1
30	1	27	1	27	0	0	0
	S. XI.		S. X.		S. IX.		G.
	S. V.		S. IV.		S. III.		Arg. Latit.

Addenda ascendendo.

	मेषतुलयो	वृषवृश्चिकयोः	मिथुनधनुषो	[illegible]
०	०।०	१।२७	१।२७	३०
१	०।४	१।२९	१।२५	२९
२	०।८	१।३०	१।२३	२८
३	०।११	१।३२	१।२१	२७
४	०।१५	१।३४	१।१९	२६
५	०।१८	१।३५	१।१७	२५
६	०।२२	१।३६	१।१५	२४
७	०।२५	१।३७	१।१३	२३
८	०।२८	१।३८	१।११	२२
९	०।३२	१।३८	१।८	२१
१०	०।३५	१।३९	१।५	२०
११	०।३८	१।३९	१।३	१९
१२	०।४२	१।४०	१।०	१८
१३	०।४५	१।४०	०।५७	१७
१४	०।४८	१।४१	०।५४	१६
१५	०।५१	१।४१	०।५१	१५
१६	०।५४	१।४२	०।४८	१४
१७	०।५७	१।४०	०।४५	१३
१८	१।०	१।४०	०।४२	१२
१९	१।३	१।३९	०।३८	११
२०	१।५	१।३९	०।३५	१०
२१	१।८	१।३८	०।३२	९
२२	१।११	१।३८	०।२८	८
२३	१।१३	१।३७	०।२५	७
२४	१।१५	१।३६	०।२२	६
२५	१।१७	१।३५	०।१८	५
२६	१।१९	१।३४	०।१५	४
२७	१।२१	१।३२	०।११	३
२८	१।२३	१।३०	०।८	२
२९	१।२५	१।२९	०।४	१
३०	१।२७	१।२७	०।०	०
	कन्या मीन	सिंह कुंभ	कर्क मकर	[illegible]

शा२फलं

Fig. 6.5 Left: A table of the *reductio* or reduction of longitude to the ecliptic for Saturn from the 1727 edition of the *Tabulae astronomicae* (de La Hire 1727, p. Tabulae 46). Right: The Sanskrit version of this table from a manuscript of the *Phiraṅgicandracchedyopayogika* (Jaipur Khasmohor 5292 f. 39r).

6.2 Regional, institutional, and intellectual trends in table-text authorship

The question of why particular *jyotiṣīs* composed or embraced particular table texts in particular localities at particular times is as large as Indian second-millennium intellectual history itself, about which we still know far too little. We can infer from evidence about the provenance of table-text manuscripts that the very rough correlations between astronomical *pakṣa* and geographical region discussed in Section 1.2.3 still more or less hold true for the astronomical tables genre. The Āryapakṣa *vākya* compositions in southern India, the Brāhmapakṣa and Gaṇeśapakṣa tables dominating northwestern regions, and the Saurapakṣa tables originating in central and eastern India (Rupa K. et al. 2014, pp. 192–194) illustrate the persistence of these overall trends. We can add to these roughly sketched patterns a few other fairly obvious broad inferences, e.g., that Sanskrit translations of Islamic astronomical tables typically proceeded from environments of courtly patronage, or that vernacular adaptations of established Sanskrit *sāraṇī* works suggest dissemination into more popular and practical spheres of activity within *jyotiṣa*.

Ultimately, however, identifying and understanding the connections among Sanskrit table texts crucially depends on tracing the multitudes of individual connections between their authors, and the prosopography of these *jyotiṣīs* is still in a very rudimentary state. But the pioneering work of, e.g., Dikshit (1969, 1981) and Pingree (1970–94, 1981), along with some additional information from manuscripts, enables us to discuss the following selected examples. Most of them are connected in some way with early modern Kāśī, illustrating the prominent role of that city as a contemporary center of Sanskrit learning and its consequent attraction for migrating Sanskrit scholars (O'Hanlon 2013).

The famous Gaṇeśa, initiator of the Gaṇeśapakṣa in Nandigrāma in about 1520 (see Section 5.6), also had accomplished astronomers for his father (Keśava of the Kauśika *gotra*, author of several works in astronomy and astrology) and brothers. Besides his own son(s) and his nephew Nṛsiṃha, Gaṇeśa is known to have taught Divākara of the Bhāradvāja *gotra* from Golagrāma on the northern bank of the Godāvarī River in eastern Mahārāṣṭra, who later moved to Kāśī. Divākara's five sons include the abovementioned brothers Viśvanātha, Viṣṇu, and Mallāri, whose nephew Nṛsiṃha along with his own three sons Divākara (II), Kamalākara, and Raṅganātha likewise contributed significantly to the flourishing study of *jyotiṣa* in that city.

At about the same time as the migration of Divākara's family, a *jyotiṣī* named Ballāla of the Devarātra *gotra* also relocated to Kāśī, from a village called Dadhigrāma on the banks of the Payoṣṇī, probably also in eastern Mahārāṣṭra. He likewise had five sons, some of whom contributed books and/or sons of their own to the cause of Sanskrit astronomy.

The output of these dozen or so astronomers in the course of about a century, even leaving aside the seminal works of Gaṇeśa on which much of it was based, is a remarkable and highly complex achievement that requires far more detailed analysis. Not the least interesting aspect of their work is the variation it reveals in the amount of attention the different authors paid to table texts. Some of the descendants of Divākara (I) and of Ballāla wrote commentaries and *udāharaṇas* on tables, especially the prolific Viśvanātha who produced no fewer than three along with multiple commentaries on *karaṇa* and astrological texts. Viśvanātha's grand-nephew Divākara (II) likewise commented on (among others) the *Pātasāraṇī* and the *Makaranda*, while his brother Kamalākara composed only a highly theoretical *siddhānta* in the classical form, along with expositions of other such works. Was this an indication of differences between the brothers in professional duties requiring different amounts of astronomical calculation, or in estimation of scholarly prestige attached to practical versus theoretical expositions, or simply in personal interests or proficiency in composing Sanskrit verse?

All such questions about individuals' involvement with the table-text genre remain entirely unanswered for almost all *brāhmaṇa jyotiṣīs* working in Sanskrit in the second millennium. (The role played by their Jaina counterparts in this genre is even more obscure; at present, we know of no Jaina authors of Sanskrit works on tables designed to aid in construction of *pañcāṅgas*, prediction of eclipses, or other crucial aspects of liturgical calendars like those with which most *brāhmaṇas* were constantly engaged.) We hope that the foregoing investigations will highlight the need for more thorough study of these issues.

6.3 The roles of scribes and users in creating and modifying table texts

The examples in Section 3.2 and elsewhere have illustrated many of the variations in format and layout that scribes can introduce when copying astronomical tables. Since astronomers in many cases made their own copies of such books for themselves or their pupils,[4] the distinction between the categories of "user" and "scribe" is not always sharply marked. If the copyists were proficient in working with tables, they might make such modifications deliberately and, in some cases, might even extend them to the basic tabular or textual content of the work.

These speculations find some corroboration in manuscripts of Sanskrit table texts. For instance, equivalent tables of the *udayāntara* "rising-difference," or component of the equation of time correction due to the obliquity of the ecliptic, in three manuscripts of the *Brahmatulyasāraṇī* are reproduced in Figures 4.28 and 6.6. Some of the differences in the ways their compilers organized and placed them are summarized in Table 6.1. These *udayāntara* tables also contain three

[4] See, e.g., Pingree (2003, pp. 164–166). The dearth of prosopographical information lamented in Section 6.2 concerning the authors of Sanskrit scientific texts is even more marked when it comes to their copyists and their readers.

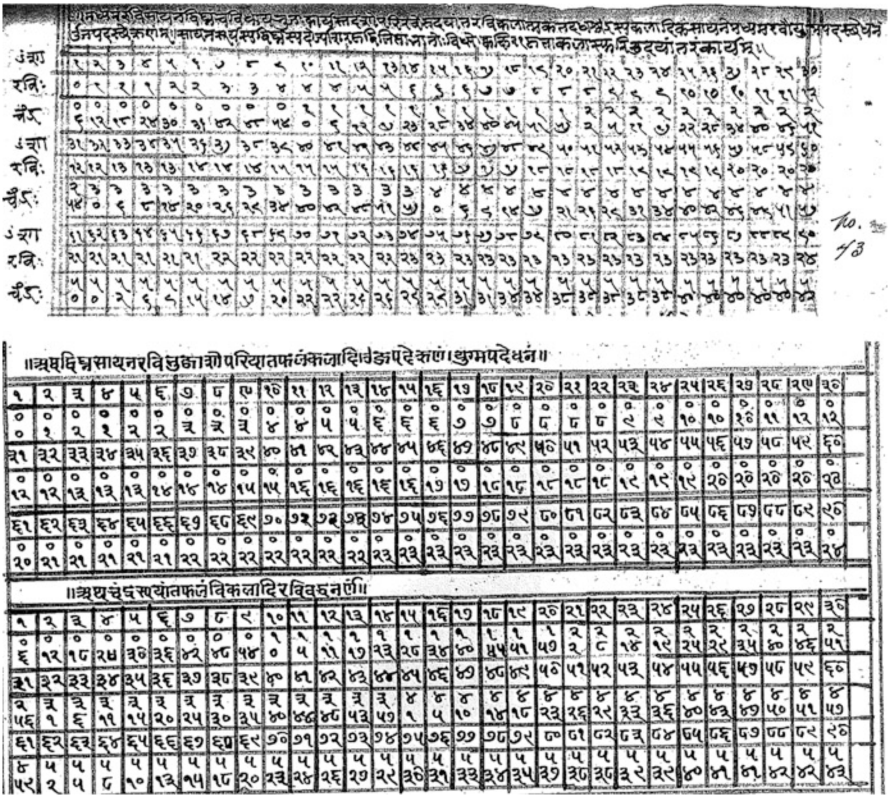

Fig. 6.6 Rising-difference (*udayāntara*) tables from two manuscripts of the *Brahmatulyasāraṇī*. Top: Smith Indic 43 f. 1r. Bottom: Smith Indic MB LVIII f. 5v.

Table 6.1 Comparison between features of the *udayāntara* table in three different manuscripts of the *Brahmatulyasāraṇī*, in argument step length, table entries, and table placement.

Manuscript	Steps	Max. value (sun/moon)	Placement
MS Smith Indic 45 (Figure 4.28)	2°	24;0/5;42,51	After the lunar *manda*-equation, followed by the correction to Mars' apogee, then Mars' *manda*-equation.
MS Smith Indic 43 (Figure 6.6, top)	1°	24/5;42	Beginning of MS, followed by the correction to Mars' apogee and then solar *manda*-equation table.
MS Smith Indic MB LVIII (Figure 6.6, bottom; separate tables for sun/moon)	1°	0;24/5;43	End of MS, after Saturn's mean longitudinal displacement.

Fig. 6.7 Title paratext of the rising-difference (*udayāntara*) table from three manuscripts of the *Brahmatulyasāraṇī*. Top: Smith Indic 45 f. 7v. Middle: Smith Indic 43 f. 1r. Bottom (two excerpts): Smith Indic MB LVIII f. 5v.

different versions of the title paratext, all derived from the following verse in the *Karaṇakutūhala* of Bhāskara (Mishra 1991, p. 29, verses 2.18), and reproduced in Figure 6.7:

dvighnasya dorjyā śarahṛd viliptā
bhānor vidhoḥ kvakṣihṛtāḥ kalās tāḥ||
svarṇaṃ ca yugmau jayadasthite 'rke
krameṇa karmetyudayāntarākhyam||18||

The sine [of the mean longitude of the sun] multiplied by 2, divided by 5, is the [correction in] arcseconds for the sun. [The same quantity in the case] of the moon is divided by 21; [the result is in] minutes. And these are [applied] positively or negatively when the sun is standing in an even or odd quadrant respectively. That is the operation called the '[equation of time] correction for ecliptic obliquity.'

The title versions in the different manuscripts are as follows:

Smith Indic 45:

atha ravicaṃdrayor udayāṃtaraṃ koṣṭakāḥ dvighnaṃ bhujāṃsopari

Now, the tabular values [for] the rising-difference [correction] pertaining to the sun and the moon [with its] superscribed [argument] the degrees of the longitudinal arc multiplied by 2.

Smith Indic 43:

madhyamaṃ ravisāyanaṃ dvighnaṃ ca vidhāya bhujaḥ kāryas tadaṃśopari raver udayāṃtaraṃ vikalātmakaṃ tadadhaś caṃdrasya kalādikaṃ sāyane madhyamaravau yugmapadasthe dhanaṃ ojapadasthe ṛṇaṃ | sāyanasūryasya dvighnasya dorjyā śarahṛd viliptā bhānoḥ vidhoḥ kvakṣi 21 hṛtā kalā sphuti udayāṃtaraṃ kāryam |

[When one] has determined the precession-increased mean [longitude] of the sun multiplied by two, a *bhuja* (i.e., an arc reduced to the first quadrant) is to be made; the rising-difference [correction] of the sun [with its] superscribed [argument] in degrees of that [is given in] arcseconds (*vikalā*). Below that, the minutes and so on of the moon. When the precession-increased mean [longitude of the] sun is situated in an even quadrant, it is positive; situated in an odd quadrant, it is negative. The sine of the precession-increased [mean longitude of the] sun multiplied by 2, [in] arcseconds (*viliptā*) [when] divided by 5 for the sun, [in] arcminutes (*kalā*) [when] divided by 21 for the moon, is to be made [as] the accurate rising-difference [correction].

Fig. 6.8 Table of epact from a manuscript of Haridatta's *Jagadbhūṣaṇa* (Smith Indic 146 f. 96v), with paratext containing a quote from the *Candrārkī* of Dinakara, presumably added by the scribe.

> Smith Indic MB LVIII:
>
> *atha dvighnasāyanaravibhujāśopari yātaphalaṃ kalādi | ojapade ṛṇaṃ | yugmapade dhanaṃ ||*
>
> *atha caṃdrasya yāṃtaphalaṃ vikalādiravivad dhanarṇaṃ |*
>
> Now, the rising-difference [correction] (*yātaphala*) [with its] superscribed [argument], the arc (*bhuja*) of the precession-increased mean [longitude of the] sun multiplied by two, is in minutes and so on. In an odd quadrant it is negative, in an even quadrant, positive.
>
> Now, the rising-difference of the moon is positive or negative similarly to [that of] the sun in arcseconds (*vikalā*) and so on.

A similar instance of (presumably) a scribe's adding supplementary quoted material to table paratext is illustrated in Figure 6.8, which shows the first part of an annual epact table from the *Jagadbhūṣaṇa* of Haridatta (Section 5.7). After the title *atha prativarṣaśuddhikoṣṭhakāḥ* ("Now, the tabular entries of annual epact") appears some text not found in Haridatta's own verses but evidently borrowed from the *Candrārkī* of Dinakara (Kolachana et al. 2018, p. 331, verse 13ab):

> *Jagadbhūṣaṇa*: *śuddhau viyadrūpakarās trayaṃ ca māsā gatā mādhavato vicintyāḥ*
>
> *Candrārkī*: *śuddhau viyadrūpakarāstadūrdhve | māsā gatā mādhavato vicintyāḥ*
>
> "If the epact [quotient with 30 happens to be] beyond zero, one or two [or three], [then] the elapsed months are to be considered [as starting] from Vaiśākha".

This rule seems to imply something equivalent to the practice of adding an intercalary month if the epact is greater than 30 *tithis*.[5]

The scribe/compiler of a table text may also, as we have seen in the comparisons in Tables 5.6 and 5.11 relating to the *Makaranda* and the *Grahalāghavasāriṇī* respectively, adjust the tables' epoch to a more convenient period by choosing to omit or augment some of the work's given multi-year cycles. Occasionally an entirely new table may even be added. Consider, for example, the mean longitude displacement tables excerpted in Figure 6.9. The top image shows part of the standard *vāṭikā* table (Section 4.1) with mean longitude increments for Mercury,

[5] Not having seen at time of writing more than one manuscript of the *Jagadbhūṣaṇa*'s tables, we cannot conclude definitively that this addition is due to a scribe rather than forming part of Haridatta's own table paratext, but it seems likely.

Fig. 6.9 Excerpts from the mean longitudinal displacement table for Mercury from a manuscript of the *Makaranda* (which includes a similar table for Venus). Top: The standard table as it appears in all MSS (Smith Indic 135 f. 14v). Bottom: Table of sexagesimal-complement values of the above table (Smith Indic 135 f. 38r).

while the entries in the bottom table (which is placed, with a similarly constructed table for Venus, at the end of the manuscript) are the sexagesimal complements of those in the former. The purpose of these additional tables is not explained.

6.4 Future directions

Among the many fundamental aspects of Indian table texts not addressed in this study due to lack of space and time, one of the most central is the nature and role of non-Sanskrit table texts. Vernacular scientific literature, as has been ably noted in, e.g., Rupa K. et al. (2014), was intertwined with Sanskrit works and undeniably has much to tell us about them. In the same vein, the corpus of ephemera constituting individual *pañcāṅga* and horoscope texts must be probed to understand the connections between the knowledge and skills maintained by practitioners of *jyotiṣa* and the content of its ultimate end products.

Even the topics that we have addressed here contain innumerable questions that remain unanswered. For example, it is still not clear exactly how or when, or why, astronomers developed the time-unit of the *avadhi* so ubiquitous in table texts. Nor do we really understand why so many computed and tabulated quantities such as parallax and equation of time are structured as separate components in Indian tables and other astronomical works, instead of being "bundled" as a combined correction as in their Ptolemaic counterparts. Seeming minutiae such as the selection (and invention) of specialized units of measurement, the initialization of table arguments at 0 rather than 1, and the use of cycle-complements of desired quantities rather than the quantities themselves are small but insistent indicators of how much still needs to be explained about Indian astronomical tables.

Underlying all these details are several deeper issues concerning the intellectual and professional development of Indian astronomy in the second millennium. For instance, were the endlessly multiplied individual variations in the structure and format of tables employed as a sort of professional "brand" for compilers of table texts? To what extent was a particular set of tables viewed, or intended to be viewed, as a particular author's composition as opposed to a customizable reference work that could be modified to fit users' differing requirements? Most importantly, we seek to know how these works fit into contemporary trends of "classical" renewal and "modern" innovation in Sanskrit intellectual history as a whole. Did astronomical table texts merely supplement the ancient genres of versified treatise and handbook, or to some extent replace them, or provide a way to bypass some of their content?[6]

The first major step in the lengthy process of elucidating these issues should be establishing the textual sources. The present study has relied on an unsystematic mix of published editions and manuscripts to explain some key features of the material, but reliable critical editions of tables as well as their accompanying instructions are urgently needed for deeper investigations. We suggest prioritizing some of the table texts discussed in Chapter 5 that have already been incompletely and/or non-critically edited: namely, the *Rājamṛgāṅka*, the *Grahajñāna*, the *Makaranda* and the *Mahādevī*. (A critical edition of the *Jagadbhūṣaṇa* will appear shortly in Melnad (Forthcoming)). Other important works that should be published in the near future include the *Śīghrasiddhi* of Lakṣmīdhara, which includes parameters

[6]The larger topic of the evolution of Sanskrit textual traditions in the mid-second millennium, and their testimony to a flourishing intellectual dynamism combined with tension between canonical authority and new ideas, has been broached in the recent project "Sanskrit Knowledge-Systems on the Eve of Colonialism," see Pollock (2002). In the specific context of *jyotiṣa*, similar questions are addressed in, e.g., Minkowski (2002).

from both the Āryapakṣa and the Brāhmapakṣa, and the two works of Rāma entitled *Rāmavinoda*—one a *karaṇa* and the other a *koṣṭhaka*—based on Saurapakṣa parameters.

Another unexplored mine of textual material is furnished by the many table texts composed in the hundred or more years following the chronological stopping point of this study in the mid-17th century. Several of these newer works (listed in Appendix A) stand out for their potential to testify to continuing innovation and adaptation within their genre. For example, the only cyclic table text other than Haridatta's *Jagadbhūṣaṇa* that we know of appears in this period: the Brāhmapakṣa *Grahasiddhi* or *Bhramaṇasāraṇī* of Trivikrama of Rājasthān. An initial inspection of this work reveals many similarities with the *Jagadbhūṣaṇa* but also many modifications to improve precision and ease of use. These cyclic table texts, despite their scarcity, are crucial to understanding the history and circulation in various scientific traditions of the planetary "goal-year period" parameters that emerged in ancient West Asia. Other later works in need of further study include the following:

***Gaṇitarāja* of Kevalarāma Pañcānana (1728)**. This Saurapakṣa mean-with-equation table text from north-east India is one of the most comprehensive sets of Sanskrit tables known, comprising detailed planetary, calendric, and eclipse tables. Pingree (1981, p. 44) has suggested that its positive-normalized equation tables may reveal inspiration from Islamic sources (although note that we have seen in Section 4.10 Makaranda's somewhat similar adjustment of table entries to make them always additive).

***Pattraprakāśa* of Viśrāmaśukla (1777)**. Another Saurapakṣa work combining planetary and calendric tables in this case composed at Kāśī with mean-to-true arrangement.

***Jayavinodasāraṇī* of Jayasiṃha.** Mahārāja Jayasiṃha of Jaipur (r. 1700–1743), an enthusiastic patron of the sciences, is credited with compiling this calendric table text as well as commissioning the Sanskrit translation *Samratsiddhānta* of the text and tables of al-Ṭūsī's Arabic recension of Ptolemy's *Almagest*.

The continued importance of Gaṇeśa's *Grahalāghava* to table compilers also should not be (longer) neglected. Many so-called *Grahalāghavasāriṇī* works well into the eighteenth century perpetuated its parameters and methods, each with its own variations in content and format. One of the *Grahalāghava*'s *cakra* dates or cycle transitions (see Section 5.6.1) was adopted as the epoch of another "spinoff" work, the 1773 *Grahāgama* of Govindasūnu.

Finally, the process of editing table texts will require extensive reconstructions and statistical analyses of the table data, not merely for detecting scribal errors but also for extracting or "squeezing" underlying parameter values. This will reveal much additional information about the technical relationships connecting table texts with *karaṇas* and *siddhāntas*, as well as the computational techniques employed by *jyotiṣa* practitioners.

Appendices

Appendix A
Inventory of Sanskrit table texts

The following inventory does not pretend to be a complete list of every surviving or known table text in Sanskrit astronomy. It is unlikely that any such list could ever be compiled, especially if the category of table text is taken to include every individual *pattra*, *koṣṭhaka* or *sāraṇī* written out by astronomers for their personal use. But the items shown here contain all the named and recognized table texts known to us, along with basic information about their date, authorship, and connections to other works.

Each listing contains the work's title as well as the following details, where available:

- epoch/composition date (Common Era dates have been converted or reconstructed from external sources)
- author
- geographical location, including latitude where known
- astronomical *pakṣa*, format, and/or "school" tradition to which the work belongs
- standard reference citations about the work
- commentators, with titles and dates of their commentaries where known
- other miscellaneous information

Ananta sudhārasasāraṇī (ca. 1525 CE)
Ananta
(Pingree 1970–94, A1.40, A5.5; Pingree 2004, pp. 29–33)

Karaṇakesarī Śaka 1603 (1681 CE)
Bhāskara
Saudāmikā (Gujarāt? $\phi = 22°;35,39$)
Brāhmapakṣa
(Pingree 1970–94, A4.328, A5.263; Pingree 1968, pp. 70–72; Montelle and Plofker 2013)

C. Montelle, K. Plofker, *Sanskrit Astronomical Tables*, Sources and Studies in the History of Mathematics and Physical Sciences,
https://doi.org/10.1007/978-3-319-97037-0

Khagataraṅgiṇī (1608 CE)
Goparāja
Saurapakṣa
(Pingree 2003, pp. 74–78)

Khecaradīpikā Śaka 1571 (1649 CE)
Kalyāṇa
(Gujarāt/Rājasthān)
Brāhmapakṣa, mean-to-true
(Pingree 1970–94, A2.25; Pingree 1968, pp. 61–2; Pingree 1981, p. 43)

Khecaraśīghrasiddhi/Grahasāraṇī Śaka 1552 (1630 CE)
Gaṅgādhara
Kāśī
Gaṇeśapakṣa, mean-with-equation
(Pingree 1970–94, A2.82–85, A4.70, A5.65; Pingree 1981, p. 43; Pingree 1973, pp. 134–141)

Kheṭakasiddhi Śaka 1500 (1578 CE)
Dinakara
Bārejya, (Bariya, Rewa Kantha, Gujarāt)
Brāhmapakṣa, mean-to-true
(Pingree 1970–94, A3.102–104, A5.139; Pingree 1968, pp. 51–3; Pingree 1973, pp. 101–112; Pingree 1981, p. 42; Pingree 2003, p. 65)

Kheṭakautūhala (1619 CE)
Sūrajit
(Ahmadabad?)
(Pingree 2003, pp. 79–80)
udāharaṇa of 1628/9 CE (Khasmohor 5602)

Kheṭataraṅgiṇī Śaka 1624 (1702 CE)
Āpadeva
Janasthāna, Mahārāṣṭra
Gaṇeśapakṣa, mean-with-equation
(Pingree 1970–94, A1.49–50, A3.15; Pingree 1981, p. 43)

Kheṭamuktāvalī Śaka 1488 (1566 CE)
Nṛsiṃha
Nandigrāma, Gujarāt
Gaṇeśapakṣa, mean-to-true
(Pingree 1970–94, A3.202–204, A4.162, A5.202; Pingree 1980; Pingree 1981, p. 42)

Gaṇakānanda (1447 CE)
Sūrya
Saurapakṣa
(Rupa 2014, p. 42)

Gaṇitamakaranda (1618 CE)
Rāmadāsa Dave
Śuddhadantī, Marwar
(Pingree 1970–94, A5.490; Pingree 2004, p. 36)

Gaṇitarāja Kali 4829 (1728 CE)
Kevalarāma Pañcānana
Navadvīpa, Bengal
Adjusted Saurapakṣa, mean-with-equation
(Pingree 1970–94, A2.63, A4.63, A5.54; Pingree 1973, pp. 158–168; Pingree 1981, p. 44)

Grahakalpataru/Maṇipradīpa Śaka 1487 (1565 CE)
Raghunātha
Kāśī
(Pingree 1970–94, A5.372; Pingree 2003, pp. 62–64)

Grahakaumudī Śaka 1510/1525 (1588/1603 CE)
Nṛsiṃha
Nandigrāma, Gujarāt
Gaṇeśapakṣa, mean-to-true
(Pingree 1970–94, A3.202–204, A4.162, A5.202; Pingree 1973, pp. 118–123; Pingree 1981, p. 42)

Grahajñāna Śaka 1054 (1132 CE)
Āśādhara
(Gujarāt)
Brāhmapakṣa, mean-to-true
(Pingree 1970–94, A1.54, A2.16, A3.16, A4.28, A5.17; Pingree 1973, pp. 69–72; Pingree 1981, p. 42; Pingree 1989)
Gaṇitacūḍāmaṇi of Harihara (*fl. ca.* 1575)

Grahaprakāśa Śaka 1584 (1662 CE)
Devadatta
Adjusted Saurapakṣa
(Pingree 1970–94, A3.119; Pingree 1973, pp. 142–148)
commentary of Devadatta

Grahaprabodha Śaka 1541 (1619 CE)
Nāgeśa
Gujarāt
Gaṇeśapakṣa, mean-with-equation
(Pingree 1970–94, A3.145–146, A4.125, A5.167; Pingree 1968, pp. 63–64; Pingree 1981, p. 43)
udāharaṇa of Yādava (Śaka 1585 = 1663 CE)

Grahalāghavavīyamadhyamaspaṣṭārkasāriṇī Śaka 1447 (1525 CE)
(Pingree 1968, p. 50)

Grahalāghavasāriṇī Śaka 1676 (1754 CE)
Gaṇeśapakṣa, mean-with-equation
(Pingree 1968, pp. 69–70; Pingree 1973, pp. 93–99)

Grahalāghavasāriṇī (*ca.* 1630 CE)
Nīlakaṇṭha
(Pingree 1970–94, A3.174, A4.142, A5.184–185; Pingree 2003, pp. 80–83)

Grahalāghavasāriṇī Śaka 1578 (1656 CE)
Prema
(Pingree 1970–94, A4.229, A5.226; Pingree 2003, pp. 87–90)

Grahasiddhi/Bhramaṇasāraṇī Saṃvat 1776 (1704 CE)
Trivikrama
Nalinapura, Rājasthān
Brāhmapakṣa, cyclic
(Pingree 1970–94, A3.93, A4.105; Pingree 1968, pp. 64–66; Pingree 1981, pp. 43–44)
udāharaṇa of Trivikrama

Grahāgama Śaka 1695 (1773 CE)
Govindasūnu
Siposi, Rājasthān
Gaṇeśapakṣa, mean-with-equation
(Pingree 1970–94, A2.143, A4.86; Pingree 1973, pp. 168–169; Pingree 1981, p. 44)

Candrārkī Śaka 1500 (1578 CE)
Dinakara
Bārejya, (Bariya, Rewa Kantha, Gujarāt)
Brāhmapakṣa, mean-to-true
(Pingree 1970–94, A3.102–104, A4.109, A5.138–9; Pingree 1968, pp. 51–3; Pingree 1973, p. 101; Pingree 1981, p. 42; Pingree 2003, pp. 65–68)
Candrārkīṭippaṇa of Dinakara

Candrārkī Śaka 1577 (1655 CE)
Acalajit
Muraripupura, Gujarāt
Brāhmapakṣa/Saurapakṣa
(Pingree 1970–94, A4.12; Pingree 1981, pp. 45–6)

Camatkārasiddhi Śaka 1549 (1627 CE)
Vīrasiṃha
Bikāner
(Pingree 1981, p. 45; Sarma 1945)

Jagaccandrikāsāriṇī Saṃvat 1725 (1668 CE)
(Jaina author)
(Pingree 2003, pp. 90–91)

Jagadbhūṣaṇa Śaka 1560 (1638 CE)
Haridatta
Mewar, Rājasthān ($\phi \approx 24$, Ujjain, Madhya Pradesh?)
Brāhmapakṣa, cyclic
(Pingree 1968, pp. 55–9; Pingree 1973, pp. 141–142; Pingree 2003, pp. 83–87; Montelle 2014)

Jayavinodasāraṇī Śaka 1657 (1735 CE)
(Jayasiṃha)
Amber Palace, Jaipur, Rājasthān
(Pingree 1968, pp. 66–7; Pingree 1981, p. 46)

Tithikalpadruma/Pañcāṅgapatraracanā Śaka 1527 (1605 CE)
Kalyāṇa
Maṅgalapura, Saurāṣṭra
Brāhmapakṣa
(Pingree 1970–94, A2.24–5, A4.47, A5.28; Pingree 1973, pp. 123–128; Pingree 1981, p. 45)

Tithikalpalatā Śaka 1281 (1359 CE)
(Pingree 1973, pp. 89–92; Pingree 2004, pp. 20–21)

Tithikāmadhenu Śaka 1279 (1357 CE)
Mahādeva
Tryambaka, Mahārāṣṭra
Āryapakṣa
(Pingree 1970–94, A4.376–77; Pingree 1973, pp. 82–89; Pingree 1981, p. 44)
commentary of Ananta (*fl. ca.* 1575)

Tithicintāmaṇi Śaka 1447 (1525 CE)
Gaṇeśa
Nandigrāma, Gujarāt
Gaṇeśapakṣa
(Pingree 1970–94, A2.100–103, A3.28, A4.74–5, A5.73; Pandeya 1938; Āpaṭe 1942b; Pingree 1968, pp. 47–50; Pingree 1973, pp. 100–101; Pingree 1981, p. 45; Ikeyama and Plofker 2001)
Harṣakaumudī of Nṛsiṃha (*b.* 1548)
Siddhāntarahasyodaharaṇa of Viśvanātha (*fl.* 1454/1458)
ṭippaṇa of Vyeṅkaṭa alias Bāpū
Maṇicandrikā of Yajñeśvara Roḍe (*fl.* 1815/1842)

Tithicūḍāmaṇi/Kāmadhenu (*ca.* 1560–1580 CE)
Rāmacandra
(Pingree 1970–94, A5.479–480; Pingree 2003, p. 65)

Tithidarpaṇa
Murāri
Kāśī
Saurapakṣa
(Pingree 1970–94, A4.441; Pingree 1973, pp. 149–153; Pingree 1981, p. 46)

Tithisāraṇī/Dinakarasāraṇī Śaka 1505 (1583 CE)
Dinakara
Bārejya, (Bariya, Rewa Kantha, Gujarāt)
Brāhmapakṣa
(Pingree 1970–94, A3.104, A5.139; Pingree 1973, pp. 112–114; Pingree 1981, p. 45 Pingree 2003, pp. 68–69)

Tithisāraṇī
Trivikrama
Nalinapura, Rājasthān
Brāhmapakṣa
(Pingree 1970–94, A3.93)
commentary of Trivikrama

Tithisāraṇī
Kevalarāma Pañcānana
Navadvīpa, Bengal
(Pingree 1970–94, A2.63)

Tithyādicintāmaṇi Saṃvat 1643 (1586 CE)
Dinakara
Unnatadurga (Uparkot in Junāgaḍh, Saurāṣṭra)
(Pingree 1970–94, A3.104–5; Pingree 1968, p. 51; Pingree 1981, p. 45)

Tyāgarti/Grahagaṇitapadakāni Kali 4813 (1712 CE)
($\phi \approx 14°$)
(Rupa 2014, pp. 40–41)

Dinacandrikā (1599 CE)
Rāghavaśarman
(Pingree 1984, pp. 24)

Dṛkpakṣasāraṇī
Kevalarāma Pañcānana
Navadvīpa, Bengal
(Pingree 1970–94, A2.63)

Daivajñavallabha (1447 CE)
Sumiśra
Nepāla
Ārdharātrikapakṣa
(Pingree 1981, p. 45)

Pañcāṅgavidyādharī Śaka 1565 (1643 CE)
Vidyādhara
Rājakoṭa, Saurāṣṭra
Āryapakṣa/Brāhmapakṣa
(Pingree 1970–94, A5.648; Pingree 1968, pp. 60–61; Pingree 1973, p. 142; Pingree 1981, p. 45)

Pañcāṅgasāriṇī (1735 CE)
Kevalarāma
Jaipur
(Pingree 2003, p. 93)

Pañcāṅgasiddhi (1529 CE)
Gaṇeśa (alias Gaṇapati)
(Pingree 1984, pp. 20)

Pattraprakāśa Śaka 1672 (1750 CE)
Viśrāmaśukla
Kāśī
Adjusted Saurapakṣa
(Pingree 1970–94, A5.660; Pingree 1973, pp. 170–5; Pingree 1981, pp. 46)

Pātasāraṇī Śaka 1444 (1522 CE)
Gaṇeśa
Nandigrāma, Gujarāt
Gaṇeśapakṣa

(Pingree 1970–94, A2.100, A4.74, A5.72; Pingree 2003, pp. 59–60)
commentary of Divākara (*fl.* 1575)
commentary of Viśvanātha (*fl.* 1612/1634)
commentary of Dinakara (*fl.* 1839)

Bṛhattithicintāmaṇi Śaka 1471 (1552 CE)
Gaṇeśa
Nandigrāma, Gujarāt
Gaṇeśapakṣa
(Pingree 1970–94, A2.104, A3.28, A4.75, A5.73; Āpaṭe 1942a; Pingree 1968, pp. 50–51; Pingree 1973, p. 101; Pingree 2003, pp. 60–61)
Subodhinī of Viṣṇu (*fl. ca.* 1575)

Brahmatulyasāraṇī Śaka 1105 (1183 CE)
(Nāgadatta?)
Brāhmapakṣa, mean-with-equation
(Pingree 1970–94, A5.166; Pingree 1968, pp. 36–37; Montelle and Plofker 2015)

Makaranda Śaka 1400 (1478 CE)
Makaranda
Kāśī
Saurapakṣa, mean-with-equation
(Pingree 1970–94, A4.341–343; Pingree 1968, pp. 39–46; Pingree 1973, p. 92; Pingree 1981, p. 42; Pingree 2003, pp. 54–59)
Makarandapañcāṅgopapatti of Ḍhuṇḍhirāja (*fl.* 1590)
Makarandavivaraṇa of Divākara (*b.* 1606)
Makarandapaddhatikārikā of Harikarṇa (*fl.* 1610)
Abhinavatāmarasa of Puruṣottama Bhaṭṭa (*fl. ca.* 1610)
Makarandodāharaṇa of Viśvanātha (*fl.* 1612/1630)
Makarandaṭippaṇa of Moreśvara (1622 CE)
Subodhikā of Kṣemaṅkara Miśra (*fl.* 1632)
Makarandakārikā of Kṛpārāma Miśra (*fl.* 1815)
vāsanā of Nīlāmbara Jhā (*b.* 18 July 1823)
udāharaṇa of Jīvanātha Jhā (*fl. ca.* 1846/1900)
Makaranda-vāsanā of Gokulanātha
Makaranda-sādhana-prakriyā of Cūḍamaṇi Cakravartin
ṭippaṇa of Vināyaka
commentary of Mākhanalāla
commentary of Trivedin
commentary of Rāma
commentary of Rāmadatta
commentary of Lakṣmīpati
commentary of Sadāśiva

Mahādevī/Grahasiddhi Śaka 1238 (1316 CE)
Mahādeva
(Gujarāt/Rājasthān)
Brāhmapakṣa, mean-to-true
(Pingree 1970–94, A4.374–76, A5.288–289; Neugebauer and Pingree 1967; Pingree 1968, pp. 37–39; Pingree 1973, p. 82; Pingree 2003, pp. 51–54)
Mahādevī-dīpikā of Dhanarāja (1635)
commentary of Divākara (*fl.* 1578)

Ravisiddhāntamañjarī Śaka 1531 (1609 CE)
Mathurānātha
(Bengal?)
Saurapakṣa, mean-with-equation
(Pingree 1970–94, A4.349, A5.274; Pingree 1981, p. 43; Pingree 1973, pp. 128–134; Jyotiṣārṇava 1911)

Rājamṛgāṅka Śaka 964 (1042 CE)
Bhojarāja
Dhārā, Western India
Brāhmapakṣa, mean-with-equation
(Pingree 1970–94, A4.256–7; Pingree 1981, p. 34; Pingree 1987b)

Rāmavinoda Śaka 1512 (1590 CE)
Rāma
Mughal court, Kāśī
Saurapakṣa, mean-to-true
(Pingree 1970–94, A5.427–428; Pingree 1973, pp. 114–118; Pingree 1981, p. 42; Pingree 2003, pp. 69–70)

Laghukhecarasiddhi Śaka 1149 (1227 CE)
Śrīdhara
Khāndeśa
Brāhmapakṣa, mean-with-equation
(Pingree 1973, pp. 73–6; Pingree 1976; Pingree 1981, p. 42)

Laghutithidarpaṇa Śaka 1587 (1665 CE)
Murāri
Kāśī
Saurapakṣa
(Pingree 1970–94, A4.442; Pingree 1973, pp. 151–153; Pingree 1981, p. 46)

Śīghrasiddhi Śaka 1200 (1278 CE)
Lakṣmīdhara
(Territory of the Yādavas of Devagiri)
Āryapakṣa/Brāhmapakṣa
(Pingree 1973, pp. 76–82; Pingree 1981, p. 44)

Siddhāntasindhu Saṃvat 1685 (1628 CE)
Nityānanda
Agra
(Pingree 1970–94, A3.173–4, A4.141, A5.184; Pingree 1981, p. 30)

Subodhasāriṇī Śaka 1479 (1557 CE)
Jayarāma
Alindrapūrī
(Pingree 1970–94, A4.96; Pingree 2003, pp. 61–62)

Sūryasiddhāntarahasya (1591 CE)
Rāghavaśarman
(Pingree 1984, p. 24)

Sūryasiddhāntasāriṇī Śaka 1665 (1743 CE)
(Pingree 2003, pp. 93–94)

Sūryasiddhāntasāriṇī/Grahaspaṣṭasāraṇī (1748 CE)
Candrāyaṇa
Mulatāna
(Pingree 1970–94, A3.46, A5.109; Pingree 2003, pp. 94–95)

Appendix B
Classification schemata and parameters

B.1 Astronomical and calendric name-lists

Fully explaining the complicated nature of traditional Indian luni-solar calendars and the challenges they pose for calendar conversion is far beyond the scope of this appendix. More detailed discussions can be found in, e.g., Sewell and Dikshit (1896), Plofker and Knudsen (2011), and Salomon (1998). As described in Section 1.1.4, Indian calendars are for the most part luni-solar, synchronizing sidereal years and synodic months; some of their commonly used eras or starting-points are noted in Table B.7. Years may be reckoned as strictly sidereal, starting from the moment or day of Meṣasaṅkrānti, the sun's entrance into the zodiacal sign/solar month Meṣa (see Table B.1). Or they may be taken to begin at the start of a synodic calendar month (see Table B.4), most often, but not invariably, the *śuklapratipad* or new moon of Caitra near the vernal equinox.

Synodic months may be considered *amānta*, beginning at new moon, or *pūrṇimānta*, beginning at full moon. An intercalary synodic month or *adhimāsa* can be inserted between any two of the twelve regular calendar months, and is typically called by the name of the month it immediately precedes, prefixed with "*adhika*". Thus an intercalary month occurring between Āśvina and Kārttika is *adhika* Kārttika, followed by "regular" or *nija* Kārttika, then Mārgaśīrṣa, etc.

Depending on the astronomical *pakṣa* used, civil days in the calendar may be reckoned from sunrise (at Laṅkā, the zero-point of the terrestrial coordinate system) or from midnight. All of these variables including calendar era, beginning of the year, month, and day, and sequence of intercalations depend on which of the many variants of regional calendars is used in a particular work or calendric calculation. Precise Julian or Gregorian equivalents of particular Indian calendar dates are thus not always readily obtainable. If the given date includes the current weekday or position in the 60-year "Jupiter cycle" (see Table B.8), identification may be easier.

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Table B.1 Sanskrit terms for the twelve 30° zodiacal signs and associated solar months, their classical equivalents, and their initial ecliptic longitudes. The actual celestial position of the signs will vary depending on whether the zodiac is considered tropical or sidereal, and the assumed location of the tropical or sidereal zero-point of the ecliptic.

	Standard name	Alternative names	Classical	λ
1	Meṣa (ram)	Triya, Aja	Aries	0
2	Vṛṣabha (bull)	Tāvura, Go	Taurus	30
3	Mithuna (couple)	Jituma, Nṛyuj	Gemini	60
4	Karkaṭa (crab)	Kulīra	Cancer	90
5	Siṃha (lion)	Leya, Mṛgarāja	Leo	120
6	Kanyā (girl)	Pārthona	Virgo	150
7	Tulā (balance)	Jūka, Vaṇij	Libra	180
8	Vṛścika (scorpion)	Kaurpi, Āli	Scorpio	210
9	Dhanus (bow)	Taukṣika, Cāpa	Sagittarius	240
10	Makara (sea-monster)	Mṛga	Capricorn	270
11	Kumbha (water-pot)	Hṛdroga, Ghaṭa	Aquarius	300
12	Mīna (fish)	Jhaṣa, Timi, Matsya	Pisces	330

Table B.2 Sanskrit names for the seven *vāras* or weekdays and their corresponding planets.

Weekday		Alternative names
Sunday (Sun)	*ravi*, *sūrya*	*arka*, *āditya*, *ina*, *dinakara*, *bhānu*, *bhāskara*, *heli*
Monday (Moon)	*candra*, *soma*	*indu*, *mṛgāṅka*, *rātrikara*, *vidhu*, *śaśi*
Tuesday (Mars)	*bhauma*	*kuja*, *maṅgala*
Wednesday (Mercury)	*buddha*	*jña*, *vid*
Thursday (Jupiter)	*guru*	*indra*, *bṛhaspati*
Friday (Venus)	*śukra*	*bhṛgu*, *śukla*, *sita*
Saturday (Saturn)	*śani*	*mṛdu*

Table B.3 Sanskrit names for the 27 13;20° *nakṣatras* and their initial (sidereal) ecliptic longitudes.

	nakṣatra	λ		*nakṣatra*	λ
1	Aśvinī	0°	15	Svāti	186°;40
2	Bharaṇī	13°;20	16	Viśākhā	200°
3	Kṛttikā	26°;40	17	Anurādhā	213°;20
4	Rohiṇī	40°	18	Jyeṣṭhā	226°;40
5	Mṛgaśiras	53°;20	19	Mūla	240°
6	Ārdrā	66°;40	20	Pūrvāṣāḍhā	253°;20
7	Punarvasu	80°	21	Uttarāṣāḍhā	266°;40
8	Puṣya	93°;20	22	Śrāvaṇa	280°
9	Āśleṣā	106°;40	23	Dhaniṣṭhā	293°;20
10	Māgha	120°	24	Śatabhiṣaj	306°;40
11	Pūrvaphālgunī	133°;20	25	Pūrvabhadrapadā	320°
12	Uttaraphālgunī	146°;40	26	Uttarabhadrapadā	333°;20
13	Hasta	160°	27	Revatī	346°;40
14	Citrā	173°;20			

Table B.4 Sanskrit terms for the twelve synodic months of the year and the corresponding six seasons. The first column contains the standard month names derived originally from the *nakṣatra* occupied by the full moon of that month, as Caitra from Citrā, Vaiśākha from Viśākhā, etc. The second column lists the ancient month names referring to their seasonal characteristics.

	nakṣatra-derived name	Seasonal name	Season
1	Caitra	Madhu	Vasanta (bright)
2	Vaiśākha	Mādhava	
3	Jyaiṣṭha	Śukra	Grīṣma (hot)
4	Āṣāḍha	Śuci	
5	Śrāvaṇa	Nabha	Vārṣa (rains)
6	Bhādrapada	Nabhasya	
7	Āśvina	Iṣa	Śarad (ripening)
8	Kārttika	Ūrja	
9	Mārgaśīrṣa/Mārgaśira	Saha	Hemanta (winter)
10	Pauṣya	Sahasya	
11	Māgha	Tapa	Śiśira (cool)
12	Phālguna	Tapasya	

Table B.5 Sanskrit names for the sixty fixed (i.e., uniquely numbered) and movable (or cyclic) *karaṇas* or half-*tithis* in a synodic month.

								karaṇa
1								Kiṃstughna (fixed)
2	9	16	23	30	37	44	51	Bava
3	10	17	24	31	38	45	52	Balava
4	11	18	25	32	39	46	53	Kaulava
5	12	19	26	33	40	47	54	Taitila
6	13	20	27	34	41	48	55	Gara
7	14	21	28	35	42	49	56	Vaṇija
8	15	22	29	36	43	50	57	Viṣṭi
58								Śakuni (fixed)
59								Catuṣpada (fixed)
60								Nāga (fixed)

The tables that follow provide the basic information about Sanskrit astronomical and calendric terminology required for identification of (most) dates and positions in tables, horoscopes, etc. For the specifics of conversion of a date between Indian and Gregorian calendars, we recommend the abovementioned sources and especially the online calendar software application "Pancanga" by Michio Yano (cc.kyoto-su.ac.jp/~yanom/pancanga).

Table B.6 Sanskrit names for the 27 *yogas*.

	yoga		*yoga*
1	Viṣkamba	15	Vajra
2	Prīti	16	Siddhi
3	Āyuṣmat	17	Vyatipāta
4	Saubhāgya	18	Varīyas
5	Śobhana	19	Parigha
6	Atigaṇḍa	20	Śiva
7	Sukarman	21	Siddha
8	Dhṛti	22	Sādhya
9	Śūla	23	Śubha
10	Gaṇḍa	24	Śukla
11	Vṛddhi	25	Brahman
12	Dhruva	26	Indra
13	Vyāghāta	27	Vaidhṛti
14	Harṣaṇa		

Table B.7 Calendar eras epochs with the equivalent BCE/CE year-numbers and commonly used dates of their epochs.

Era	Conventional epoch date(s)
Śaka, Śālivāhana	Meṣasaṅkrānti/Caitra-*śuklapratipad* 78 CE
Vikrama/Vikramāditya Saṃvat	Meṣasaṅkrānti/Caitra-*śuklapratipad* 57 BCE
Kaliyuga, Kali	Midnight/sunrise Meṣasaṅkrānti (18 February) 3102 BCE
(Islamic calendar)	1 Muḥarram AH 1 = sunset 15 July 622 CE

Table B.8 Sanskrit names for the successive years of the 60-year Jupiter cycle.

	Jupiter year		Jupiter year		Jupiter year
1	Prabhava	21	Sarvajit	41	Plavaṅga
2	Vibhava	22	Sarvadhārin	42	Kīlaka
3	Śukla	23	Virodhin	43	Saumya
4	Pramoda	24	Vikṛta	44	Sādhāraṇa
5	Prajāpati	25	Khara	45	Virodhakṛt
6	Aṅgiras	26	Nandana	46	Paridhāvin
7	Śrīmukha	27	Vijaya	47	Pramādin
8	Bhāva	28	Jaya	48	Ānanda
9	Yuvan	29	Manmatha	49	Rākṣasa
10	Dhātṛ	30	Durmukha	50	Anala
11	Īśvara	31	Hemlamba	51	Piṅgala
12	Bahudhānya	32	Vilamba	52	Kālayukta
13	Pramāthin	33	Vikārin	53	Siddhārthin
14	Vikrama	34	Śārvarī	54	Raudra
15	Vṛṣa	35	Plava	55	Durmati
16	Citrabhānu	36	Śubhakṛt	56	Dundubhi
18	Tāraṇa	38	Krodhin	58	Raktākṣa
19	Pārthiva	39	Viśvāvasu	59	Krodhana
20	Vyaya	40	Parābhava	60	Kṣaya

Table B.9 Sanskrit names and abbreviations for planetary synodic phenomena.

Phenomenon	Sanskrit term	Abbreviation	Greek symbol
West rising, reappearance	*udaya paścima*	*u pa*	Ξ
Retrogradation, first station	*vakra*	*va*	Φ (sup. planets) Ψ (inf. planets)
West setting, disappearance	*asta paścima*	*a pa*	Ω
East rising, reappearance	*udaya pūrva*	*u pū*	Γ
Direct motion, second station	*mārga*	*mā*	Ψ (sup. planets) Φ (inf. planets)
West setting, disappearance	*asta pūrva*	*a pū*	Σ

B.2 Parameter sets of the *pakṣas*

The values of mean daily motion listed in the following tables (B.10–B.14) are truncated at the ellipsis rather than rounded to the nearest integer sexagesimal digit. These parameter sets are modeled on the similar tables presented by Pingree (1978a), but recomputed from the original data in the cited published editions.

Table B.10 Āryapakṣa parameters derived from *Āryabhaṭīya*, *daśagītika* 3–4 (Shukla 1976, p. 6).

Āryapakṣa
Length of period: *mahāyuga* = 4,320,000 years
Civil days in period: 1,577,917,500
Year length: 365;15,31,15 days
Epoch: sunrise

Planet	Revolutions	Mean daily motion
Sun	4,320,000	0°;59,8,10,13,3,31...
Moon	57,753,336	13°;10,34,52,39,18,56...
Lunar node	−232,226	−0°;3,10,44,7,49,44...
Lunar apogee	488,219	0°;6,40,59,30,7,38...
Mars	2,296,824	0°;31,26,27,48,54,22...
Mercury's *śīghra*	17,937,020	4°;5,32,18,54,36,24...
Jupiter	364,224	0°;4,59,9,0,38,51...
Venus's *śīghra*	7,022,388	1°;36,7,44,17,4,45...
Saturn	146,564	0°;2,0,22,41,41,32...

Table B.11 Ārdharātrikapakṣa parameters derived from *Khaṇḍakhādyaka* 1.8–10, 13–14 (Chatterjee 1970, pp. 49–50, 91–92).

Ārdharātrikapakṣa
Length of period: *mahāyuga* = 4,320,000 years
Civil days in period: 1,577,917,800
Year length: 365;15,31,30 days
Epoch: midnight

Planet	Revolutions	Mean daily motion
Sun	4,320,000	0°;59,8,10,10,37,48...
Moon	57,753,336	13°;10,34,52,6,50,56...
Lunar node	−232,226	−0°;3,10,44,7,41,54...
Lunar apogee	488,219	0°;6,40,59,29,51,10...
Mars	2,296,824	0°;31,26,27,47,36,54...
Mercury's *śīghra*	17,937,000	4°;5,32,17,45,23,13...
Jupiter	364,220	0°;4,59,8,48,36,56...
Venus's *śīghra*	7,022,388	1°;36,7,44,13,7,53...
Saturn	146,564	0°;2,0,22,41,36,36...

Table B.12 Brāhmapakṣa parameters derived from *Paitāmahasiddhānta* III.5 (Pingree 1967–8, p. 478), *Brāhmasphuṭasiddhānta* 1.14–22 (Dvivedī 1901–1902, pp. 5–7).

Brāhmapakṣa Kalpa length: 4,320,000,000 years Civil days in a kalpa: 1,577,916,450,000 Year length: 365;15,30,22,30 days Epoch: sunrise		
Planet	Revolutions	Mean daily motion
Sun	4,320,000,000	0°;59,8,10,21,33,30...
Moon	57,753,300,000	13°;10,34,52,46,30,13...
Lunar node	−232,311,168	−0°;3,10,48,20,6,41...
Lunar apogee	488,105,858	0°;6,40,53,56,32,54...
Mars	2,296,828,522	0°;31,26,28,6,47,45...
Mercury's *śīghra*	17,936,998,984	4°;5,32,18,27,45,31...
Jupiter	364,226,455	0°;4,59,9,8,37,23...
Venus's *śīghra*	7,022,389,492	1°;36,7,44,35,18,27...
Saturn	146,567,298	0°;2,0,22,51,43,56...

Table B.13 Saurapakṣa parameters derived from *Sūryasiddhānta* 1.29–33, 37 (Pāṇḍeya 1991, pp. 7–8).

Saurapakṣa Length of period: *mahāyuga* = 4,320,000 years Civil days in period: 1,577,917,828 Year length: 365;15,31,31,24 days Epoch: midnight		
Planet	Revolutions	Mean daily motion
Sun	4,320,000	0°;59,8,10,10,24,12...
Moon	57,753,336	13°;10,34,52,3,49,8...
Lunar node	−232,238	−0°;3,10,44,43,10,4...
Lunar apogee	488,203	0°;6,40,58,42,31,5...
Mars	2,296,832	0°;31,26,28,11,8,56...
Mercury's *śīghra*	17,937,060	4°;5,32,20,41,51,16...
Jupiter	364,220	0°;4,59,8,48,35,47...
Venus's *śīghra*	7,022,376	1°;36,7,43,37,16,52...
Saturn	146,568	0°;2,0,22,53,25,46...

Table B.14 The mean daily motions have been derived from *Grahalāghava* 1.6–8 (Jośī 1994, p. 18). Their source is Gaṇeśa's prescribed *dhruvas*, namely, the complement up to 360° or 360°-remainder of the longitudinal excess over complete revolutions for each of the planets per 4016-day interval. To recompute the mean daily motions, we have added this excess to the appropriate integer number of complete revolutions and divided the sum by 4016. As these *dhruvas* are precise only to arcminutes (arcseconds in the case of the sun and the moon), the resulting mean daily motions are only approximate.

Gaṇeśapakṣa		
Planet	*dhruva*	Mean daily motion
Sun	0^s 1°;49,11	0°;59,8,10,10,...
Moon	0^s 3°;46,11	13°;10,34,52,4,...
Lunar node	7^s 2°;50	−0°;3,10,47,12,...
Lunar apogee	9^s 2°;45	0°;6,40,55,16,...
Mars	1^s 25°;32	0°;31,26,28,26,...
Mercury's *śīghra*	4^s 3°;27	4°;5,33,57,32,...
Jupiter	0^s 26°;18	0°;4,59,8,0,...
Venus's *śīghra*	1^s 14°;2	1°;36,9,17,34,...
Saturn	7^s 15°;42	0°;2,0,23,18,...

Appendix C
Sanskrit technical terms

C.1 The Sanskrit alphabet

The table in Figure C.1 lists the *akṣaras* or syllabic phonemes of the Classical Sanskrit alphabet in *nāgarī* script, transliterated according to the IAST (International Alphabet of Sanskrit Transliteration) system. The traditional ordering of the *akṣaras* from *a* to *h* proceeds in this phonetic array from left to right and from top to bottom. The glossary in Section C.3 follows this alphabetical order.

In our transliterations we also employ three additional symbols common in the *nāgarī* script: the *avagraha*' indicating an elided initial *a*-vowel, the *anusvāra ṃ* standing in for any of the nasal consonants, and the *visarga ḥ* representing a final *s*. The single *daṇḍa* | and double *daṇḍa* ‖ are the standard punctuation marks.

C.2 Technical vocabulary for tables

Most of the terms listed in Section C.3 are part of the standard technical vocabulary of *jyotiṣa*, and are defined and explained in detail in many editions of Sanskrit treatises. In this section we explore in more depth the use of a few specialized terms pertaining especially to table-text works.

C.2.1 Names for table-text compositions

There appears to be no single standard Sanskrit word for an astronomical table text. Although individual tables vary greatly in content and format, there seems to be no systematic distinction between those called *koṣṭhaka* and those called *sāraṇī*. Nor

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Vowels	Simple vowels	अ	आ	इ	ई	उ	ऊ	ऋ	ॠ	ऌ	ॡ
		a	ā	i	ī	u	ū	ṛ	ṝ	ḷ	ḹ
	Diphthongs			ए	ऐ	ओ	औ				
				e	ai	o	au				
Consonants	Velar	क	ख	ग	घ	ङ					
		k	kh	g	gh	ṅ					
	Palatal	च	छ	ज	झ	ञ					
		c	ch	j	jh	ñ					
	Retroflex	ट	ठ	ड	ढ	ण					
		ṭ	ṭh	ḍ	ḍh	ṇ					
	Dental	त	थ	द	ध	न					
		t	th	d	dh	n					
	Labial	प	फ	ब	भ	म					
		p	ph	b	bh	m					
	Semivowels	य	र	ल	व						
		y	r	l	v						
	Sibilants	श	ष	स							
		ś	ṣ	s							
	Aspirate	ह									
		h									

Fig. C.1 The *akṣaras* of the Sanskrit alphabet.

1	2	3	4	5	6	7	8	9	0
१	२	३	४	५	६	७	८	९	०

Fig. C.2 Modern standard forms of *nāgarī* decimal numerals.

have we found any clear pattern of nomenclature among authors or commentators to denote a set of astronomical tables regarded as a book in its own right, as opposed to an individual *koṣṭhaka* or *sāraṇī* table itself.[1]

[1]The secondary literature tends to follow the flexible approach of the sources themselves. Pingree, for example, references the Sanskrit table-text genre sometimes by the term *sāraṇī* (Pingree 1968, p. 3), sometimes by *koṣṭhaka* (Pingree 1981, p. 41), and sometimes just as "astronomical tables" with no Sanskrit equivalent (Pingree 1973). In manuscript catalogues he tends to use *koṣṭhaka* as the subject category for this type of texts, but individual works may be called by either name or neither. The word *koṣṭhakakāra* (Pingree 1981, p. 44) appears to be Pingree's own coinage; at least we have not seen it elsewhere. Poleman (1938) refers to such works as "tables." Sarma and Sastry (2002) list several works whose titles contain words such as *sāraṇī*, *koṣṭhaka*, and *vākya*, but do not designate a separate genre in *jyotiṣa* for them.

Table C.1 Usage examples for *koṣṭhaka/koṣṭha*.

koṣṭhaka/koṣṭha meaning	Example source	Excerpt
Table text (in title)	*Mahādevī*, Smith Indic MB 4946 LXI f. 91, colophon	*iti śrīmahādevīkoṣṭhaka saṃpūrṇaḥ*
	Candrārkī, Jaipur Khasmohor 5247 f. 10v, colophon	*iti caṃdrārkīkoṣṭaka saṃpūrṇam*
Individual table/tables	*Brahmatulyasāraṇī*, verse 4 (Montelle and Plofker 2015, pp. 11–12)	*kendrasya doraṃśamitiś ca koṣṭe*
	Tithicintāmaṇi, UPenn 390 1859, f. 12v	*iti sūryamahānakṣatrakoṣṭhakāni saṃkramaṇakoṣṭhakāni samāptāni*
	Jagadbhūṣaṇa, Poleman 4869, f. 1r, title	*jagadbhūṣaṇasaṃjñakasya jyotirgranthasya koṣṭhakāni*
Table cell/cells	*Brahmatulyasāraṇī*, verse 6 (Montelle and Plofker 2015, pp. 16–17)	*bhāgāṅkasaṃkhyāgatakoṣṭakaṃ*
	Candrārkī, verse 8, RORI 5482 f. 2r	*tasya nāḍyā gatir nighnā taraṇer nijakoṣṭajā*
	Candrārkī, verse 12, RORI 5482 f. 2v	*tau yuktau nijakoṣṭeṣu yāvat koṣṭamitir bhavet*
	Jagadbhūṣaṇa, verse 1.19, LDI 6182 f. 1v	*granthābdavṛndaṃ nijacakramityā bhajet tataḥ śeṣamitāṅkakoṣṭe*
	Karaṇakesarī, 1.3 (Montelle and Plofker 2013, p. 17)	*mitaiḥ koṣṭhakair aṅgulādiḥ śaraḥ syāt*

Table C.2 Usage examples for *grantha*.

Example source	Excerpt
Makaranda, Vyāsa 12/30 Bodleian f. 21v, colophon	*iti śrīsaurapakaṣīyamakaraṃdagraṃthaḥ saṃpurṇaḥ*
Jagadbhūṣaṇa, verse 1.19, LDI 6182 f. 1v	*granthābdavṛndaṃ nijacakramityā bhajet*
Jagadbhūṣaṇa, LDI 6182 f. 5v, colophon	*saṃpūrṇo yaṃ jagadbhūṣaṇasaṃjñako graṃthaḥ*
Mahādevī, JVS 70 1103 f. 13v, table title	*mahādevīgraṃthe sūryaspaṣṭa avadhamukhe paṃktiḥ*
Viṣṇu Daivajña commentary on *Bṛhattithicintāmaṇi* (see Section 5.6.3) (Āpaṭe 1942a, p. 1)	*bṛhaccintāmaṇisaṃjñaṃ granthaṃ*

koṣṭhaka. The word *koṣṭhaka* (frequently spelled with a non-aspirated dental consonant *koṣṭaka*) and its variant *koṣṭha* seem to be used interchangeably to designate a table text, an individual table within a table text, or a cell within an individual table. See Table C.1.

grantha. The general term *grantha* or "book" is very commonly used by scribes in colophons to mean the work they have just finished copying, irrespective of what its specific textual genre may be. This usage is sometimes applied to table texts as well, as illustrated in Table C.2.

Table C.3 Usage examples for *pattra/patra*.

Example source	Excerpt
Candrārkī, verse 1, RORI 5482 f. 1v	*. . . vakṣye sūryacandrodbhavaṃ ca | patraṃ pañcāṅgābhidhaṃ*
Gaṇitacūḍāmaṇi, verse 114 (Pingree 1989, p. 32)	*kutūhalād rājamṛgāṅkād vā āśādharāt khecarasiddhitas tu || pañcāṅgapatrād vidadhīta . . .*

Table C.4 Usage examples for *sāriṇī/sāraṇī*.

Example source	Excerpt
Brahmatulyasāraṇī, Smith Indic 29, f. 6v, colophon	*iti brahmatulyasāraṇīślokaḥ*
Brahmatulyasāraṇī, Jaipur Khasmohor 5253(a) f. 1r, colophon	*iti śrīkarṇakutūhalasāraṇyāḥ*
Subodhāsāraṇī, Jaipur Khasmohor 5519, f. Av, colophon	*iti subodhāsāraṇī samāptāḥ*
Makaranda, RORI 5498 f. 45v, colophon	*samāpteyaṃ makaraṃdasya sāraṇī liṣatu*
Makaranda, Nepal 5.5639 f. 1r, title page	*atha makarandasāriṇi prārabhyate*
Grahalāghavasāriṇī of Nīlakaṇṭha, verse 1 (Pingree 2003, p. 80)	*. . . grahalāghavasāriṇī kriyate nīlakaṇṭhena . . .*
Grahalāghavasāriṇī of Prema, verse 1 (Pingree 2003, p. 88)	*premo grahārthaṃ grahalāghavasya laghukriyāṃ sāraṇikāṃ prakurve*
Jaipur Puṇḍarīka Jyotiṣa 48 f. 3r, colophon and Jaipur Puṇḍarīka Jyotiṣa 47 f. 3v, colophon (Pingree 2003, p. 93)	*iti śrījyotiṣarāyakevalarāmakṛtā paṃcāṃgasāraṇī*

pattra/patra. This term, literally meaning "leaf, folio," is widely used by scribes and cataloguers in titles or headings of individual tables or small sets of tables, apparently used as standalone reference works concerning a particular astronomical quantity. They generally pertain to calendrics and timekeeping. Some other examples of its usage are shown in Table C.3.

sāraṇī/sāriṇī. This word appears in the title of works to indicate a table-text compilation, and also references individual tables. See Table C.4.

C.3 Glossary of Sanskrit terms

aṃśa "part"; degree; fractional part

akṣa terrestrial latitude

aṅka number, numeral, digit; word-numeral 9

aṅga "limb"; ascendant; word-numeral 6

aṅgula "finger-breadth"; (linear) digit, unit of linear measure for quantities such as lunar latitude or disk diameter, equivalent to about three arc-minutes (see Chapter 2, Note 9)

atha "now"; frequently used to begin table titles

adhimāsa intercalary month

adhyāya chapter

antara	difference, distance; interval for linear interpolation
antardaśā	in astrology, subdivision of a *daśā*, life-stage, the division of the native's life into periods related to the planets
a pa	abbreviation of *asta-paścima*
a pū	abbreviation of *asta-pūrva*
abda	year
abdapa	"lord of the year"; the excess measured in integer and fractional days by which a sidereal year exceeds 52 integer weeks or $52 \times 7 = 364$ days; the weekday beginning the year
arka	Sun; word-numeral 12
avadhi	14-day interval; the time required for the mean sun to traverse 13;20° or one *nakṣatra* in longitude, a little less than 14 days (see Sections 2.3.2 and 5.4.1)

Example source	Excerpt
Grahasiddhi, 7, RAS Tod 24 f. 1v	*krameṇāvadhayo 'bdāder manubhir manubhir dinaiḥ*
Jagadbhūṣaṇa, verse 2.2, LDI 6182 f. 2r	*caturdaśāhāntaritāḥ sameśakāle sphuṭā bhāvadhiṣu grahāḥ syuḥ*

avasthā	"situation, status"; in astrology, a particular characteristic of a planet
aṣṭakavarga	"*varga* of eight"; in astrology, a system for determining the effects of planets' positions and ascendant upon one another
asta	setting
asta-paścima	"west setting"; acronycal setting of a planet, last visibility in the west; see Table B.9
asta-pūrva	"east setting"; heliacal setting of an (inferior) planet, last visibility in the east; see Table B.9
ahargaṇa	accumulated days between a given epoch and a given date
āgama	"lore"; used in book titles
indu	"drop"; Moon
ucca	"high"; orbital apogee; in astrology, exaltation
utkramajyā	R vers of an arc/angle, equivalent to the difference between the trigonometric radius R and the R cos of the arc
uttara	north
udaya	"rising"; heliacal rising of a planet; ascension
udaya-paścima	"west rising"; acronycal rising of an (inferior) planet, first visibility in the west (see Table B.9)
udayapūrva	"east rising"; heliacal rising, first visibility in the east (see Table B.9)

udayāntara/yātaphala	"rising-difference"; a time correction due to the difference between an arc of mean solar longitude on the ecliptic and its right ascension (see Section 2.1.5)
unnata	"elevation"; complement of the zenith distance *nata*; arc of celestial equator corresponding to time since sunrise or until sunset
unmīlana	moment of emersion in an eclipse, the end of totality
u pa	abbreviation of *udaya-paścima*
upakaraṇa/upakarṇa	offset in mean longitude for the beginning of the current year (see Section 5.6.3); computational technique (see Section 5.2)
upari	"above"; argument value, specifically at top of table column

Example source	Excerpt
Karaṇakesarī, RORI 12792 f. 2r, title	. . . *avadhyopariḥ*. . .
Karaṇakesarī, RORI 12792 f. 2v, title	. . . *atha caṃdrachinnāṃguloparicaṃdrasya madhyasthiti*. . .
Brahmatulyasāraṇī, Smith Indic 43 f. 12r, title	*śīghrakeṃdrāṃśopari-śukrasya śīghraphalaṃ*

u pū	abbreviation of *udaya-pūrva*
ṛṇa	subtractive; decreasing, negative (see Section 3.2.1)
eṣya	"future"; next table entry, as opposed to *gata* or previous entry, in interpolation procedure (see Section 2.2.1)
kaṭapayādi	alphanumeric notation for representing numbers
kanda	"bulb, root"; in the *Makaranda*, a calendar day/time corresponding to a mean time-unit (see Table 5.7)
karaṇa	astronomical handbook; a time interval equal to half a *tithi*
karṇa	hypotenuse; geocentric distance of planet (see Sections 2.1.2 and 2.1.3)
kalā	arcminute
kali/kaliyuga	last and worst division of a *mahāyuga*; the present age; a historical era with epoch 3102 BCE (see Table B.7)
kalpa	time interval equal to 4,320,000,000 years; lifetime of the universe
kuja	Mars
kṛṣṇapakṣa	"dark half"; half of a synodic month from full moon to new moon
ketu	lunar node; descending node; the ninth of the celestial bodies or *navagraha*
kendra	orbital anomaly or arc between mean longitudes of planet and orbital apogee (see Sections 2.1.2 and 2.1.3); time offset

	between the end of some astronomical cycle and the start of a year (see Section 5.2)
koṭi	complement of an arc/angle *bhuja*; upright side of a right triangle
koṭiphala	scaled *R* sin of the complement of an arc/angle
koṣṭha/koṣṭhaka	set of tables, a table, a tabular cell; numerical entry in a tabular cell; see Section C.2.1
krānti	declination, angular distance north or south of celestial equator
kṣepa	additive quantity; constant difference; epoch offset in mean longitude, epact, etc.

Example source	Excerpt
Candrārkī, verse 4, RORI 5482 ff. 1v–2r	*śāko vihīno gaganābhraghasraiḥ nighno guṇaiḥ kṣepayuto dhruvaḥ syāt*
Mahādevī, BORI 497 f. 8v, table title	*guro koṣṭhakā sakṣepā sabījā*
Grahajñāna, RAS Tod 36e f. 4r, paratext	*kṣepakabījasaṃskāravinā*
Laghukhecarasiddhi, verse 4 (Pingree 1976, p. 4)	*kṣepānvitāḥ*
Grahasiddhi, verse 21, RAS Tod 24 f. 1v	*nāgāgnyarkona śākaghnā guṇāḥ kṣepayutā dhruvāḥ*

khaṇḍa	"portion"; increment, difference between two successive values
khecara	"sky-mover"; planet
gata	"past"; previous table entry, as opposed to *eṣya* or next entry, in interpolation procedure (see Section 2.2.1)
gati	"motion"; velocity; mean amount of ascension per degree of the corresponding zodiacal sign (see Section 4.5)
gatiphala	planetary velocity-correction due to orbital equation (see Sections 2.1.2 and 2.1.3)
gamya	"future"; next table entry in interpolation procedure (see Section 2.2.1)
guccha	"blossom, flower, shrub"; in the *Makaranda*, the calendar day/time of an initial mean time-unit (see Table 5.7)
guṇaka	multiplier; longitudinal increment to be multiplied by a number of time-units
guru	"heavy, great"; great, long; teacher; Jupiter; in prosody, heavy syllable
gṛha	"house"; in astrology, an astrological house
gocara	planet; in astrology, transit
gomūtra/gomūtrika	a multiplication technique
gola	"sphere"; spherics; armillary sphere
grantha	book, work; see Section C.2.1
graha	planet

ghaṭī/ghaṭikā	time-unit equal to one-sixtieth of a day or 24 minutes
ghaṭī	sixtieth; a "sexagesimorion" or 6° arc-unit interval in longitude (see Sections 4.2.3 and 5.4.1)
cakra	"cycle"; cycle, period of some number of days/years/unit of measure; especially a period of 4016 days or nearly 11 years (see Section 4.1); in astrology, a diagram with astrological/divinatory significance
catuṣṭayaikya	"sum of four"; the combination of the *udayāntara*, *bhujāntara*, *cara*, and *deśāntara* time-corrections (see Section 2.1.5)
candra	Moon
cara	"variable"; the half-equation of daylight; ascensional difference
carakaraṇa	"movable *karaṇa*"; the sequence of seven *karaṇas* or half-*tithis* that repeats eight times in a synodic month (see Table B.5); see also *sthirakaraṇa*
carakhaṇḍa	ascensional difference
cālaka	interval for linear interpolation
cālana	interval for linear interpolation
cintāmaṇi	"thought-jewel"; used in book titles
cūḍāmaṇi	"crest-jewel"; used in book titles
chāyā	shadow; tangent
janma	in astrology, birth, nativity
janmapattra	in astrology, birth-chart, nativity horoscope diagram
jātaka	"nativity"; in astrology, genethlialogy/horoscopes
jīvā	"bow-string"; sine; chord
jñāna	knowledge; used in book titles
jyā	"bow-string"; sine; chord
jyotiṣa	"astral sciences"; calendrics, astronomy, astrology
ṭippaṇa	commentary
tattva	truth, reality, essence; used in book titles
tantra	astronomical treatise; model; system
tājika	in astrology, a type of genethlialogy adapted from Arabic/Persian sources
tithi	lunar day, one thirtieth of a lunar month
triṃśāṃśa	in astrology, subdivision of a zodiacal sign into thirty
traikya	"sum of three"; the combination of the *udayāntara*, *bhujāntara*, and *cara* time corrections (see Section 2.1.5)
dakṣiṇa	south
daṇḍa	vertical stroke; punctuation mark
darpaṇa	mirror; used in book titles
daśā	"condition, period of life"; in astrology, a life-stage, the division of the native's life into periods related to the planets
dina	day; "day" of a planet, i.e., time from its rising to setting
dinamāna	length of daylight

dīpikā	"lamp, illumination"; used in book titles
dṛṣṭi	"sight"; in astrology, aspect
deśāntara	"place-difference"; a time-correction to account for terrestrial longitude (see Section 2.1.5)
dreṣkāṇa	in astrology, subdivision of a zodiacal sign into three
dvādaśāṃśa	in astrology, subdivision of a zodiacal sign into twelve
dvisaṃskṛti	"two corrections"; equation of time or the combination of the *udayāntara* and the *bhujāntara* corrections (see Section 2.1.5)
dhanaṃ	increasing; additive, positive (see Section 3.2.1)
dhruva/dhruvaka	"fixed [quantity]"; linear increment; epoch offset for desired year (see Sections 4.1 and 5.2.1); oblique ascension (see Section 4.5);

Meaning	Example source	Excerpt
Epoch offset for desired year	*Candrārkī*, verse 4, RORI 5482 ff. 1v–2r	*śāko vihīno gaganābhraghasraiḥ nighno guṇaiḥ kṣepayuto dhruvaḥ syāt*
	Grahasiddhi, verse 6, RAS Tod 24 f. 1v	*śuddhyabdeśaguṇadhruvau*
Cumulative oblique ascensions	*Grahalāghavasāriṇī*, Smith Indic 26 f. 5r, row heading	*dhruva*
Epact complement	*Grahajñāna*, verse 6 (Pingree 1989, p. 6)	*svakīyair dhruvakair vihinā*
	Tithicintāmaṇi, verse 5 (Ikeyama and Plofker 2001, pp. 264–5)	*śuddhiḥ khaṣaṭśodhitanāḍikā syāt tithidhruvo ...*

dhruvāṅka	in the *Grahalāghava*, a term referring to the epoch offset for the 11-year cycle; the number of completed 11 year cycles since the epoch (see Section 5.6.1)
nakṣatra	"star, constellation"; one of the 27 constellations in the moon's path; a 13°;20 arc of the ecliptic; the time it takes the moon to travel 13°;20 in longitude (see Section 2.3.2 and Table B.3)
nata	"depression"; zenith distance, complement of the elevation *unnata*; arc of celestial equator corresponding to time before noon (in morning) or since noon (in afternoon)
nati	latitudinal parallax
navāṃśa	in astrology, subdivision of a zodiacal sign into nine
navagraha	"nine planets"; in astrology, the group of celestial bodies comprising the two luminaries, the five star-planets, and the ascending and descending lunar nodes
nāḍī/nāḍikā	time-unit equal to one-sixtieth of a day or 24 minutes
nimīlana	moment of immersion, beginning of totality in an eclipse

nīca	"low"; perigee; in astrology, dejection
pakṣa	"wing"; synodic fortnight or period of 15 *tithis* bounded by new and full moon; astronomical school
paṅkti	"row"; one table in a set of tables of a given quantity

Example source	Excerpt
Jagadbhūṣaṇa, verse 2.2, LDI 6182 f. 2r, verse	*śeṣamitāṅkapaṅktyām*
Karaṇakesarī, RORI 12792 f. 1r/v, table title extracts	. . . *labdhapaṃkti*. . .*śeṣapaṃkti*. . .
Mahādevī, JVS 70 1103 f. 13v, table title	*mahādevīgraṃthe sūryaspaṣṭa avadhamukhe paṃktiḥ*
Grahajñāna, verse 8 (Pingree 1989, p. 7)	*tattulyapaṅktyaṃśarāḥ sphuṭās te*

pañcāṅga	"five-limbed"; calendar, almanac; specific ephemeral text created for a particular year tracking successive *vāras*, *tithis*, *nakṣatras*, *yogas*, and *karaṇas*; see Section C.2.1
pattra/patra	"leaf, folio/page"; a table or set of tables particularly concerning calendrics or corrections to time; see Section C.2.1
paddhati	manual; path, course, line; used in book titles
parvan	instant of syzygy; an eclipse
parveśa	in astrology, deity assigned to a specific eclipse
pala	time-unit equal to one-sixtieth of a *ghaṭikā*, or 24 seconds
pāṭī	"board"; computation text; arithmetic
pāta	see *mahāpāta*
pāda	"foot"; one fourth; a quarter-verse
piṇḍa	"lump"; cumulative value as a sum of *khaṇḍas*
prakāśa	"light"; elucidation, explanation; used in book titles
prati	every, per, for each
pratikoṣṭhaka	"for each table entry"
prabodha	awakening, knowledge; used in book titles
praśna	"question"; in astrology, interrogation
prāṇa	"breath"; time-unit equal to one-sixth of a *vighaṭī*/*vighaṭikā*
phala	result; astronomical equation or correction, esp. equation of anomaly; interval for linear interpolation
bārhaspatya	Jupiter cycle, "Jovian years"
bāhu	"arm"; arc or angle; horizontal side of a right-angled triangle
bīja	"seed"; a small correction to adjust astronomical parameters; algebra
bindu	"dot, spot, drop"; in astrology, a symbol indicating malefic/benefic impact
buddha	Mercury
bṛhaspati	Jupiter

bhāva	in astrology, astrological house
bhukta	"elapsed"; a past time interval; *gata* or previous table entry in interpolation
bhukti	(daily) velocity
bhuja	"arm"; an arc or angle; table argument measured in units of arc; an arc reduced to the first quadrant; horizontal side of a right triangle
bhujaphala	scaled R sin of an arc/angle; second component of the equation of time correction (see Section 2.1.5)
bhujāntara	"[positional] arc-difference", a time-correction due to longitudinal difference between mean and true sun (see Section 2.1.5)
bhūtasaṃkhyā	word-numeral system for representing numbers
bhūṣaṇa	ornament, decoration; embellishment; used in book titles
bhoga	"advance"; mean longitudinal displacement
bhogya	"advance, future"; mean longitudinal displacement; *eṣya* or next table entry in interpolation
bhauma	Mars
mañjarī	blossom; used in book titles
maṇḍala	"circle"; orbit; cycle, as in a planetary synodic cycle
madhya	"middle"; mean; midpoint of any interval, such as the day (noon), an eclipse, a *mahāpāta*, etc.
madhyama	mean
manda	"slow"; relating to planetary eccentric anomaly (see Section 2.1.2)
mandāṅka	"*manda*-number"; scaled *manda*-equation (*mandaphala*) values (see Table 5.8)
marda	half-duration of totality of an eclipse
mahāpāta	a particular symmetric configuration of the sun and moon
mahāyuga	"great age"; time interval equal to 4,320,000 years
mā	abbreviation for *mārga*
mānasa	spirit, mind; used in book titles
mārga	"path, on course"; period of direct motion of a planet; second station (see Table B.9)
mālā	garland; used in book titles
māsa	month
mitra	"friend"; in astrology, friendship between planets
muhūrta	"moment"; time-unit equal to one-thirtieth of a day or 48 minutes; catarchic astrology or the identification of auspicious moments for particular actions
mṛgāṅka	"deer-marked"; Moon
yantra	"instrument"; astronomical instrument; in astrology, a diagram with astrological/divinatory significance
yātaphala	see *udayāntara*
yāmya	south

yātrā	in astrology, determination of auspicious times for military operations, a branch of catarchic astrology
yuga	era; time cycle
yoga	"sum, conjunction, combination"; interval of time in which the sum of the solar and lunar longitude increases by 13°;20 (see Table B.6); in astrology, various configurations of planet, house, and aspect
yojana	unit of distance, on the order of 1/5000 of earth's circumference
ratna	"jewel"; in divination, the determination of auspicious characteristics of gems; used in book titles
ramala	geomancy, a form of divination
ravi	Sun
rātri	night
rātrikara	"night-maker"; Moon
rāmabīja	see *bīja*
rāśi	"heap"; a quantity/amount; zodiacal sign; a 30° arc
rāśi-bheda	"dividing of signs"; in astrology, subdivision of the zodiac
rāhu	lunar node; ascending node; eclipse
rekhā	"line"; in astrology, a symbol indicating malefic/benefic impact
lagna	ascendant; ascension
laghu	"easy, light"; easy, simplified, short; designation for abbreviated/condensed version of a text/rule/set of tables (see Section C.2.1); in prosody, light syllable
laṅkā	Laṅkā; zero-point of terrestrial longitude and latitude
laṅkodaya	"rising at Laṅkā"; right ascension
lakṣaṇa	"mark, sign"; a branch of astrology concerning the interpretation of marks on the body
labdha	"obtained"; quotient from division; result of operation
lambana	longitudinal parallax
va	abbreviation for *vakra*
vakra	"reverse"; period of retrograde motion of a planet; first station (see Table B.9)
varga	in astrology, subdivision of the zodiac
varṣeśa	"lord of the year"; the excess measured in integer and fractional days that a sidereal year exceeds a year of fifty-two 7-day weeks
valana	angle of deflection associated with eclipses
vallī	in the *Makaranda*, increments to linearly increasing quantities such as civil days or lunar anomaly (see Table 5.7)
vākya	"sentence"; numerical data verbally encoded in *kaṭapayādi*
vāṭikā	in the *Makaranda*, mean longitudinal displacement (see Table 5.7)
vāra	weekday

vāstu	"house, building"; in divination, determination of auspicious features of buildings; geomancy, the interpretation of patterns formed from scattered rocks/soil
vikalā	time-unit equal to one second; an arcsecond
vikrama**/*vikramāditya***	designation for Saṃvat era (see Table B.7)
vikṣepa	lunar latitude
vighaṭī/vighaṭikā	time-unit equal to one-sixtieth of a *ghaṭī*, or 24 seconds
vitribhalagna	"ascendant less three signs"; nonagesimal or midpoint of the ecliptic semicircle above the horizon
vidyādharī	"knowledge-bearer"; used in book titles
vidhu	Moon
vināḍī/vināḍikā	time-unit equal to one-sixtieth of a *nāḍī*, or 24 s
vinoda	amusement, pastime, diversion; used in book titles
vipala	time-unit equal to one-sixtieth of a *pala*/*vighaṭī*
vilagna	nonagesimal or midpoint of the ecliptic semicircle above the horizon
vivaraṇa	explanation; commentary
vivāha	"marriage"; in astrology, determination of auspicious times for marriage, a branch of catarchic astrology
vivṛti	commentary
viveka	investigation, knowledge, discussion; used in book titles
vaidhṛti	one of the two *mahāpātas* or symmetric configurations of sun and moon
vyaṅgula	unit of measurement equal to one-sixtieth of an *aṅgula*
vyatipāta	one of the two *mahāpātas* or symmetric configurations of sun and moon
śaka	historical era beginning in 78 CE (see Table B.7)
śaṅku	"stick"; gnomon; *R* sin of altitude
śatru	"enemy"; in astrology, enmity between planets
śani	Saturn
śara	"arrow"; planetary latitude, esp. lunar; word-numeral 5
śaśi	"containing a hare"; Moon
śākuna	"bird"; in astrology, ornithomancy or the interpretation of bird behavior
śānti	in astrology, an act to ward off the impending ill-effects of an omen/forecast
śālivāhana	alternative name for Śaka era (see Table B.7)
śiromaṇi	"crest-jewel"; used in book titles
śīghra	"fast"; relating to planetary synodic anomaly (see Section 2.1.3)
śīghrāṅka	"*śīghra*-number"; scaled *śīghra*-equation value (see Table 5.9)
śukra	Venus
śukla	Venus

śuklapakṣa	"bright half"; half of the synodic month from new moon to full moon
śuklapratipad	beginning of a synodic month
śuddhi	epact
śeṣa	"remainder"; remainder from subtraction or division
saṃvat	for *saṃvatsara*, year; historical era beginning in 57 BCE (see Table B.7)
saṃvatsara	year
saṃhitā	in astrology, an omen; divination from omens
saṃkrānti	solar transit or solar entrance into signs or *nakṣatras*
saṃdhi	in astrology, a junction between houses
sanna	(Arabic) "year"; usually signifying a foreign calendar system such as *hijarī* or Gregorian
sama	"equal, same"; in astrology, neutrality between planets
sammīlana	moment of immersion in an eclipse, beginning of totality
sādhana	accomplishment; demonstration; computation
sāmudrika	in astrology, physiognomy and palmistry/chiromancy, or the interpretation of bodily marks/features/expressions
sāraṇī/sāriṇī	set of tables, a table; see Section C.2.1
siddhānta	astronomical treatise; doctrine
siddhi	attainment; determination
sūkṣma	accurate
sūrya	Sun
soma	Moon
saumya	north
saurabha	in the *Makaranda*, corrections due to anomaly (see Table 5.7)
sthāna	a significant figure; a place or digit in a sequence of digits
sthiti/sthityardha	half-duration of, e.g., an eclipse, a *pāta*
sthirakaraṇa	"fixed-*karaṇa*"; one of the four *karaṇas* or half-*tithis* occurring around new moon (see Table B.5); see also *carakaraṇa*
sthūla	"coarse, rough"; approximate value
sparśa	moment of first contact in an eclipse
spaṣṭa	accurate; true
sphuṭa	accurate; true
svapna	"sleep, dream"; in astrology, oneiromancy or the interpretation of dreams
svara	"sound, tone, noise"; in astrology, pneunomancy or the interpretation of breathing
hasta/hasta-rekhā	"hand, palm"; in astrology, palmistry or the interpretation of lines (*rekhā*) on the hand; a unit of linear measure equivalent to the length of the forearm or 24 *aṅgulas*

hāyana	in astrology, anniversary horoscopes
hāra	divisor
hijarī	Islamic calendar
hṛti	divisor
horā	(Greek) "hour"; in astrology, horoscopy, also subdivision of a zodiacal sign into two; time-unit equal to 1 hour or 1/24 of a day

Appendix D
Credits and Acknowledgements

The following abbreviations appear in the manuscript shelf-marks referenced in this book. See also the more detailed descriptions of selected manuscript collections in Section 3.1.

Baroda Oriental Institute, Maharaja Sayajirao University of Baroda, Vadodara, India. See Nanbiyar (1950). We thank this institution for permission to reproduce the images shown in Figures 2.18, 4.1, 4.6, 4.33, 4.50, 4.62, and 4.63.

BORI Bhandarkar Oriental Research Institute, Pune, India. See Bhandarkar Oriental Research Institute (1990–1991). We thank this institution for permission to reproduce the images shown in Figures 2.18, 3.2, 3.3, 4.8, 4.21, 4.48, 4.51, 4.54, 4.55, 4.59, 4.60, 5.16, 5.17, and 5.18.

British Library OR Oriental Manuscripts Collection, British Library, London, UK. See Rieu (1881). We thank this institution for permission to reproduce the image shown in Figure 6.3.

CSS Chandra Shum Shere Collection, Bodleian Library, Oxford University, Oxford, UK. See Pingree (1984). We thank this institution for permission to reproduce the images shown in Figures 4.36, 4.39, and 4.49.

IO India Office Collection, British Library, London, UK. See Eggeling (1896). We thank this institution for permission to reproduce the images shown in Figures 2.14, 2.16, 3.6, 4.18, 5.1, 5.2, 5.4, 5.7, and 5.9.

Jaipur Khasmohor Khasmohor Collection, Maharaja Man Singh II Museum, Jaipur, India. See Pingree (2003). We thank this institution for permission to reproduce the images shown in Figures 4.26, 4.30, 4.65, 6.4, and 6.5.

Jaipur Puṇḍarīka Jyotiṣa Puṇḍarīka Jyotiṣa Collection, Maharaja Man Singh II Museum, Jaipur, India. See Pingree (2003).

Jodhpur Fort Maharaja Mansingh Pustak Prakash, Jodhpur, India. See Kshirsagar and Vyāsa (1986). We thank this institution for permission to reproduce the image shown in Figure 4.21.

C. Montelle, K. Plofker, *Sanskrit Astronomical Tables*, Sources and Studies in the History of Mathematics and Physical Sciences,
https://doi.org/10.1007/978-3-319-97037-0

JVS Jaina Vidya Sansthan (Shri Mahavirji), Jaipur, India. Handwritten manuscript inventory; at present no published catalogue of these manuscripts is known to us. We thank this institution for permission to reproduce the images shown in Figures 5.13, 5.14, 5.15, 6.1, and 6.2.

LDI Lalbhai Dalpathbhai Institute of Indology, Ahmedabad, India. See Shah (1963–68). We thank this institution for permission to reproduce the image shown in Figure 4.19.

Leipzig Universitäts-Bibliothek, Leipzig, Germany. See Aufrecht (1901). We thank this institution for permission to reproduce the image shown in Figure 4.21.

Nepal National Archives of Nepal, Kathmandu, Nepal. See Nepalese-German Manuscript Cataloguing Project (n.d.). We thank this institution for permission to reproduce the image shown in Figure 4.64.

Plofker Photocopies of manuscripts purchased by Kim Plofker in Jaipur and donated to the collection of the Jaina Vidya Sansthan; at present only the identification numbers in the original purchase list are available to identify the manuscripts.

Poleman H. I. Poleman's 1938 census of North American Indie manuscripts (Poleman 1938).

Pune Aanandashram Sanstha Library, Pune, India. At present no published catalogue of these manuscripts is known to us. We thank this institution for permission to reproduce the image shown in Figure 4.32.

RAS Tod Tod Collection, Royal Asiatic Society, London, UK. See Barnett (1940). We thank this institution for permission to reproduce the images shown in Figures 3.7, 4.7, 4.16, 4.19, 5.3, and 5.11.

RORI Rajasthan Oriental Research Institute, Jodhpur, India. See Rajasthan Oriental Research Institute (1963–2007). We thank this institution for permission to reproduce the images shown in Figures 3.1, 3.3, 3.5, 3.8, 3.10, 3.11, 3.12, 3.13, 4.17, 4.21, 4.23, 4.29, 4.37, 4.38, 4.41, 4.42, 4.43, 4.45, 4.46, 4.47, 4.52, and 4.53.

Smith Indie Smith Indie Collection, Columbia University, New York, USA. See Plofker (2007). We thank this institution for permission to reproduce the images shown in Figures 2.15, 2.17, 3.2, 3.4, 3.9, 3.11, 3.14, 3.15, 3.16, 3.18, 4.2, 4.5, 4.9, 4.11, 4.12, 4.13, 4.14, 4.22, 4.28, 4.31, 4.35, 4.70, 4.71, 4.72, 4.73, 5.5, 5.6, 5.8, 5.10, 5.12, 6.6, 6.7, 6.8, and 6.9.

SRCMI Sri Ram Charan Museum of Indology, Jaipur, India. See Sharma (1986–1994). We thank this institution for permission to reproduce the image shown in Figure 4.25.

Tokyo Tokyo University Library, Tokyo, Japan. See Matsunami (1965). We thank this institution for permission to reproduce the images shown in Figures 4.27 and 4.66.

UC MB Macmillan Brown Collection, University of Canterbury Library, Christchurch, New Zealand. See University of Canterbury Macmillan Brown Library (n.d.). We thank this institution for permission to reproduce the images shown in Figures 4.67, 4.68, and 4.69.

UPenn Collection of Indic Manuscripts, Kislak Center for Special Collections, Rare Books and Manuscripts, University of Pennsylvania, Philadelphia, USA. See University of Pennsylvania Rare Book and Manuscript Library (n.d.). We thank this institution for permission to reproduce the images shown in Figures 4.44, 4.56, 4.57, 4.58, 4.61, 5.19, and 5.20.

Vyāsa Vyas-Weisz Collection, Bodleian Library, Oxford University. See Minkowski (2010).

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